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Abstract. The purpose of this research is to propose an optimization method for the stochastic disassembly lot-sizing problem under uncertainty of lead time. One type of end-of-life (EoL) product is disassembled to satisfy a dynamic and known demand of items over a planning horizon. The tactical problem is considered as a random optimization problem in order to minimize the expected total cost. A sample average approximation (SAA) approach, is developed to model the studied random optimization problem and minimize the average total cost. The effectiveness of the solution approach has successfully tested and proved.

Keywords: Disassembly lot-sizing · Random lead time · Stochastic programming · Monte Carlo simulation.

1 Introduction

Managing uncertainty is becoming one of the most important challenge in optimizing the disassembly process. Uncertainty leads to several difficulties in production planning and inventory management. The sources of uncertainty are diverse and can be found at several levels of the disassembly process: variability of demand, probabilistic recovery rates of items, delivery times, quality problems, etc. In this paper we consider the situation where demand on items/parts must be satisfied from the disassembly of EoL products. According to the classification proposed by [2], the studied problem enters into the classification of disassembly lot sizing problem (DLS) problem that consists in finding when and how much EoL products to disassemble to meet demand for the items while minimizing the costs associated with the disassembly system.

Without trying to do an exhaustive review of the literature, we highlight the works on DLS problem under uncertainty. The stochastic literature review can be split into four different cases: (i) the uncertainty of demand [1,5], (ii) the uncertainty of disassembly yield (see for example the paper of [4,3]), (iii) the uncertainty of yield and demand [7] and (iv) the uncertainty of disassembly lead time (DLT) [9,10]. This latter can be defined as the time difference between placing a disassembly order and receiving the disassembled items. The DLT is crucial parameter in disassembly planning. However, their variability strongly affect the disassembly system efficiency and it can affect the customer orders with a random availability of disassembled items. Managing disassembly operation under uncertainty of DLT is very challenging in practice. Indeed, when an EOL product is disassembled, it is fundamental to ensure that it is the right model to be supplied under uncertain DLT in order to manage disassembly and delivery process. In order to position our research in the existing literature, we will only review in detail the previous works on the stochastic DLS problem under uncertainty of DLT. Only two papers populated this category. The paper of [9] is the first to treat this problem type. The case of two-level disassembly system is studied under unlimited disassembly capacity. The problem is formulated as a minimization problem and then converted to a Monte Carlo-mixed integer programming model. The Model is used to determine the optimal quantity for EoL products in order to minimize the average total cost over the planning horizon. Recently, [10] proposed a generalization of the discrete Newsboy formulae to find the optimal release date when the disassembly lead time of the EoL product is random variable. This study deals with a single-period disassembly to-order problem with known and fixed demand for components, when the disassembly capacity is unlimited.

To close to real industrial planning approach, the capacity restriction on disassembly resources is an important consideration. As for approach, a sample average formulation is necessary to approximate the expected objective value. This paper is a continuation of our previous preliminary work [9]. The contributions of this study are as follow:

1. The problem is extended to a capacitated disassembly lot sizing (CDLS) problem and formulated as a minimization problem and converted to a Monte Carlo-Mixed-Integer Programming model (MC-MIP);
2. The sample average approximation algorithm based on Monte Carlo (MC) optimization approach is proposed.

In this article, we study the case of a two-level disassembly system with a lot-sizing policy. The disassembled item requests are known and must be delivered on predefined delivery dates. To meet the items requirements, a quantity of EoL product Z_t is ordered at the period t. Once the disassembly order is released, a random real L_t is found in that period and the disassembled items are received at period $t^{'} = t + L_t$. Note that, a setup cost is generated if any disassembly operation is released in period t. The DLT at each period are independent random random discrete variables with known probability distributions and varying between L^- and L^+. The randomness of the lead time is a classic problem for industrial companies. If the date of recovery of an item during the disassembly of the EoL product is lower than its planned delivery time, this item will be stored until this date. This situation usually occur in companies and generates holding costs. In the same manner, if an item is not received in time, a backlogging cost is incurred for it. Regarding the capacity, the disassembly quantity is limited by a certain capacity time in each period over the planning horizon. The rest of this paper is organized as follows. In section 2, the proposed stochastic programming model of the CDLS problem is described. MC optimization approach is presented in section 3. Section 4 reports some preliminary results. Our conclusions are drawn in the final section.

2 Problem Formulation

To solve the studied problem under random lead time, a stochastic MIP is proposed to minimize the expected total cost ($\mathbb{E}(TC)$). The list of notations used in this paper is given in Table 1:

Table 1: Notation.

