# Promotion of Cooperation in Public Goods Game by Socialized Speed-Restricted Movement

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***Abstract*—This paper studies the formation of cooperation in mobile networks following an Evolutionary Game Theory (EGT) approach. Motivated by real-world human motions in mobile social networks, we propose a Socialized Speed-Restricted Mobility (SSRM) model to simulate users' movement. Interactions among mobile users are formulated as public goods game. To validate the SSRM model, we first derive its approximated degree distribution, and prove that exponential and power-law degree networks can be constructed from SSRM. Then we conduct extensive simulations to study the evolution of cooperation. In contrast to the recent work which concluded that random and homogeneous movement is harmful to cooperation, we find cooperation is in fact promoted in mobile networks driven by SSRM. This is mainly due to a reduction of randomness in users' neighborhood, as well as the degree heterogeneity. Results in this paper are helpful in understanding cooperative behaviors in mobile social networks. Our EGT framework can be further used to study the performance of specific cooperation-based protocols.**

## I. Introduction

In recent years, we have witnessed the proliferation of mobile devices and users. This trend encourages a broad portfolio of new applications and technologies including wireless cooperation. Cooperative protocols allow mobile users to collaborate and overcome the most critical challenges in mobile environments such as energy consumption [9], connectivity [16], and spectrum capacity issues [17]. Examples of wireless cooperation can be found in Disruption Tolerant Networks (DTN) [10] and cellular networks [20], as well as other applications like cooperative video streaming [14], crowdsourcing [24], etc.

While central to the success of a mobile society, cooperation does not come for free. Selfish users may choose not to contribute as it consumes their own resources (such as energy and bandwidth). Due to selfishness, the number of cooperators is diminished and it results in the "tragedy of the commons" [19]. To stimulate cooperation, credit or reputation-based systems have been developed [11] [1]. The idea is to reward cooperative actions by virtual payments or raising user's reputation. However, deploying these mechanisms incurs additional costs. In credit-based system, centralized management of currency and tamper-proff hardware need to be implemented, while in reputation-based system a network-wide reputation monitoring and update must take place.

In addition to the above cooperation enforcement schemes, a considerable amount of efforts have been devoted to game theoretic analysis of cooperation [15], [18], [23]. Among them, *Evolutionary Game Theory* (EGT) has proved to be an efficient approach to investigate cooperative behaviors [27]. EGT is used to model an evolutionary (Darwinian) process where the more fit individuals pass on their strategies to more offspring and increase their representation in the population. The typical setup is the following: two strategies are concerned, Cooperator (C) or Defector (D); fitness is assumed to be the payoff obtained from pairwise or group interactions defined in the form of social dilemma games; a reproduction (or strategy update) model that favors individuals with higher fitness; the predominant strategy at equilibrium state implies an evolutionarily stable strategy.

In literature, many factors are found to have an impact on the evolution of cooperation, including network structure, payoff heterogeneity and strategic complexity [22]. These studies have assumed a static network topology. In most recent research, the network topology was deemed dynamic due to users' movement. The evolution is thus more complex as the time-varying network structure forces the users to interact with different groups of people. In [4], the players move randomly with constant velocity and participate in group interactions. The correlation between cooperation and velocity is reported: an intermediate degree of mobility was found to be optimal for the evolution of cooperation. [2], [6], [7] employ similar network models but study pairwise interactions. They obtain consistent results as [4].

The shift from static network to mobile network is an important step towards more realistic network conditions, yet the underlying mobility patterns have been regarded as homogeneous. In fact, all the above-mentioned works assume users to have constant moving speed and they can travel across the whole network. However, it has been well known that real-life *mobile social networks* are heterogeneous [8], [21], [29], e.g. users frequently visit and stay around a few "home locations", and their moving ranges are confined by heterogeneous gyration radii. These properties are currently missing in this line of research.

In this paper, we study the formation of cooperation in mobile networks following the EGT framework. Different from previous work, we propose a mobility model called Socialized Speed-Restricted Mobility (SSRM) model to represent realistic human motions. Motivated by mobile social networks, in SSRM, every user is initially located at a randomly picked "home position". Then a moving area $A_i$ centered at the home location is drawn from some probability

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distribution to restrict its movement. SSRM is able to characterize both homogeneous and heterogeneous mobility patterns by adjusting the distribution of $A_i$. User randomly moves to a new position within the area per time step. Simultaneously to the movement, every user encounters instant neighbors if their distance is smaller than a certain communication range. We assume each user has a memory of $\tau$ time steps, and is able to accumulate the neighbors encountered during this period of time. This neighbor collection process can be seen as an exploration of the neighborhood surrounding the home position. After every $\tau$ time steps, users interact with the accumulated neighbors by playing the *Public Goods Game* (PGG), which is a common model for group interaction. Users reproduce by updating their strategies based the obtained payoff. This is seen as one round of evolution. We are interested in finding the predominant strategy (C or D) after a long term of evolutions. This paper makes the following contributions:

– We show that the SSRM model generates similar structure as mobile social networks. By deriving an approximation to the degree distribution, we prove that two distributions that commonly appear in social networks: *exponential* and *power-law* distribution, can be reproduced from SSRM for large $\tau$.

– An extensive simulation-based study on the evolution of cooperation is conducted:

  ◦ We first focus on the simultaneous evolution of mobility and strategy by letting $\tau = 1$. This is the most popular setting in recent works [2], [4], [6], [7]. Under SSRM, we find similar result as previous works, i.e. cooperation is enhanced at infinitesimal mobility, but damaged when mobility is moderate.

  ◦ Then $\tau$ is increased to separate the mobility and reproduction. This pictures a more realistic evolution as users are given more time to explore the neighborhood. We show that cooperation is promoted in this case due to a reduction in the randomness of neighborhood. It is also found that degree heterogeneity further encourages cooperation compared to homogeneous movement.

