Game Theoretic Analysis of Citizens Broadband Radio Service

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***Abstract*—The Citizens Broadband Radio Service (CBRS) is a spectrum sharing framework on the 3.5 GHz tier with three priority tiers: the incumbents, priority commercial users (PAL), and general commercial users (GAA). Thus, commercial users compete for resources within the second and third priority tiers. The interaction between commercial providers and customers is complicated by the presence of the incumbents, who impact the availability of spectrum but bypass the market entirely. In particular, PAL customers are themselves subject to preemption even with the priority purchase. In this paper, we propose a game-theoretic framework to shed light into the equilibrium outcomes and the impact of the incumbents into these. We determine that there exist several possible equilibrium regions, including one with a unique mixed equilibrium which is stable in the evolutionary stable strategy sense, and others featuring unstable mixed equilibria and stable pure equilibria. We show that for fixed parameters, the maximum possible revenue a provider can obtain is associated with a stable equilibrium and is thus guaranteed. However, changes in incumbent behavior can result in phase changes which have a sizable impact on the maximum potential revenue.**

***Index Terms*—Shared spectrum, network economics, game theory, queuing theory.**

I. Introduction

Increased spectrum demand to facilitate the roll out of new wireless technologies has spurred the development of spectrum sharing to leverage the fact that while in practice, many previously allocated bands have sparse geographic and temporal use. One such framework is the Citizens Broadband Radio Service (CBRS) on the 3.5GHz band. CBRS defines three priority tiers of users [1], [2]:

1) Incumbents: granted by default to defense related users (typically the United States Navy);
2) Priority Access: determined by allocation of Priority Access Licences (PALs);
3) General Authorized Access (GAA): licensed by rule.

The CBRS also defines a Spectrum Access System (SAS) to coordinate spectrum availability. In particular, ensuring that lower priority users do not interfere with higher priority users.

There is clearly robust demand for the available spectrum, with telecommunications and utility providers spending a collective 4.5 Billion US$ to secure PALs in the 2020 FCC Auction 105 [3]. Potential customers who want to make use of the spectrum must decide whether to do so under the GAA provision, or make use of resources offered by a PAL holder. The decision is driven by whether the benefits of minimizing the costs of service preemption is greater than the costs of access to the PAL holder licensed spectrum. However, the presence and nature of the incumbents means that service preemption cannot be avoided entirely even with priority access, impacting how much the PAL holder may charge others to lease their spectrum.

As discussed in Section II, while there is prior work analyzing strategic interactions in CBRS, the impact of the incumbents' presence on the decision-making process of customers has not yet been studied, to our knowledge. Within the CBRS framework, we aim to address the following:

- How does the cost of preemption impact the customer decision between which priority tier to utilize?
- Can a PAL holder be guaranteed their maximum possible revenue in a meaningful sense?
- How do changes in incumbent behavior impact the commercial market?

Our contributions are as follows. First, we formalize the customer tier decision and provider price decision within a queuing game framework [4], considering incumbent and customer users as continuous streams arriving to the system. Our model explicitly accounts for the preemption costs and the traffic load of incumbents. Second, we determine that there exist several possible equilibrium regions, including some with mixed equilibria which are stable in the *Evolutionary Stable Strategy* sense, and others featuring multiple unstable mixed equilibria. Third, we determine that despite the possibility of regions with unstable equilibria, the maximal provider revenue for fixed parameters is always associated with a stable equilibrium state. Fourth, we demonstrate that alterations in the behavior of the incumbents alone can force a phase change between equilibrium regions, which can potentially have a significant impact on provider revenues.

The remainder of the paper is organized as follows. In Section II we provide an overview of related works. In Section III we detail our system model and the associated game. In Section IV we analyze the resulting possible equilibria states, which we leverage in Section V to evaluate how this impacts the PAL holder's behavior. Due to space constraints, some of the proofs are omitted.

