Application of Pinching Method to Quantify Sensitivity of Reactivity Coefficients on Power Defect

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Abstract Reactor power affects the temperature of fuel and coolant of a nuclear power plant. The change in temperature of fuel and coolant modify the reactivity of a nuclear reactor. The change in reactivity due to the change of reactor power is known as power defect. The power defect depends on the various parameters such as reactivity coefficients due to thermal hydraulics feedback, the response of fuel and coolant temperature due to variation of reactor power, etc. The reactivity coefficients are significantly varied due to different operating conditions, changes in fuel characteristics and fuel burn-up during fuel residence time inside the reactor. This wide variation of reactivity coefficient are in general technically specified in the form of lower and upper bound for safety analysis purpose. A thought experiment has been carried out considering those reactivity coefficients contain stochastic variability and ignorance (i.e., lack of knowledge). The uncertainty involved in reactivity coefficients are captured by defining them with probability box (p-box). After propagating the p-box of reactivity coefficients through the theoretical model of power defect, the p-box of power defect has been generated. In the pinching method, one of two reactivity coefficients will be fixed at their average value and observation on the change of area of p-box of power defect has been made for sensitivity analysis. Based on the reduction of area of p-box of power defect, the sensitivity of these two reactivity coefficients has been analyzed. The parametric studies of variation of sensitivity for five different power drops (i.e., 10%, 25%, 50%, 75% and 100%) have been studied and quantified in this paper. It is found that the reactivity coefficient due to coolant temperature is more sensitive than reactivity coefficient due to the fuel temperature on

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power defect. It is also found that the sensitivity does not depend on amount of power drop.

Keywords Power Defect · Probability-box · pinching method · nuclear power plant · reactivity · thermal hydraulic feedback

1 Introduction

The basic principle of power manoeuvrings of a nuclear power plant (NPP) is reactivity management. The reactivity is the measure of the rate of nuclear fission reactions occurring inside a reactor. Positive reactivity signifies that reactor is in supercritical sate. Reactivity equals to zero means reactor in critical state i.e., self sustaining chain reaction of nuclear fission. Negative reactivity implies that the reactor is in subcritical state, i.e., shut-down condition. The power raises with increase of reactivity and decrease of reactivity lowers the power. Apart from the physical reactivity devices such as control rods, neutron poison addition/removal system, the variation of reactivity is possible through thermal hydraulic feedback mechanisms such as temperature of fuel and coolant. In the pressurized light water reactor system, the reactivity coefficient of fuel temperature (RCFT) and reactivity coefficient for coolant temperature (RCCT) are desired to be negative by design to enhance the inherent safety feature [1]. The characteristics of negative reactivity coefficients enables decrement of reactivity during heating up and increment of reactivity during cooling down. If there is a need to shut down the reactor from its hot full power state, then there will be a change of reactivity that may reduce the effective control rod worth due to the cooling effect of fuel and coolant. Hence, power defect [2] should be considered during evaluation of reactivity margin available in shut-down condition. The reactor is operated from very low power during the start-up operation to its full power operation. Again, with continuous extraction of power leads to the change in fuel depletion and build-up of neutron absorbing fission products. The reactivity coefficients are significantly varied due to different operating conditions, changes in fuel characteristics and fuel burn-up during fuel residence time inside the reactor. This wide variation of reactivity coefficient are in general technically specified in the form of lower and upper bound for safety analysis purpose. A thought experiment has been carried out considering those reactivity coefficients contain random variability and ignorance (i.e., lack of knowledge). The variability of a parameter is analysed by probabilistic method whereas ignorance by possibilistic approach such as interval arithmetic [3], fuzzy set theory [4], evidence theory [5], etc. However, combined effect can be studied by second order Monte-carlo method [6], fuzzy-stochastic response surface [7] formulation and modelling with probability box (aka p-box) [8]. The hybrid uncertainty involved in reactivity coefficients are captured by defining them with p-box, which is bounded by lower and upper cumulative distribution functions (CDFs). After propagating the p-box of reactivity coefficients through the theoretical model of power defect, the p-box of power defect has been generated. In the pinching method

[9], one of two reactivity coefficients will be fixed at their average distribution. The change in area of the p-box of an input variable affects the area of the output variable. The fractional change of area of p-box of the output variable from its unpinched condition is the indicator for the sensitivity of the pinched input variable[8]. The calculation methodology for power defect, notation of probability box, random sampling methodology from a normal distribution, methodology for sensitivity analysis, parametric studies and results obtained are discussed in the subsequent sections.