We believe our findings are helpful to understand cooperative behaviors in mobile social networks. It is also worth mentioning that although mobile interactions in this paper are modeled as PGG game, our EGT framework could be used to study specific cooperation-based mobile applications once they are formulated in the PGG paradigm such as in [5], [30].

The rest of this paper is structured as follows. Section II introduces the system model. In Section III, we derive the approximation to the degree distribution of SSRM model. The results are verified in Section IV. Section V is devoted to simulation-based study on the evolution of cooperation. Finally, conclusion is drawn in Section VI.

## II. System Model

The mobile network under consideration is composed of users that move according to the SSRM model. To study their cooperative behaviors, we consider that each user in its movement accumulates neighbors through a collection process, and engages them to participate in a group interaction modeled by PGG. Users update their game strategies afterwards to improve the fitness. Details are presented in the following.

### A. Network Geometry

We consider the network extension to be a 2-dimensional circular space $\mathcal{A}$ with area $A = \pi R^2$. A population of $N(A)$ mobile users are randomly distributed in the network according to Poisson Point Process (P.P.P.) of intensity $\phi$, i.e. $\mathbb{P}[N(A) = k] = \frac{(A\phi)^k}{k!} e^{-A\phi}$. Without loss of generality, we assume $\phi = 1$ throughout this paper, thus the density of mobile users in the network $\rho = N(A)/A \sim 1$[1] when $A$ is sufficiently large.

### B. Socialized Speed-Restricted Mobility (SSRM) Model

Mobile users move under a speed-restricted model. In the initial stage (time step $t = 0$), all users are uniformly and randomly located at an initial position in the network. Then we assign each user a circular moving area $\mathcal{A}_i$ centered at the initial position, whose radius $R_i$ restricts its moving speed. Once the network starts to operate, users will move randomly and independently within their own circular area. The location of each user, denoted as $X_i^t$, will be totally reshuffled from one time step $t$ to $t + 1$.

The size of $\mathcal{A}_i$ is denoted as $A_i = \pi R_i^2$. There are two ways to determine each user's $A_i$ in SSRM model:

**Homogeneous case**: every user has the same moving range $R_i = R_{uni}$ and moving area $A_i = A_{uni} < A$.

**Heterogeneous case**: $A_i$s are treated as independent and identically distributed (iid) random variables (*rv*) with probability density function (pdf) $f_{A_i}(a)$. In this paper, we consider $A_i$ to be either *Exponential* or *Pareto rv*, i.e.

$$f_{A_i}^{exp}(a) = \frac{\lambda}{\pi} e^{-\frac{\lambda}{\pi} a}, a \geq 0, \tag{1}$$

and

$$f_{A_i}^{pareto}(a) = \frac{\alpha(\pi\mu)^\alpha}{(\pi\mu + a)^{\alpha+1}}, a \geq 0, \tag{2}$$

where $\lambda, \mu$ and $\alpha$ are positive parameters that control the degree of heterogeneity in $A_i$. Both of the two cases imply that users are more likely to stay in a fixed area around some home location, and the heterogeneous model further guarantees the moving areas are different across the users. Therefore the SSRM model complies with moving patterns found in mobile social networks [8], [21], [29].

### C. Public Goods Game and Strategy Update

In addition to the initial position, each user is randomly assigned an initial strategy $s_i^0$ from C ($s_i^0 = 1$) and D ($s_i^0 = 0$). Mobile users have a constant communication range $C_r$, so that user $i$ and $j$ are mutual neighbors if $\left\|X_i^t - X_j^t\right\| \leq C_r$, where $\|\cdot\|$ denotes the Euclidean distance. At each time step $t$, user $i$ moves to a new random location in $\mathcal{A}_i$ and encounters a set of neighbors $\mathcal{N}_i^t$.

Apart from the mobility model, we define a neighbor collection process $\mathcal{T}$ that incorporates $\tau$ time steps. Parameter

[1] $f(n) \sim g(n)$ means that $\lim_{n\to\infty} f(n)/g(n) = 1$.

$\tau$ acts as the memory of mobile users. During the collection process, user $i$ continues its random movement but keeps the history of the encountered neighbors. Therefore the set of neighbors user $i$ accumulates in $\mathcal{T}$ is $\mathcal{N}_i^{\mathcal{T}}(\tau) = \bigcup_{t=t_0}^{t_0+\tau-1} \mathcal{N}_i^t$. Users play a round of PGG after every collection process as follows. First, each user $i$ forms a group with its neighbors $j \in \mathcal{N}_i^{\mathcal{T}}(\tau)$. Suppose that $d_i$ is the size of $\mathcal{N}_i^{\mathcal{T}}(\tau)$ and $d_i + 1$ is the group size. A single PGG game is played for every group in the network. Therefore every user $i$ participates in exactly $d_i + 1$ games. We assume that each user has a fixed amount of resource $c$ to contribute, and a Cooperator allocates $c_i = \frac{c}{d_i+1}$ to every group it is involved in, while Defector contributes nothing. In a single PGG game, the group accumulates the contributions of Cooperators and multiplies it by an enhancement factor $r$. The resulting public goods are equally distributed among all the participants. The payoff obtained by each user from a single PGG is equal to its share of the public goods minus its contribution. As every user is engaged in multiple PGG games, the total payoff obtained by user $i$ after $\mathcal{T}$ is:

$$P_i^{\mathcal{T}}(\tau) = \sum_{j \in \mathcal{N}_i^{\mathcal{T}}(\tau) \cup \{i\}} \frac{\sum\limits_{l \in \mathcal{N}_j^{\mathcal{T}}(\tau) \cup \{j\}} r c_l s_l^{t_0}}{d_j + 1} - (d_i+1) c_i s_i^{t_0} \quad (3)$$