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II. Related Work

CBRS is a topic of interest for research in light of the ongoing PAL allocations and resulting deployments. However, while there is existing literature regarding analyzing interactions on the CBRS band, these predominately focus on provider-level decision making which does not directly account for incumbent behavior. Examples include the decision on which tier to operate [5]; impact of small-cell resource allocation on deployment decisions by multiple providers [6]; the problem of sub-leasing PAL access to other providers [7]; competitions among providers for users given access to varying tiers [8]; the impact of resource sharing on the decision of which base stations to keep active during low traffic periods [9]; and minimizing the impact of free-riders within a shared spectrum scenario [10]. None of these works focus on the decision-making process of the customers of whether to utilize service backed by the PAL or GAA tier *given the presence of higher priority incumbent tier traffic*. Furthermore, our economic analysis captures the stochastic nature of traffic. Notably, our work shows that the variance of the service time has a significant impact on the equilibrium outcomes.

The use of queuing games to analyze priority purchasing decisions in the face of potential delays is a recurring topic in the literature [11]–[13]. Models with Poisson distributed arrivals and general service distribution in particular are commonly used in modeling cognitive radio as noted in prior surveys [14]. The question of a customer priority purchasing decision under general service distribution has been previously considered under a two-class scenario [15], [16]. In our paper, we consider how the presence of the incumbent class, which is not subject to purchasing decisions, impacts the customers. We further evaluate how customers' sensitivity to preemption impacts their decision making process. This is especially important as the customers cannot escape preemption altogether due to the incumbents. We also introduce a dynamic game played over multiple time intervals, and show how changes in incumbent behavior impacts provider revenues.

III. Model and Problem Statement

We now formally define our CBRS system model, and the associated priority purchasing game. The game features a single provider and three tiers of users: the incumbent users who automatically have highest priority; and the customers who are further subdivided into the priority users paying a provider to utilize PAL tier resources, and general users who opt for service under on the GAA tier.

A. System Model

Based on the CBRS specification, we consider a scenario where a single 10 Mhz channel on the 3.5 Ghz band is available. A provider holding a PAL for this channel is present, as well as a Spectrum Access System (SAS) in the background managing spectrum availability and communicating with the users [1]. From highest to lowest priority, the tiers of user present are the incumbents in, priority customers pc, and general customers gc. Incumbents are predesignated, and may utilize the spectrum at any time. Otherwise, customers decide whether to pay a fee C to lease capacity from the PAL holder on a short term basis in exchange for being granted priority customer status. The concept of short term leasing of shared resources exists within the cloud computing space today [17]; further, the ability to sub-lease PAL spectrum, including limited overlap, has been considered in a slightly different context in prior work, specifically hinging on geographic subdivision [7]. If the customer chooses to do so, they may utilize the spectrum so long as no incumbents are present. If a customer does not pay C, then they become a general customer under GAA tier provisions, and may utilize spectrum only if no higher priority users are present.

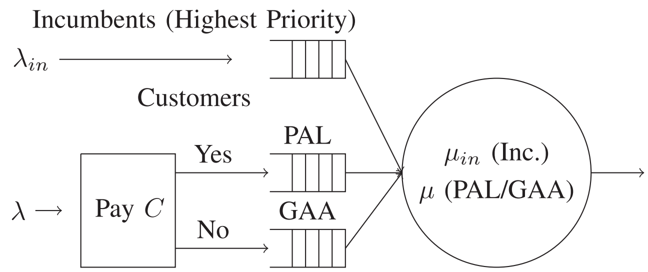


Fig. 1: CBRS expressed as a queuing model. Incumbents are predefined as such, and immediately join the highest priority queue upon arrival. Customers choose between paying a priority access fee C to the 2nd priority PAL tier queue, or forgo the payment and default to the 3rd priority GAA tier queue. Users enter service based on priority; First Come, First Serve is in effect for users within each priority tier. Users depart after completion of service.

The incumbents and customers form continuous streams of users arriving to the system. Users are served in priority order, with the SAS indicating when users may transmit or must yield to higher priority traffic. As a result, we apply a queuing model, as illustrated in Figure 1. That is, we assume that arrivals follow a Poisson process, service times follow a general distribution, and that there is a single server for all arrivals. The assumption of Poisson arrivals is justified by prior measurement studies; these same studies suggest that the common assumption of exponentially distributed service is not reasonable in practice, hence our adoption of general service distribution [18]. Because the SAS is indicating when users may transmit or must yield, we assume that the queue is preemptive in nature. As there are existing spectrum hand off procedures which hold lower priority transmissions until they can be resumed [19], we assume that this system is specifically as a multi-class preemptive-resume priority queuing system [20]. That is, a system where customers resume from the point of interruption once the SAS indicates the channel is free following preempted service.