Parameters	
$\mathcal{T}$	Set of time periods of the planning horizon
$\mathcal{N}$	Set of items
Ω	Set of possible scenarios
t	Index of period t, $t \in \mathcal{T}$
i	Index of item i, $i \in \mathcal{N}$
ω	Index of scenario ω, $\omega \in \Omega$
R_i	Number of units of i obtained from disassembling the EoL product
$D_{i,t}$	External demand for item i in period t
$I_{i,0}$	Starting inventory of item i
L_t^ω	Random disassembly lead time in period t under scenario ω
h_i	Per-period inventory holding cost of one unit of item i
s_t	Per-period setup cost in period t
b_i	Per-period backlogging cost of one unit of item i
G	Disassembly operation time
C_t	Available capacity in period t
M	A large number
Functions	
$\mathbb{E}(.)$	Expected value
p_ω	Probability distribution for scenario ω, $\sum_{\omega \in \Omega} p_\omega = 1$
Decision variable	
Z_t	Disassembly quantity ordered in period t
Y_t	Binary indicator of disassembly in period t
$H_{i,t}^\omega$	Inventory level of item i at period t under scenario ω
$B_{i,t}^\omega$	Backordered quantity of item i at period t under scenario ω

We note that the total number of scenarios is $|\Omega| = \prod_{t\in\mathcal{T}}(L^+ - L^- + 1)$. Thus, $\mathbb{E}(TC)$ can be calculated by considering all possible values of L_t^ω and the objective function, expressed in Eq. (1), can be modeled as follows:

$$min\ \mathbb{E}(TC) = \sum_{t\in\mathcal{T}}\Big(\sum_{\forall i\in\mathcal{N}}\sum_{\omega\in|\Omega|}\frac{1}{|\Omega|}\Big(h_i.H_{i,t}^\omega + b_i.B_{i,t}^\omega\Big) + s_t.Y_t\Big) \tag{1}$$

Eq. (2) defines the inventory balance for each item i at the end in each period t under scenario ω:

$$I_{i,t}^\omega = H_{i,t}^\omega - B_{i,t}^\omega \qquad \forall i\in\mathcal{N}, \forall t\in\mathcal{T}, \forall\omega\in\Omega \tag{2}$$

where

$$I_{i,t}^\omega \equiv I_{i,0} + \sum_{\tau=1}^{t-L_\tau^\omega} R_i.Z_\tau - \sum_{\tau=1}^{t} D_{i,\tau}$$

Constraints (3) represents the modeling constraint for disassembly indicator of the EOL product in period t:

$$Z_t \leq Y_t.M \qquad \forall t\in\mathcal{T} \tag{3}$$

Constraints (4) represents the capacity constraint in each period t:

$$G.Z_t \leq C_t \qquad \forall t\in\mathcal{T} \tag{4}$$

Constraints (5 to 7) provides the conditions on the decision variables:

$$Z_t \geq 0 \qquad \forall t\in\mathcal{T} \tag{5}$$

$$H_{i,t}^\omega, B_{i,t}^\omega \geq 0 \qquad \forall i\in\mathcal{N}, \forall t\in\mathcal{T}, \forall\omega\in\Omega \tag{6}$$

$$Y_t \in \{0,1\} \qquad \forall t\in\mathcal{T} \tag{7}$$

The complexity of the problem may increase exponentially if a large set of $|\Omega|$ is considered to represent the stochastic DLT and the resolution of (1) to (7) becomes impossible. The total cost can be estimated by an empirical average ($\bar{TC}$) by using: *Monte Carlo (MC) simulation.* It can provide an estimate of the criterion that converge to $\mathbb{E}(TC)$ calculated by the stochastic MIP.

Proposition 1. *The MIP model can be converted to the following MC-MIP model:*

$$\bar{TC} = \sum_{\forall t\in\mathcal{T}}\Big(\sum_{\forall i\in\mathcal{N}}\sum_{\omega\in\vartheta}\frac{1}{K}\Big(h_i.H_{i,t}^\omega + b_i.B_{i,t}^\omega\Big) + s_t.Y_t\Big) \tag{8}$$

$$I_{i,t}^\omega = H_{i,t}^\omega - B_{i,t}^\omega \qquad \forall i\in\mathcal{N}, \forall t\in\mathcal{T}, \forall\omega\in K \tag{9}$$

$$H_{i,t}^\omega, B_{i,t}^\omega \geq 0 \qquad \forall i\in\mathcal{N}, \forall t\in\mathcal{T}, \forall\omega\in K \tag{10}$$

subject to (2), (3),(4), (5) and (7).