Next, individuals update their strategies based on the obtained payoff. Every user $i$ chooses one of its neighbors $j$ at random. The probability that $i$ takes $j$'s strategy in the next collection process is

$$\mathbb{P}[s_i(t_0 + \tau) = s_j(t_0)] = \frac{\max\{P_j^{\mathcal{T}}(\tau) - P_i^{\mathcal{T}}(\tau), 0\}}{\max\{P_k^{\mathcal{T}}(\tau) - P_l^{\mathcal{T}}(\tau)\}}, \forall k, l. \quad (4)$$

From equation (4), we see that users tend to adopt strategies of successful neighbors whose payoff is higher than their own. This is in line with [26] [4]. Up to this point, one round of evolution is completed. The same process is repeated until the number of cooperators becomes stable.

## III. Approximated Degree Distribution of SSRM Model

In this section, we investigate the degree distribution of SSRM model. The purpose is twofold: first, to confirm SSRM indeed generates similar network structure as mobile social network; second, to facilitate our analysis in Section V. The latter is motivated by [22], [26], which have shown that degree distribution can greatly affect the level of cooperation in static networks.

At the end of a collection process, the network connectivity is given by $\mathcal{N}_i^{\mathcal{T}}(\tau)$s. The degree of user $i$ is $d_i$. For the sake of tractability, given the moving area $A_i$, we approximate $d_i$ with the number of users whose initial positions locate in $\mathcal{A}_i$, and denote it with $\hat{d}_i$. Since the population is generated by P.P.P process, conditioned on $A_i = a$, we have $\hat{d}_i = N(a) = Poisson(a)$, i.e. $p_{\hat{d}_i|A_i}(k|a) = \frac{a^k}{k!} e^{-a}$. This approximation is reasonable because users whose initial positions locate in $\mathcal{A}_i$ have higher probability to encounter $i$ compared to others. Therefore they are more likely to appear in $\mathcal{N}_i^{\mathcal{T}}(\tau)$ when $\tau$ is large. This will be further verified in numerical simulations.

With respect to the notations in Section II and the assumption made above, we have the following theorem.

**Theorem 1.** *After every neighbor collection process $\mathcal{T}$, the approximated degree distribution of our considered mobile network is Mixed Poisson Distribution [13] with mixing distribution $f_{A_i}(a)$:*

$$\begin{aligned} p(k) &= \mathbb{P}(\hat{d}_i = k) \\ &= \int_0^\infty f_{A_i}(a) \frac{e^{-a} a^k}{k!} da, \qquad k = 0, 1, ... \end{aligned} \quad (5)$$

*Proof.* The probability mass function (pmf) of degree distribution, $p(k)$, can be interpreted as the fraction of users with $\hat{d}_i = k$ when $N(A)$ is sufficiently large. We denote this fraction as $f(k)$, and

$$f(k) = \frac{1}{N(A)} \sum_i \mathbb{1}_{\hat{d}_i = k} = \sum_{a \in \bigcup_i A_i} \underbrace{\frac{n_a}{N(A)}}_{\theta_a} \cdot \underbrace{\frac{\sum\limits_{A_i = a} \mathbb{1}_{\hat{d}_i = k}}{n_a}}_{\varphi_{k,a}},$$

where $\mathbb{1}_e$ is the indicator function and is equal to 1 if $e$ is true and 0 otherwise; $n_a$ ($\theta_a$) is the number (fraction) of users with $A_i = a$; similarly, $\varphi_{k,a}$ is the fraction of users with $\hat{d}_i = k$ among the $n_a$ users. By taking the limit of $A$, we obtain

$$\begin{aligned} p(k) &= \lim_{A \to \infty} f(k) = \lim_{A \to \infty} \sum_{a \in \bigcup_i A_i} \theta_a \cdot \varphi_{k,a} \\ &= \int_0^\infty f_{A_i}(a) \cdot p_{\hat{d}_i|A_i}(k|a) da \\ &= \int_0^\infty f_{A_i}(a) \frac{e^{-a} a^k}{k!} da, \qquad k = 0, 1, ... \end{aligned} \quad (6)$$

□

In the rest of this section, we derive the approximated degree distribution for homogeneous and heterogeneous mobility model.

### A. Homogeneous Mobility Model

**Corollary 1.** *In the homogeneous case, $\hat{d}_i$ is a Poisson* rv *with pmf*

$$p_{uni}(k) = \frac{e^{-A_{uni}} {A_{uni}}^k}{k!}, \qquad k = 0, 1, ... \quad (7)$$

*Proof.* In this case, every user has the same moving area $A_{uni}$. Therefore $f_{A_i}(a) = \delta(a - A_{uni})$, where $\delta(\cdot)$ is the Dirac delta function. The distribution of $\hat{d}_i$ can be easily obtained from equation (6).

□

### B. Heterogeneous Mobility Model

We consider the *Exponential* and *Pareto* density function as in equation (1) and (2).