As arrivals are continuous, the number of users of each type is not fixed a priori. Further, there is no physical queue for users to arrive to, rather the queue is determined by the requests made to the SAS for spectrum availability. As a result, users will not have precise information about the

Parameter	Definition
λ, λ_{in}	Arrival rate of customers and incumbents, respectively.
μ, μ_{in}	Service rate of customers and incumbents, respectively (equal to 1 over mean service length).
ρ, ρ_{in}	Traffic load of customers and incumbents, respectively (equal to arrival rate over service rate).
K, K_{in}	Service variance parameter, such that the second moment of service equals K/μ^2 K_{in}/μ_{in}^2 for customers and incumbents, respectively.
V_d	The value placed on the cost of waiting for completion of service, per time unit.
V_p	The value placed on the cost of preemption by a higher class user.
α, β, γ	Substitutions for the repeating expressions $1-\rho_{in}$, $1-(\rho_{in}+\phi\rho)$, and $1-(\rho_{in}+\rho)$, respectively
D_{pc}, D_{gc}	Random variable representing the total system delay for primary customers and general customers, respectively.
ϕ	Fraction of customers who are priority customers.
C	Cost of joining the PAL tier and becoming a priority customer.
$\mathcal{R}$	Average provider revenue per time unit.

TABLE I: Definition of parameters and variables related to the queuing game model for CBRS. The first six are input parameters which are traffic parameters and costs based on the customers' own valuation of service. The next three are derived from the analysis. The cost C (and by extension the average revenue R) is the parameter which the provider seeks to optimize.

number of users currently present in the system. As a result, we have an *unobservable* queue where customers make their priority purchasing decision based on knowledge of derived queuing statistics [12]. We also employ other standard queuing assumptions: specifically that users' service distributions are independent and identically distributed (IID) and that the system is stable, that is new users do not arrive faster than the current users can be serviced [11], [12].We assume that the incumbents and customers have differing statistical parameters, but that users within each stream are homogeneous.

The parameters and variables of interest to the purchasing decision are defined in Table I. In particular, we note that the arrival rates λ and λ_{in}, service rates μ and μ_{in}, PAL tier fee C, and the value placed on waiting for service V_d are all positive. Per the assumption of stability of the system, the overall traffic load $\rho+\rho_{in}$ must be less than 1; by extension this implies that ρ and ρ_{in} are individually strictly between 0 and 1. Finally, the parameters K and K_{in} arise from the fact that while we assume general service, as shown in the sequel knowledge of the second moment of service is necessary in order to make the purchasing decision. Thus, K and K_{in} are defined in terms of the second moment and therefore $K, K_{in} \geq 1$ follows. Of note, $K = 1$ corresponds to a deterministic distribution, and a value of 2 corresponds to an exponential one.

B. Priority Purchasing Game

With the model established, we may now formalize the game that results from the competition for resources. Our analysis aims to address the following: (i) how the customers make the priority purchasing decision, (ii) what is the cost the provider needs to set to maximize its revenue, and (iii) what are the impacts of incumbent behavior and the cost of preemption on these decision making processes. To do so, we formally define the action spaces for the provider and customer and the impacts of the players' decisions on their net benefit.

For the provider, their action space is the level at which to set the PAL tier access fee C. We assume that this price does not fluctuate from moment to moment based on the exact number of users present. Rather, the provider sets C based on their knowledge of the current queuing statistics and only updates if changes to these statistics are detected.

Aside from the fee C paid if joining the priority class, the other costs to the customers are self-imposed: the cost of waiting for the completion of service, and the cost of preemption. If $E[D_{pc}]$ and $E[D_{gc}]$ are the expected total system delay for priority and general customers, then the expected costs of waiting in each class will be these quantities multiplied by V_d, the per-time unit value on waiting for completion of service. Similarly, the cost of preemption is the value placed on preemption V_p multiplied by the number of preemptions. However, the expected number of preemptions is equal to the traffic rate of any higher class users.