Proof. The MC-MIP is implemented with the following steps:

- Step 1: $\forall t \in \mathcal{T}$, generate the random samples for L_t^ω for $\omega \in \vartheta$ where ϑ is a set of random samples such that $\vartheta \subset \Omega$ and $|\vartheta| = K$,
- Step 2: Each expected inventory level, noted by $\mathbb{E}(I_{i,t})$, is estimated by $\bar{I}_{i,t}$.

Of particular interest is to study the Monte Carlo optimal solution convergence, we describe in the next section the convergence provided by the SAA algorithm based on MC optimization approach.

3 "Exact" Monte Carlo Optimization Method

In this section we present an almost exact method to solve the stochastic problem, referred to as the true optimization problem using MC-MIP. This approach is also known as the stochastic counterpart or the sample path method [8]. The evidence from this study points towards the idea incorporates an optimal algorithm to solve the problem under a large set of samples [6]. Thus,the excepted total cost developed in Eq. (1) can be approximated by SAA based on MC optimization steps:

- Step 1: Solve the MC-MIP model: Let X^* be the optimal solution and $\bar{TC}^*$ the obtained optimal average cost,
- Step 2: $\forall t \in \mathcal{T}$, generate $\mathcal{B}$ large random samples of L_t: $L_t^1, \ldots, L_t^{\mathcal{B}} | \omega \in \mathcal{B}$,
- Step 3: Evaluate the "Exact" optimal cost $\hat{TC}_{\mathcal{B}}$ of the optimal solution X^*, using Eq. (11):

$$\hat{TC}_{\mathcal{B}} = \sum_{\forall t \in \mathcal{T}} \Big(\sum_{\forall i \in \mathcal{N}} \sum_{\omega \in \mathcal{B}} \frac{1}{\mathcal{B}} \Big(h_i . H_{i,t}^\omega + b_i . B_{i,t}^\omega \Big) + s_t . Y_t \Big) \tag{11}$$

subject to (9), (4) and (10).

Remark 1. According to [6], "By the law of large numbers, $\bar{I}_{i,t}$ (respectively $\bar{TC}$) converges with probability 1 to $\mathbb{E}(I_{i,t})$ (respectively $\mathbb{E}(TC)$) as K increases".

4 Computational Experiments and Result

In order to investigate the speed convergence of the solution provided by the MC optimization, this section presents numerical results that prove our findings. All approaches were solved using optimization software for small instances. All formulations are implemented in C using Concert Technology and are solved by IBM CPLEX 12.4 on a PC with processor Intel (R) Core ™ i7-5500 CPU @ 2.4 GHz and 8 Go RAM under Windows 10 Professional.

We considered a finite planning horizon with 5 periods and a disassembly system with 3 items (See Fig. 1). The number in parentheses represents the number of items obtained from the EoL product. The number of scenarios K

takes a value in [2, 10, 30, 50, 150, 200, 400, 600, 1000]. For each K, we performed 100 independent runs of the MC-MIP to provide 100 optimal solutions X^* and the related average costs. The "exact"cost of each X^* was evaluated using a big number of scenarios (10^5 samples), i.e., $\bar{TC}_{\mathcal{B}} \approx \mathbb{E}(TC)$ with $\mathcal{B}$ =10^5 (See [6]). To obtain $\mathbb{E}(TC)$, the stochastic MIP was solved with $|\Omega| = 3125$ scenarios and the $\mathbb{E}(TC)$ takes a value equal to 70955 (see Fig. 2).
The tests are carried out under the parameter costs listed in Table 2. The inventory and backlogging costs of one unit of item i are 3 and 87, respectively. Starting inventory for each item is assumed to be zero. We note that the capacity is 180 for all periods.

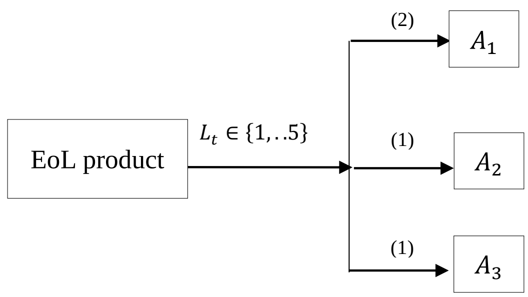

Fig. 1: A two-level disassembly system.

Table 2: Characteristics of the data set.