**Corollary 2.** *For exponential moving area, the pmf of $\hat{d}_i$ is geometric($\frac{\lambda}{\lambda+\pi}$), thus has exponential tail.*

*Proof.* In this case, the degree distribution is

$$
\begin{aligned}
p_{exp}(k) &= \int_0^\infty f_{A_i}^{\exp}(a)\frac{e^{-a}a^k}{k!}da \\
&= \frac{1}{k!}\int_0^\infty a^k f_{A_i}^{\exp}(a)e^{-a}da \\
&= \frac{1}{k!}\int_0^\infty a^k \mathcal{L}_{\exp}(s)da\Big|_{s=1} \\
&= \frac{1}{k!}(-1)^k\frac{\partial^k\mathcal{L}_{\exp}(s)}{\partial s^k}\Big|_{s=1},
\end{aligned}
$$

where $\mathcal{L}_{exp}(s)$ is the Laplace transform of $f_{A_i}^{exp}(a)$. Replace $\mathcal{L}_{exp}(s)$ with $\frac{\lambda/\pi}{\lambda/\pi+s}$ we obtain

$$p_{exp}(k) = \frac{\lambda}{\lambda+\pi}(\frac{\pi}{\lambda+\pi})^k, \qquad k = 0, 1, ...$$

Obviously, the degree is geometric distribution with parameter $p \triangleq \frac{\lambda}{\lambda+\pi}$. The Complementary Cumulative Distribution Function (CCDF) is

$$\bar{F}_{exp}(k) = (1-p)^k = e^{-\ln\frac{1}{1-p}k} = e^{-\Lambda k}.$$

Therefore the degree distribution exhibits exponential tail. □

**Corollary 3.** *For Pareto moving area, the pmf of $\hat{d}_i$ is asymptotically equal to $f_{A_i}^{pareto}(k)$, thus has power-law tail at infinity.*

*Proof.* When the moving area is drawn from Pareto distribution, notice that

$$f_{A_i}^{pareto}(a) = \alpha(\pi\mu)^\alpha\frac{a^{\alpha+1}}{(\pi\mu+a)^{\alpha+1}}\cdot a^{-(\alpha+1)} \triangleq C(a)\cdot a^\gamma,$$

where $C(a) = \alpha(\pi\mu)^\alpha\frac{a^{\alpha+1}}{(\pi\mu+a)^{\alpha+1}}$ is a slowly varying function [3] with respect to $a$ since

$$\lim_{a\to\infty}\frac{C(\xi a)}{C(a)} = \lim_{a\to\infty}\xi^{\alpha+1}\frac{(\pi\mu+a)^{\alpha+1}}{(\pi\mu+\xi a)^{\alpha+1}} = 1, \forall\xi > 0.$$

In addition, $\gamma = -(\alpha+1) < -1$. We can apply Theorem 2.1 in [28] to obtain the following tail distribution of $p_{pareto}(k)$:

$$p_{pareto}(k) \sim C(k)k^\gamma = \frac{\alpha(\pi\mu)^\alpha}{(\pi\mu+k)^{\alpha+1}}, \qquad k\to\infty$$

The CCDF is

$$\bar{F}_{pareto}(k) \sim (\frac{\pi\mu}{\pi\mu+k})^\alpha \sim (\pi\mu)^\alpha k^{-\alpha},$$

which exhibits power-law tail when $k$ is large. □

## IV. Numerical Validation of Degree Distribution

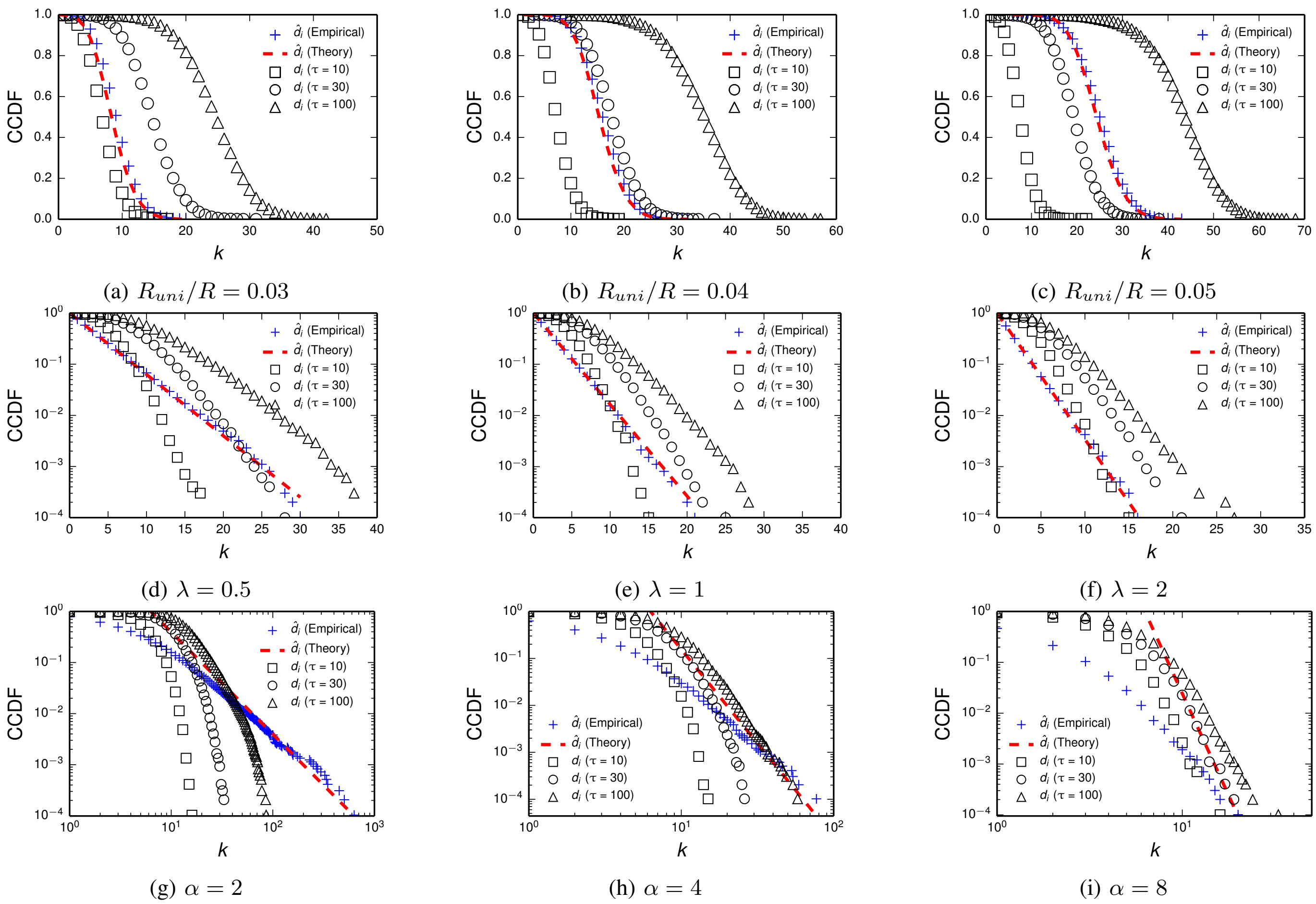
We verify our theoretical results in Section III through numerical simulations. First, we create a circular space with area $A$ and generate $N(A)$ users from $Poisson(A)$. Then the users independently and randomly choose their initial positions within the space. This process is simulating P.P.P. of density 1 on the circular plane. Next, every user is assigned a moving area around its initial position according to our mobility model. We acquire the empirical distribution of degree approximation $\hat{d}_i$ by gathering $\hat{d}_i = \sum_j \mathbb{1}_{\|X_i^0 - X_j^0\| \le R_i}, \forall i$. The actual degree distribution is obtained from $\mathcal{N}_i^{\mathcal{T}}(\tau)$s by simulating the collection process as in II-C.