The possible system states which may arise depend on the fraction $\phi \in [0,1]$ of customers opting for priority over general status. As a result, ϕ represents the customers' collective strategy. We are particularly interested in which states ϕ result in *(Nash) equilibrium* states under steady state conditions. ϕ is equilibrium strategy if customers are indifferent between their options, which occurs whenever the following holds:

$$V_d E[D_{pc}] + V_p \rho_{in} + C = V_d E[D_{gc}] + V_p(\rho_{in} + \rho\phi). \quad (1)$$

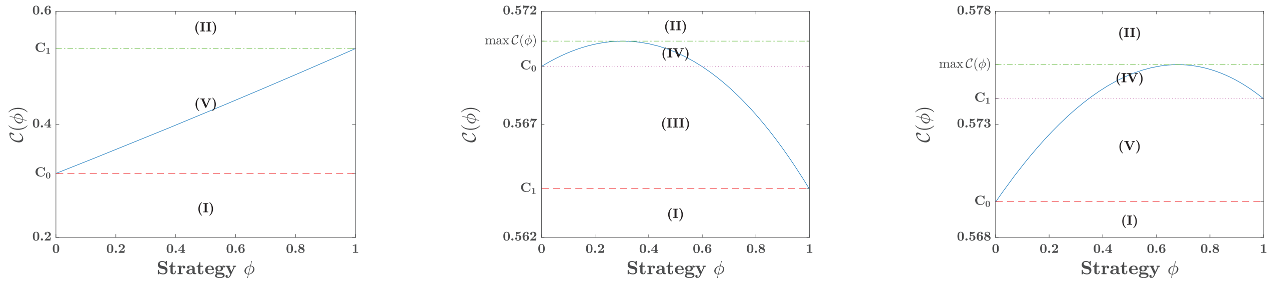
WLOG, we let $V_d = 1$, so that costs are normalized in terms of the cost of the system delay. As we assume steady state conditions, we may apply our parameters to well defined expressions for the system delay in multi-class preemptive-resume priority queues [20, p. 175]:

$$\begin{aligned} E[D_{pc}] &= \frac{1}{\mu\alpha} + \frac{K_{in}\rho_{in}/\mu_{in} + K\rho\phi/\mu}{2\alpha\beta}; \\ E[D_{gc}] &= \frac{1}{\mu\beta} + \frac{K_{in}\rho_{in}/\mu_{in} + K\rho/\mu}{2\beta\gamma}. \end{aligned} \quad (2)$$

Solving Equation (1) for C and applying the definitions in Equation (2), we may define a best response function $\mathcal{C}(\phi)$:

$$\mathcal{C}(\phi) = \frac{\rho\Big(K_{in}\mu\rho_{in} + 2\mu_{in}\phi\gamma + K\mu_{in}(\alpha - \phi\gamma)\Big)}{2\mu\mu_{in}\alpha\beta\gamma} + V_p\rho\phi. \quad (3)$$

As seen in the next section, the possible equilibria associated with a PAL tier access fee C will depend on the relation between C and the function $\mathcal{C}(\phi)$. Analyzing the function will in turn enables us to address the questions which were raised at the beginning of this subsection.



(a) Low variance $K = 1.3$, $V_p = 1.15$ (b) High variance $K = 3.9$, $V_p = 1.15$ (c) High variance $K = 3.9$, $V_p = 1.25$

Fig. 2: Example plots of $\mathcal{C}(\phi)$, labeled with the ranges of C for which specific equilibrium regimes are possible, as defined in Theorem 1. These examples feature incumbent queuing parameters of $\rho_{in} = 0.05$, $\mu_{in} = 0.033$ and $K_{in} = 1.855$; a customer traffic rate of $\rho = 0.1$ and service rate of $\mu = 0.628$. We vary K and V_p to demonstrate different possible regions. Regimes (I) and (II) are always possible if C is set low or high enough. In the low variance ($K = 1.3$) case, the monotone increasing nature of $\mathcal{C}(\phi)$ results in Regime (V) being the only other possibility. In the high variance ($K = 3.9$) case, the unimodal nature of $\mathcal{C}(\phi)$ results in a region where regime (IV) predominates, alongside either (III) or (V) depending on which of C_0 or C_1 is the minimum value of the function; this will be determined by V_p in conjunction with the other parameters.