2 Calculation Methodology

2.1 Calculation of Power Defect

The variation of reactivity depends of various parameters such as fuel temperature, coolant temperature, coolant density, fuel burn-up, thermal hydraulics feedbacks, etc. and hence the power defect also depends on various properties such as thermal hydraulic feedbacks, control characteristics, etc. For simplicity, here the power defect is estimated for variation of two parameters such as fuel temperature and coolant temperature. Hence, let us consider that the reactivity is a function of fuel temperature (T_F) and Coolant temperature (T_C) as represented in Eqn. (1).

$$\rho = \rho\left(T_F, T_C\right) \tag{1}$$

To account for the changes in reactivity due to changes in reactor power which in turn depends on changes in fuel temperature and coolant temperature, the change of reactivity can be represented in Eqn. (2).

$$d\rho = \left(\frac{\delta\rho}{\delta T_F}\frac{\delta T_F}{\delta P} + \frac{\delta\rho}{\delta T_C}\frac{\delta T_C}{\delta P}\right)dP \tag{2}$$

where, P is reactor power; ρ is reactivity; $\frac{\delta\rho}{\delta T_F}, \frac{\delta\rho}{\delta T_C}$ are the RCFT and RCCT respectively; and $\frac{\delta T_F}{\delta P}, \frac{\delta T_C}{\delta P}$ are response of fuel temperature (PRFT) and coolant temperature (PRCT) due to the change of reactor power respectively. In this study, RCFT and RCCT are considered as hybrid uncertain variables defined by p-box. Random sampling-based algorithm is used to generate the p-box of power defect from the reactivity coefficients.

2.2 Notation for Probability Box

Probability bounding analysis can be carried out by defining probability box or simply 'p-box'. P-box is a structure for probability distribution of a hybrid uncertain variable X i.e., $F_X := \left[\underline{F_X(x)}, \overline{F_X(x)}\right]$. $\left[\underline{F_X(x)}, \overline{F_X(x)}\right]$ denote the set of monotonically increasing functions $F_X(x)$ of real variable X in the interval $[0, 1]$ such that $\underline{F_X(x)} \leq F_X(x) \leq \overline{F_X(x)}$ [10]. The functions $\overline{F_X}$ and $\underline{F_X}$ are interpreted as bounds on CDFs. The region specified by the pair of distributions $\left[\underline{F_X(x)}, \overline{F_X(x)}\right]$ is known as the "probability box" or "p-box". This means that if $\left[\underline{F(x)}, \overline{F(x)}\right]$ is a p-box for a random variable X whose distribution $F_X(x)$ is unknown except that it is within the p-box, then $\underline{F_X(x)}$ is a lower bound on $F_X(x)$, which is the (imprecisely known) probability that the random variable X is smaller than x. Likewise, $\overline{F_X(x)}$ is an upper bound on the same probability. From a lower probability measure P for a random variable X , one can compute the upper and lower bounds on the distribution functions using Eqns. (3) and (4).

$$\overline{F}_X(x) = 1 - \underline{P}(X > x) \tag{3}$$

$$\underline{F}_X(x) = \underline{P}(X \leq x) \tag{4}$$

When the bounds of the p-box coincide for every $x \in X$, i.e., $\underline{F_X(x)} = \overline{F_X(x)}$, the corresponding p-box degenerates into a single CDF, as it is usual in standard probability theory.

The values for RCFT and RCCT are generally reported as range of values in the safety analysis. The interval for RCFT and RCCT considered in this study are $[-3.2, -2.1]$ and $[-63.0, -4.0]$ in unit of $pcm/^{\circ}C$ respectively to demonstrate the application of pinching methodology in carrying out the sensitivity analysis. In the p-box formulation, lower and upper bound of each reactivity coefficients are considered as random variables and following a normal distribution. The normal distribution is defined with bounding value as mean and 10% of bounding value as standard deviation for this sensitivity analysis. The graphical representation of the p-box for RCFT and RCCT are shown in the Fig. 1.