Our simulation uses the following configuration: $A = 10000$, communication range $C_r = 0.5$, $\tau = [10, 30, 100]$. For the homogeneous distribution, we consider $R_{uni}/R$ to be in the range $0.03 \sim 0.05$. For exponential moving area, we consider $\lambda$ from $0.5 \sim 2$. For Pareto moving area, we consider $\mu = 2$ and $\alpha$ from $2 \sim 8$. Every simulation is repeated 10 times to reflect an accurate empirical distribution.

Simulation results for the homogeneous case are shown in Fig. 1 (a)-(c). Looking at the theoretical and empirical CCDF of $\hat{d}_i$, we see that our result in Corollary 1 is accurate. Moreover, we compare the distribution of $\hat{d}_i$ and $d_i$ at different $\tau$. We are interested in the optimal value of $\tau$, denoted as $\tau_{opt}$, at which $d_i$ is best approximated by $\hat{d}_i$. First, in each of the subfigures, we observe that when $\tau$ increases, the CCDF of $d_i$ shifts to the right, causing a higher average degree. This is intuitive since larger $\tau$ means a longer collection process, thus the average number of encountered nodes is increased. Secondly, $A_{uni}$ affects $\tau_{opt}$. For example, in (a), $\hat{d}_i$ is well aligned with $d_i$ at $\tau = 10$, but deviates from $d_i$ when $\tau = 30$ or 100. Increasing $A_{uni}$, the CCDF of $\hat{d}_i$ gets closer to $d_i$ at $\tau = 30$. Notice that $\hat{d}_i$, the number of users covered by $A_{uni}$, will surely increase with $A_{uni}$. In the meantime $d_i$ increases as well because $\mathcal{N}_i^{\mathcal{T}}(\tau)$ becomes larger as $X_i(t)$s are more spread out, and $\mathcal{N}_i^t$s are more diverse for a larger $A_{uni}$. However, $\hat{d}_i$ is more directly influenced by $A_{uni}$ than $d_i$, which explains the phenomenon shown above.

When the moving areas are exponentially distributed, the semi-logarithmic plots in Fig. 1 (d)-(f) show that our theoretical result for $\hat{d}_i$ in Corollary 2 matches the empirical result very well. As for $d_i$, we observe similar phenomenon as in the homogeneous case: 1) larger $\tau$ leads to a longer tail in the degree distribution; 2) the larger the average moving area, the larger the value of $\tau_{opt}$. In this case, the average moving area $\propto \frac{1}{\lambda}$. Therefore $\tau_{opt} \simeq 30$ when $\lambda = 0.5$, and $\tau_{opt} \simeq 10$ when $\lambda = 1$ or 2.

Lastly, we consider the Pareto moving area. According to Corollary 3, we are only able to capture the asymptotic behaviour of $\hat{d}_i$. Therefore we see in the log-log graph (Fig. 1 (g)) that when $\alpha = 2$ and the magnitude of the tail is $10^2$, $\hat{d}_i$ is in good agreement with the power-law distribution. Otherwise $\hat{d}_i$ deviates from the power-law CCDF as the tail is short. Besides, the distribution of $d_i$ at different $\tau$ and the value of $\tau_{opt}$ at different $\alpha$ have the same trend as before.




(a) $R_{uni}/R = 0.03$ (b) $R_{uni}/R = 0.04$ (c) $R_{uni}/R = 0.05$

(d) $\lambda = 0.5$ (e) $\lambda = 1$ (f) $\lambda = 2$

(g) $\alpha = 2$ (h) $\alpha = 4$ (i) $\alpha = 8$

Figure 1: Simulation results for degree approximation. Homogeneous: (a)-(c). Exp: (d)-(f). Pareto: (g)-(i).

The numerical results suggest that although our theoretical distributions of $\hat{d}_i$ from Section III are correct, how well they can approximate the actual degree $d_i$ depends on the average moving area as well as $\tau$. In the following simulations we stick to $\tau = 30$, which provides relatively good approximation for the considered parameters. Even if $\hat{d}_i$ deviates from $d_i$, we see from Fig. IV (d)-(i) that $d_i$ still has exponential or pareto tail, i.e. SSRM is able to reproduce real-life degree distribution in mobile social networks.