IV. Equilibria Analysis

We now turn to the question which equilibrium states are possible for a given cost C. As the customer priority purchasing decision is a reaction to C and the current queue statistics, a revenue maximizing provider may use their knowledge of the possible equilibrium states to set the cost accordingly. We analyze the function $\mathcal{C}(\phi)$ to determine both the existence and stability of equilibria.

A. Equilibrium Existence

In determining the existence of equilibria states for a given cost C, we note the customers' action space is the binary decision of which class to join. As a consequence, there are three possible equilibrium types:

1) *All upgrade*: $\phi = 1$, i.e. all customers are priority;
2) *None upgrade*: $\phi = 0$, i.e. all customers are general;
3) *Some upgrade*: $\phi \in (0, 1)$ of customers are priority, the rest are general. Such values of ϕ are found through Equation (1) for fixed queuing parameters.

And based on the relationship between C and $\mathcal{C}(\phi)$, we may make high level assertions of the possible equilibria types:

Lemma 1: Let $\mathcal{C}(\phi)$ be as defined as in Equation (3) for fixed incumbent and customer queuing parameters, and further define $C_0 = \mathcal{C}(0)$ and $C_1 = \mathcal{C}(1)$. The following equilibria states are possible based on the value of C relative to the values of $\mathcal{C}(\phi)$:

1) If $\min \mathcal{C}(\phi) < C < \max \mathcal{C}(\phi)$, at least one *some upgrade* equilibrium is possible.
2) If $C < C_1$, a *all upgrade* equilibrium is possible.
3) If $C < \min \mathcal{C}(\phi)$, an *all upgrade* is the unique equilibrium state.
4) If $C > C_0$, a *none upgrade* equilibrium is possible.
5) If $C > \max \mathcal{C}(\phi)$, a *none upgrade* is the unique equilibrium state.

From the results of Lemma 1, we may derive the exact conditions for regions in which specific combinations of equilibria types exist from an analysis of $\mathcal{C}(\phi)$ as it was defined in Equation (3), given fixed parameters. We begin by defining the following quantities which determine the possible equilibrium regions. Specifically, these are boundary values on the value of preemption $\mathsf{V}_\mathsf{p}^{(\mathsf{L})}$,$\mathsf{V}_\mathsf{p}^{(\mathsf{M})}$, $\mathsf{V}_\mathsf{p}^{(\mathsf{H})}$, and a threshold ρ^T for the customer traffic load:

$$\mathsf{V}_\mathsf{p}^{(\mathsf{L})} = \frac{\mu_{in}\alpha(K(\gamma - \rho) - 2\gamma) - K_{in}\mu\rho\rho_{in}}{2\mu\mu_{in}\alpha^3\gamma}, \tag{4}$$

$$\mathsf{V}_\mathsf{p}^{(\mathsf{M})} = \frac{\mu_{in}\alpha(K(\gamma - \rho) - 2\gamma) - K_{in}\mu\rho\rho_{in}}{2\mu\mu_{in}\alpha^2\gamma^2}, \tag{5}$$

$$\mathsf{V}_\mathsf{p}^{(\mathsf{H})} = \frac{\mu_{in}\alpha(K(\gamma - \rho) - 2\gamma) - K_{in}\mu\rho\rho_{in}}{2\mu\mu_{in}\alpha\gamma^3}, \tag{6}$$

$$\rho^T = \frac{(K - 2)\mu_{in}\alpha^2}{2(K - 1)\mu_{in}\alpha + K_{in}\mu\rho_{in}}. \tag{7}$$

Theorem 1: Let $\mathcal{C}(\phi)$ be as defined in Equation (3); $\mathsf{V}_\mathsf{p}^{(\mathsf{L})}$, $\mathsf{V}_\mathsf{p}^{(\mathsf{M})}$, and $\mathsf{V}_\mathsf{p}^{(\mathsf{H})}$ as defined in Equations (4), (5), and (6); and ρ^T as defined in Equation (7). There are five possible equilibrium regimes which may occur, conditioned on the value of C set by the provider and the queuing parameters for each class as follows:

(I) If $C < \min \mathcal{C}(\phi)$, *all upgrade* is the sole equilibrium.