2.3 Random Sampling from Normal Distribution

Random sampling techniques include conversion of probability density function (PDF) to CDF, mapping with that of uniform random number distribution in the interval $[0, 1]$. Requirement of mapping on uniform distribution is based on the fact that most of the computer based random number generators generate uniform random number ($i.e., \xi$) in closed interval $[0, 1]$. In the random sampling method ξ will be used to draw a sample from other distributions utilising the property of CDF of $0 \leq F(x) \leq 1.0$. After mapping of CDF of uniform distribution with desired distribution, inverse CDF corresponding to the uniform random number ($i.e., \xi$) gives the random number belonging to the desired PDF. Graphically, these steps are explained for continuous distribution function in the Fig. 2.

In this paper, variability of the reactivity coefficients in power defect assessment is considered as normal distribution. This unbounded symmetric unimodal distribution is mathematically defined as a two parameters distribution function. These two parameters are location (i.e., mean) and scale (i.e., standard deviation). A normal distribution with mean (μ) and standard deviation

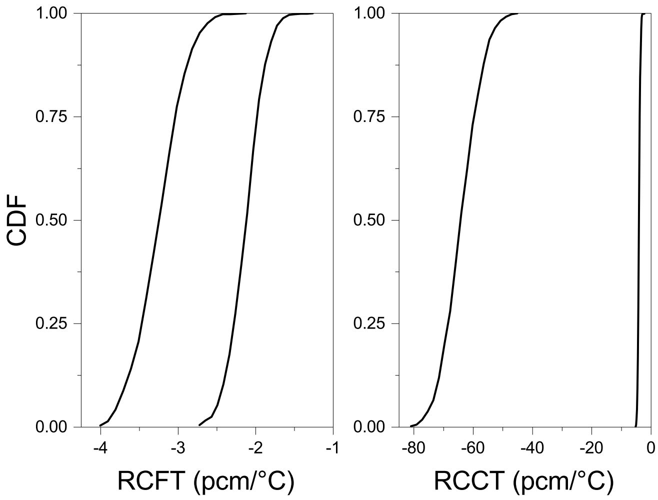

Fig. 1 Representation of RCFT and RCCT in p-box form

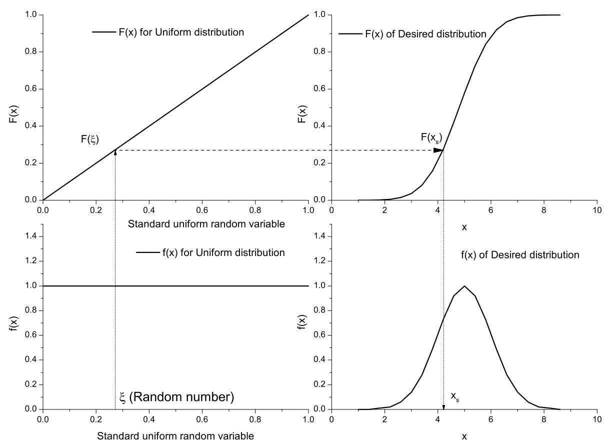

Fig. 2 General sampling techniques for continuous PDF

(σ) is mathematically represented as $N(\mu, \sigma)$. The probability distribution of the distribution $N(\mu, \sigma)$ is given in Eq. (5).

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{5}$$

There are many approaches to draw a random sample for the normal distribution such as inverse transform sampling, Ziggurat algorithm [11] [12], BOX-

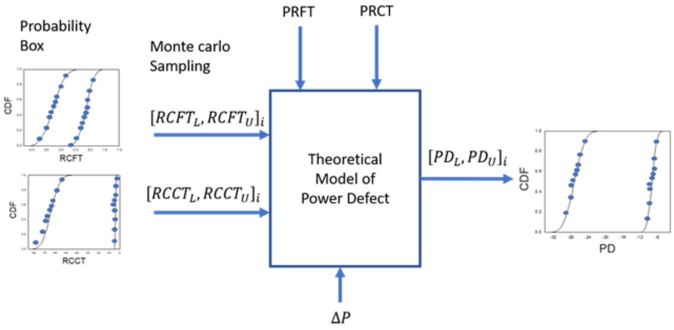

Fig. 3 Methodology for probability box analysis for a theoretical model

Muller algorithm [13], Wallace method [14]. Generation of a sample from the given normal distribution involves two steps process. First step is to generate random sample form a standard normal distribution $N(0,1)$. Second step is to transform the generated random sample to the desired normal distribution using Eqn. (6).

$$x = x_s\sigma + \mu \tag{6}$$

The random number i.e. x_s for distribution $N(0,1)$ is generated using Box-Muller algorithm, which requires two uniformly distributed random numbers in range $[0,1]$. The algorithm is shown in Eq. (7) and Eq. (8).

$$x_{s,i} = \sqrt{-2ln\xi_i}cos\left(2\pi\xi_{i+1}\right) \tag{7}$$

$$x_{s,i+1} = \sqrt{-2ln\xi_i}sin\left(2\pi\xi_{i+1}\right) \tag{8}$$

This algorithm is very simple to implement and also fast.