## V. Simulation Study on Evolution of Cooperation Under SSRM Model

In this section, we study the evolution of cooperation in the defined mobile network through simulations. The simulation is divided into epochs of $\tau$ time steps. Each epoch corresponds to a neighbor collection process. The movement and evolutions are simulated based on Section II. Whether cooperation can survive is indicated by the fraction of cooperators $\rho_c$ after a long term of evolutions, given some fixed enhancement factor $r$. If $\rho_c$ is close to 1 at relatively small $r$, we say cooperation survives more easily, or is promoted by the underlying network conditions. In practice, $r$ is normalized with respect to the average group size $\langle d \rangle + 1$ [4], [22], [26]. The resulting factor is denoted as $\eta = \frac{r}{\langle d \rangle + 1}$. It has been shown that ideally cooperation prevails at $\eta = 1$ in an infinite, well-mixed population. This serves as a benchmark in our evaluations. In the simulations, we compute $\rho_c$ after 10000 epochs with respect to $\eta$. The network size $A = 1000$. Other configurations are kept the same as the previous section. Without loss of generality, we set $c = 1$.

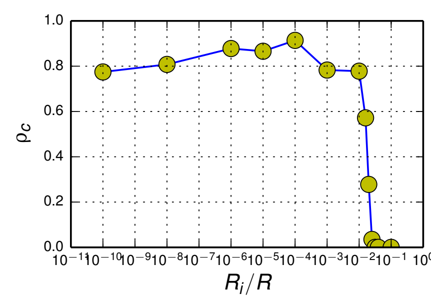
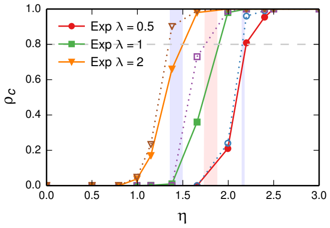
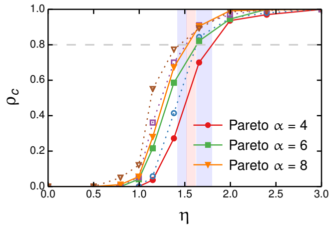



Figure 2: Effects of mobility on cooperation rate. $C_r = 1.5$ to guarantee network connectivity. $\eta = r/(\pi C_r^2 + 1) = 0.9$.

We consider two different evolution processes by adjusting the value of $\tau$. In the first case, $\tau = 1$ so that the movement and strategy update occur simultaneously. We find SSRM has the same impact on cooperation as previous models in [4] [2] [7]. Then we turn to the evolution with larger $\tau$, which leads to more realistic movement (as validated in

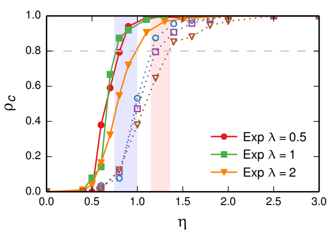



(a) Exponential case.

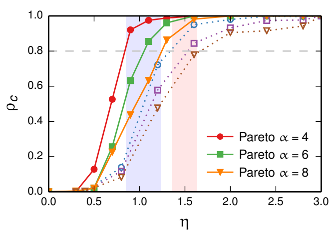



(b) Pareto case.

Figure 3: Cooperation rate $\rho_c$ as a function of $\eta$ in heterogeneous mobile networks when $\tau = 1$. *The dotted curves correspond to homogeneous networks. Curves with the same markers have the same average moving range. The shaded area projects the critical range of $\eta$ for which $\rho_c$ reaches 0.8.*




(a) Exponential case.




(b) Pareto case.

Figure 4: Cooperation rate $\rho_c$ as a function of $\eta$ in heterogeneous mobile networks when $\tau = 30$.

Section IV). Moreover, the separation between movement and strategy update is more consistent with human behavior as movement and neighbor collection are microscopic processes that happen frequently, while strategy update (adaption) takes much longer time [12]. Simulations reveal that heterogeneous mobility model significantly promotes cooperation compared to the homogeneous model. Further investigation is conducted by correlating such enhancement with the behavior of high degree users in heterogeneous mobile networks.

### A. Simultaneous Evolution of Mobility and Strategy

We first study the case where $\tau = 1$. Users move, play PGG, and update strategies simultaneously every time step. In homogeneous SSRM, we vary the value of $R_{uni}/R$ to reflect different level of mobility. When $R_{uni}/R$ is very small, the mobile network is reduced to a static one. Users interact with constant group of neighbors and the group size is *Poisson*($\pi C_r^2 + 1$). Increasing $R_{uni}$ introduces more randomness in the evolution, as the set of neighbors changes every time step. It has been shown in [4] and [7] that a slight increase in mobility improves users' ability to discover clusters of cooperators, while a higher level of mobility strongly reduces the formation of large cooperator clusters due to invasion of defectors. Fig. 2 shows the cooperation rate $\rho_c$ as a function of mobility level. Our result in a homogeneous mobile network is consistent with previous studies.

From Fig. 2, cooperation is maximally promoted at $R_{uni}/R = 10^{-4} \ll 1$. From this point on, $\rho_c$ drastically decays. This result is rather pessimistic because in mobile social networks, the average moving range can be much larger, i.e. $\langle R_i \rangle = O(R)$. Those fast moving users are detrimental to the formation of cooperation as they may invade already-grouped cooperator clusters.

To verify this, we examine simultaneous evolution in heterogeneous mobile networks. According to our model, the moving area has heavy tail distribution. Therefore few users travel in a large area while the majority moves in a very small range. In the mean time we keep the average moving range relatively large. We choose $\lambda = [0.5, 1, 2]$ for the exponential model, and $\mu = 2$, $\alpha = [4, 6, 8]$ for the pareto model. With these parameters, $\langle R_i \rangle / R$ is $(2.6 \sim 7) \cdot 10^{-2}$, which correspond to points of low cooperation in Fig. 2. We plot $\rho_c$ with respect to $\eta$. In comparison, we also plot $\rho_c$ of a homogeneous network with the same average moving range. Results are shown in Fig. 3.