(a) Demands and setup costs.

Period	1	2	3	4	5
Demand A1	17	28	34	15	9
Demand A2	56	25	0	76	12
Demand A3	12	58	74	13	69
s_t	100	0	200	100	300

(b) Disassembly lead time probability distribution.

ω	1	2	3	4	5
$Pr(L_t = \omega)$	0.245	0.48	0.255	0.01	0.01

Let us now compare the convergence of $\bar{TC}$. Fig. 2 shows that $\hat{TC}_{\mathcal{B}}$ decreases when K is increasing. The findings of this study support the idea that $\bar{TC}$ converges to the optimal solution $\mathbb{E}(TC)$ and is quite robust if a large number of samples is used, for example if $K \geq 200$. Our work has led us to conclude that our proposed optimization method can provide the convergence of the solution to the optimal one as the set of scenarios increases.

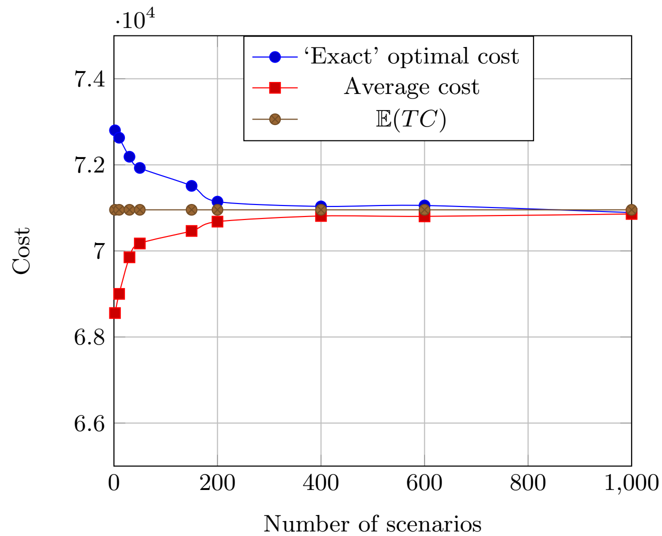

Fig. 2: Convergence cost.

As expected, our experiments show that for $K = 10^3$, the MC-MIP approach can generate a good approximate solution for the stochastic problem. As previously noted, we realized 100 independent runs of the same data set of the MC-MIP with 10^3 scenarios. Then, 100 optimal estimated solutions are obtained (Sol_{1000}). Among these solution, the Best Known Solution (BKS) is selected.

As Fig. 3 witnesses, the results founded are stable and the average gap from BKS is no more than 4% for all runs.

5 Conclusion

In this work, we studied a stochastic CDLS problem with several items disassembled from one type of EoL. We assumed that the items-demand is known. The DLT are an independent random variables whose probability distribution is known and bounded. The combination of MC-MIP approach has been developed. A SAA based on MC optimization provides almost optimal solutions using a modest number of scenarios in order to minimize the average total cost. Our model can easily be implemented in practice to define the DLT in a disassembly planning and control system. Our experiments show clear that the average total cost can be tends to the expected one as the number of scenarios increases. The stability of the proposed model is also verified. In the future, we will try to treat the randomness of demand and/or disassembly yield in the study of the multi-level disassembly systems and multi type EOL products.

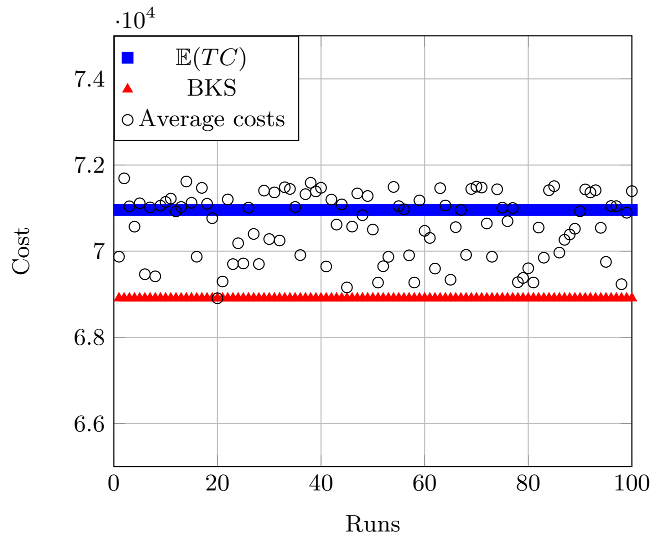

Fig. 3: Validation of MC simulation approach.

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