2.4 Methodology for Sensitivity Analysis

Interval form of the RCFT and RCCT are converted to p-box by assuming normal distribution with bound value as mean and 10% bound value as standard deviation for sensitivity analysis. Monte-Carlo sampling [15] of size 1000 from the both lower and upper bounding distributions of p-box has been drawn. Each pair of samples (i.e., lower and upper) are propagated through the theoretical model power defect to obtain the p-box for power defect. The methodology for p-box analysis is graphically presented in the Fig. 3.

The area between two cumulative distribution functions (CDFs) is the measure of uncertainty. If one of the p-box of input variables is pinched, i.e., fixed to its average distribution, then the area of the p-box of the output variation will reduce. The amount of reduction of area can be give relative measure of sensitivity. Let's assume the area of the p-box of an output variable without pinching of input variable as a base case. And reduced area with pinching of input variable is a pinched case. Then sensitivity of the input variable can be measured by using Eqn. (9).

$$sensitivity\,(\%) = 100 \times \left(1 - \frac{Area\,[Pinched]}{Area\,[Base]}\right) \tag{9}$$

3 Results and Discussions

3.1 Temperature Response with Respect to the Power Variation

System thermal hydraulic code RELAP5 [16] has been used to model reactor systems with primary heat transport system, reactor core, pressurizer, secondary heat transport system, etc. and estimate the fuel temperature and coolant temperature response with variation of reactor power through steady state simulation for each power level.

At the ten discrete power levels i.e., 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, 90% and 100% of full power, the discrete values of fuel temperature and coolant temperature are obtained and fitted with line equation with R^2 value is 0.999. The formulae for the fitted curve of fuel temperature and coolant temperature response are given in Eqns. (10) and (11) respectively [2].

$$T_F = 270.8756 + 5.21779P \tag{10}$$

$$T_C = 276.10307 + 0.30274P \tag{11}$$

Equations (10) and (11) are used to obtain , slope of the response of fuel temperature(PRFT) and coolant temperature(PRCT) with respect to the power variation respectively, to be used in Eqn. (2) for power defect estimation. The values of PRFT and PRCT are 5.21779 and 0.30274 respectively.

3.2 Estimation of Probability Box for Power Defect

P-box for power defect i.e., the change of reactivity with change in reactor power, has been generated from probability box definition of reactivity coefficients. P-box for power defect has been generated for five different power drops (i.e., 10%, 25%, 50%, 75% and 100%). The p-box for RCFT and RCCT have been shown in the left and middle figure respectively on the top layer of Figure 4.

The right figure of the top layer depicts the p-box of power defect corresponding to power drop of 75% full power. The input variables, RCFT and RCCT are not pinched on the top layer figures. Therefore, p-box obtained for the power defect is the base case. The area of the p-box of power defect for the base case is found to be 17.92.

The variation of the p-box area of power defect for the base case with five discrete power drops i.e., 10%, 25%, 50%, 75% and 100% of full power has been analyzed and given in the Fig. 5.