Not surprisingly, the level of cooperation increases with $\eta$, as large value of $r$ rewards contributors and thus encourages cooperation. However, to reach a high cooperation level $\rho_c = 0.8$, $\eta$ must be at least 1.5 and sometimes even larger than 2, while in Fig. 2, cooperation prevails at $\eta = 0.9$ when moving range is much smaller. Moreover, we have the following observations in Fig. 3a and 3b:

**1)** Cooperation emerges more easily as we increase $\lambda$ or $\alpha$ (or equivalently, decrease the average moving area).

**2)** Keeping their average moving ranges the same, homogeneous and heterogeneous mobility models have similar impact on the cooperation level as their critical $\eta$ are almost the same.

Observation 1) is consistent with our findings in Fig. 2. To explain 2), recall that when $\tau = 1$, no matter in homogeneous or heterogeneous networks, the degree distribution is Poisson with the same mean $\pi C_r^2 + 1$. When the average moving range is large, users always encounter and interact with different groups of people characterized by the same distribution. Heterogeneity in moving area does not make a big difference in this scenario.

### B. Increasing $\tau$ Stimulates Cooperation

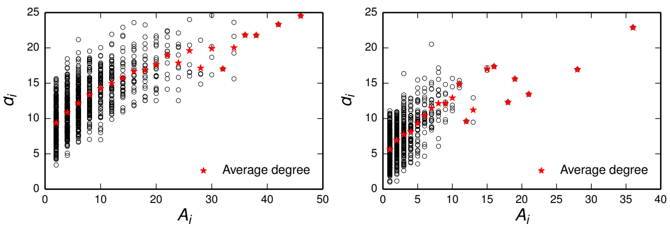
Now we focus on the evolution of cooperation after a prolonged collection process. We set $\tau = 30$ and $C_r = 0.5$. As validated in section IV, the chosen parameters generate degree $d_i$ that can be well approximated by exponential and pareto distribution. We repeat the simulation in Fig. 3. Results are shown in Fig. 4.

Comparing Fig. 3 and 4, we clearly see that Cooperators survive more easily when $\tau$ is increased, no matter in homogeneous or heterogeneous networks (although in Fig. 3, we bind the critical $\eta$ for curves with the same marker in one shaded area, and in Fig. 4 for curves with the same line-style). Moreover, the cooperation level is improved even more in heterogeneous networks. In exponential degree network, cooperation prevails at $\eta \leq 1$ and in pareto degree network it prevails at $\eta \sim 1$. This is almost as good as the optimal case in Fig. 2.

First, to explain why cooperation is promoted once we increase $\tau$, consider the randomness of neighbor set $\mathcal{N}_i^{\mathcal{T}}(\tau)$ in two different collection processes. When $\tau = 1$, $\mathcal{N}_i^{\mathcal{T}}(1) = \mathcal{N}_i^{t_0}$, which is a collection of users inside the range $C_r$ around

Table I: Stability $\mathcal{S}$ comparison

| Model | Top 10% | | Bot 10% | |
|---|---|---|---|---|
| | $\tau = 1$ | $\tau = 30$ | $\tau = 1$ | $\tau = 30$ |
| Exp ($\lambda = 0.5$) | 0.059 | 0.348 | 0.17 | 0.614 |
| Exp ($\lambda = 1$) | 0.092 | 0.47 | 0.263 | 0.706 |
| Exp ($\lambda = 2$) | 0.147 | 0.571 | 0.369 | 0.773 |
| Pareto ($\alpha = 4$) | 0.115 | 0.531 | 0.335 | 0.753 |
| Pareto ($\alpha = 6$) | 0.168 | 0.6 | 0.4216 | 0.815 |
| Pareto ($\alpha = 8$) | 0.2 | 0.638 | 0.48 | 0.8358 |




(a) Exponential case. (b) Pareto case.

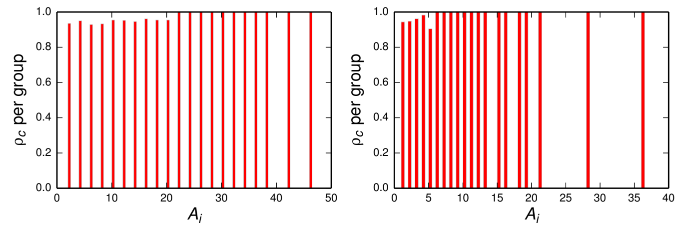
Figure 5: Degree and moving area correlation.

a random location. For a larger $\tau$, this set becomes a union of multiple such $\mathcal{N}_i^{\mathcal{T}}(1)$s. Since every user has a restricted moving area $\mathcal{A}_i$, when $\tau$ increases, this union will not grow infinitely but is largely limited to the neighbors that frequently appear in the moving area, i.e. those whose initial positions are inside $\mathcal{A}_i$. Therefore the randomness is reduced when we increase $\tau$. As a consequence, the network topology is more stable. To quantify the randomness, we define the following stability metric:

$$\mathcal{S} = \frac{\sum_{m=1}^{M} \left|\mathcal{N}_i^{\mathcal{T}_m}(\tau)\right|}{M\left|\bigcup_{m=1}^{M} \mathcal{N}_i^{\mathcal{T}_m}(\tau)\right|}.$$