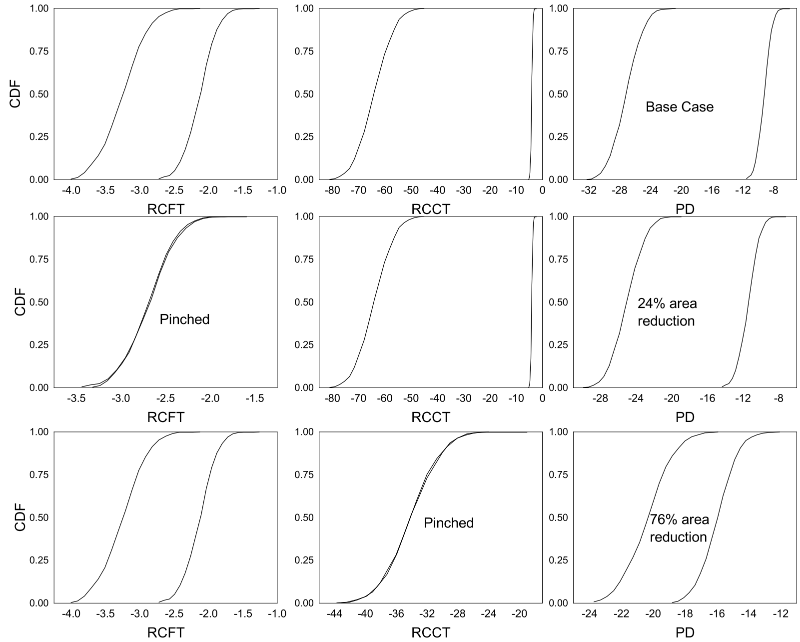

Fig. 4 Reduction of uncertainty through pinching

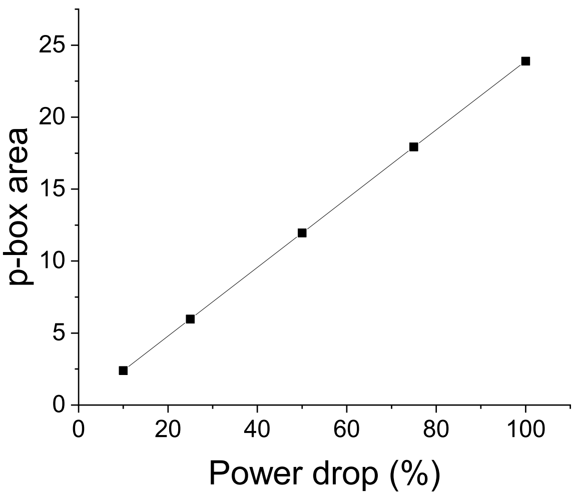

Fig. 5 Variation of p-box area of power defect with power drop

Table 1 Variation of sensitivity with power drop

Power drop %	sensitivity,%	
	RCFT Pinched	RCCT Pinched
10	25.40	75.55
25	25.40	75.55
50	25.40	75.55
75	25.40	75.55
100	25.40	75.55

It is found that the variation of p-box area is linearly proportional to the power drop. It is also found that the increment p-box area of amount 0.239 per unit percent of power drop.

In the middle layer, the p-box of RCFT is pinched to single normal distribution with mean and standard deviation are -2.65 and 0.265 respectively. The pinched distribution of RCFT has been shown in left figure of middle layer of Fig. 4. The p-box of RCCT is kept unchanged. The p-box of power defect with RCFT in pinched condition has been generated and shown in right figure in the middle layer in Fig. 4. It is found that area of the p-box of power defect is reduced by 24% of base case.

In the bottom layer, the p-box of RCCT is pinched to single normal distribution with mean and standard deviation are -33.5 and 3.35 respectively. The pinched distribution of RCCT has been shown in middle figure of bottom layer of Fig. 4. The p-box of RCFT is kept unchanged. The p-box of power defect with RCCT in pinched condition has been generated and shown in right figure in the bottom layer in Fig. 4. It is found that area of the p-box of power defect is reduced by 76% of base case.

3.3 Sensitivity Analysis

Sensitivity in terms of the reduction of p-box area due to the pinching of input variables such as RCFT and RCCT has been estimated using Eqn. (9). Sensitivity analysis for five discrete power drops, i.e., 10%, 25%, 50%, 75% and 100% of full power has been carried out. The result obtained from this analysis is shown in the Table 1. It is found that the sensitivity does not change with the power drop.

4 Conclusions

The methodology for estimation of sensitivity of the theoretical model for power defect has been developed and demonstrated using p-box and pinching technique. In this study, interval bound data of input variables are transformed into p-box formulation. P-box of input variables propagated through the theoretical model of power defect using Monte Carlo based sampling techniques. Finally, the p-box of the output variable, i.e., power defect, has been generated for sensitivity analysis. Pinching method, where single distribution with average properties is used on the desired reactivity coefficient, is applied to study the variation of p-box area of power defect of the nuclear reactor. It is found that the area of p-box of power defect is linearly increasing with power drop. However, the sensitivity of input variable does not change with power drop. It is found that the RCCT is more sensitive than RCFT on power defect.

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