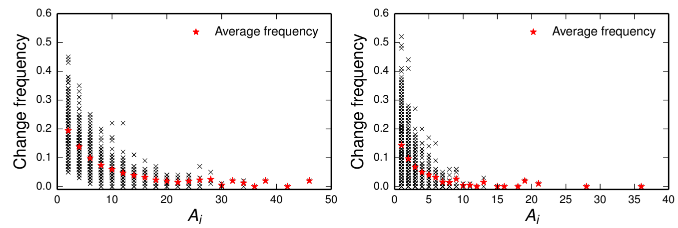
$\mathcal{S}$ is equal to the average size of $\mathcal{N}_i^{\mathcal{T}}(\tau)$ in $M$ epochs divided by the size of their union. One can easily prove that $0 < \mathcal{S} \leq 1$. $\mathcal{S} = 1$ when the $\mathcal{N}_i^{\mathcal{T}}(\tau)$ does not change. The more diverse (random) the $\mathcal{N}_i^{\mathcal{T}}(\tau)$, the smaller the $\mathcal{S}$. Comparison in term of $\mathcal{S}$ is shown in Table. I. We take $M = 100$ and present the average stability of 10% users with largest (top) and smallest (bot) moving areas. Obviously, stability in $\mathcal{N}_i^{\mathcal{T}}(\tau)$ is increased for larger $\tau$.

With network randomness being controlled, the difference between the two mobility models lies in the degree heterogeneity. Recall $\mathcal{N}_i^{\mathcal{T}}(30)$ has exponential or power-law tail approximation, which is heavier than poisson tail. Therefore there are more high degree users in the heterogeneous models than in the homogeneous one. Those users, together with their neighbors, form clusters that are analogous to social communities. According to previous studies [26] [25], degree heterogeneity and clustering promote cooperation on a static network. Although we are concerning a mobile network, it has been shown that the restricted moving areas and a large $\tau$ limit the randomness induced by mobility. Consequently the benefit of degree heterogeneity appears and we observe the promotion of cooperation in heterogeneous mobile networks.




(a) Exponential case. (b) Pareto case.

Figure 6: Cooperation rate in moving area groups.




(a) Exponential case. (b) Pareto case.

Figure 7: Strategy update frequency in moving area groups.

Next, we take a closer look at the evolution process in heterogeneous mobile networks. Of particular interests are the cooperative behaviors of high degree users.

### C. Cooperative Behaviors of Heterogeneous Mobile Users

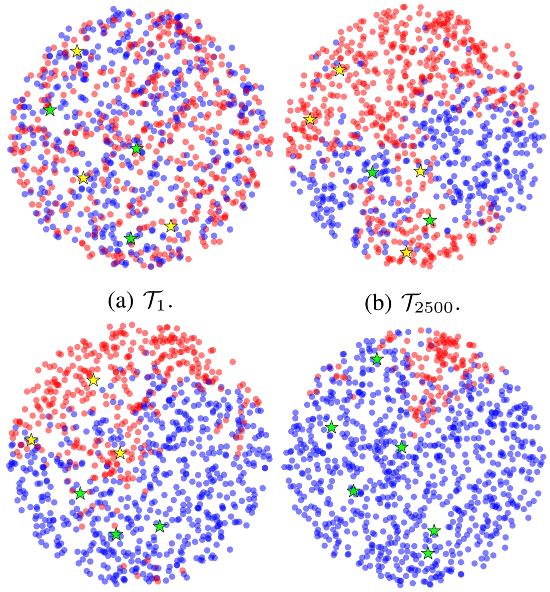
We focus on two instances of heterogeneous mobile networks at $\tau = 30$:

**1)** Exponential with $\lambda = 0.5$.
**2)** Pareto with $\mu = 2$, $\alpha = 4$.

We first examine the correlation between degree $d_i$ and moving area $A_i$. We group users based on their moving areas $A_i$, and plot their average degrees over 100 epochs. The average degree in every group is displayed as well. As shown in Fig. 5, user's degree is positively correlated with its moving area. In the following, we refer to the users with large moving areas as "hub"s, as they are usually the centers of communities in mobile social networks.

Next, we look at the strategy distribution across the moving area groups, after the system enters equilibrium stage. Fig. 6 shows the average cooperation rate in different groups at $\eta = 1$. We see that at steady state, the hubs all become cooperators, and defectors only exist among non-hub users. In Fig. 7, we plot the average strategy update frequency. It is shown that the strategies of hubs rarely change, while non-hub users are more easily affected by neighbors.

Lastly, we present the snapshots (Fig. 8) of the evolution process in 4 different stages. Due to the limit of space, only exponential network is considered here. At $\mathcal{T}_1$, there are equal amount of C and D randomly spread in the space. The marked users are 3 hub Cooperators and 3 hub Defectors. After 7500 rounds of evolution, all of the hubs evolve to cooperators. In the meantime, we clearly see that the neighborhood around hubs is dominated by cooperators, leaving defectors concentrated in regions far away from the cluster of cooperators.



(a) $\mathcal{T}_1$. (b) $\mathcal{T}_{2500}$. (c) $\mathcal{T}_{5000}$. (d) $\mathcal{T}_{7500}$.

Figure 8: Snapshots in exponential mobile network. *Cooperators (defectors) are labeled red (blue). Users marked as stars are cooperators (green) and defectors (yellow) at hubs.*

## VI. Conclusion

In this paper, we employ the EGT framework to study cooperative behavior in spatial PGG over a mobile system. A socialized mobility model SSRM is developed to drive the movement. PGG and strategy update occur after every neighbor collection process. We first verify that the SSRM model produces exponential and power-law degree distributions for a long neighbor collection process. It is also found that a long collection process reduces randomness in network connectivity. These two facts explain the main finding in this paper: cooperation is significantly promoted in mobile social networks due to the degree heterogeneity and regular moving patterns. In future, we will use our framework to study specific cooperative protocols.

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