(II) If $C > \max \mathcal{C}(\phi)$, *none upgrade* is the sole equilibrium.

(III) If $K > 2$, $\rho < \rho^T$ AND
  a) $\min \mathcal{C}(\phi) < C < \max \mathcal{C}(\phi)$ and $V_p < \mathsf{V}_\mathsf{p}^{(\mathsf{L})}$, OR
  b) $\min \mathcal{C}(\phi) < C < C_0$ and $\mathsf{V}_\mathsf{p}^{(\mathsf{L})} < V_p < \mathsf{V}_\mathsf{p}^{(\mathsf{M})}$,

  a *some upgrade* is the sole equilibrium.

(IV) If $K > 2$, $\rho < \rho^T$, AND
  a) $C_0 < C < \max \mathcal{C}(\phi)$ and $\mathsf{V}_\mathsf{p}^{(\mathsf{L})} < V_p < \mathsf{V}_\mathsf{p}^{(\mathsf{M})}$, OR
  b) $C_1 < C < \max \mathcal{C}(\phi)$ and $\mathsf{V}_\mathsf{p}^{(\mathsf{M})} < V_p < \mathsf{V}_\mathsf{p}^{(\mathsf{H})}$,

  there are three equilibria: two *some upgrade* and one *none upgrade*

(V) Otherwise, there are three possible equilibria states: an *all upgrade*, a *none upgrade*, and a single *some upgrade*.

Proof: The existence of regimes (I) and (II) follow directly from Lemma 1. Therefore, assume $\min \mathcal{C}(\phi) < C < \max \mathcal{C}(\phi)$. We further note that the function $\mathcal{C}(\phi)$ will be continuous on the domain $\phi \in [0,1]$. To determine the behavior of the function, we may analyze the derivative $\mathcal{C}'(\phi)$:

$$\mathcal{C}'(\phi) = \frac{\rho\Big(K_{in}\mu\rho_{in}\rho + \mu_{in}\alpha(2\gamma - K(\gamma-\rho))\Big)}{2\mu_{in}\mu\alpha\beta^2\gamma} + V_p\rho. \quad (8)$$

Evaluating the sign of the derivative, we determine that there are three possible behaviors for the function:

1) Monotone decreasing if $K > 2$, $\rho < \rho^T$, and $V_p < \mathsf{V}_\mathsf{p}^{(\mathsf{L})}$.
2) Unimodal with unique maximum if $K > 2$, $\rho < \rho^T$, and $\mathsf{V}_\mathsf{p}^{(\mathsf{L})} < V_p < \mathsf{V}_\mathsf{p}^{(\mathsf{H})}$.
3) Monotone increasing otherwise.

Applying Lemma 1 again, we note that in the monotone decreasing case, $\min \mathcal{C}(\phi) = C_1$ and $\max \mathcal{C}(\phi) = C_0$, and there will be exactly one solution to $\mathcal{C}(\phi) = C$. Therefore, it follows from Lemma 1 that there is a single equilibrium possible, which will be a *some upgrade* type, i.e. regime (III). We can make a similar argument to show that one equilibrium of each type must be possible in the monotone increasing function case, satisfying regime (V).

However, if the function is unimodal, it transitions from increasing to decreasing at some $\phi^{max} \in (0,1)$. Thus, there are values of C for which multiple solutions to $\mathcal{C}(\phi) = C$ exist. The intervals for which this is true, and what other equilibria will be possible, will depend on whether $\min \mathcal{C}(\phi)$ is equal to C_0 or C_1. For $\min \mathcal{C}(\phi) < C < \max\{C_0, C_1\}$ only one solution to $\mathcal{C}(\phi) = C$ exists in the domain $\phi \in [0,1]$, thus exactly one *some upgrade* equilibrium will be possible. If $\min \mathcal{C}(\phi) = C_1$, then $C_1 < C < C_0$ implies neither pure equilibrium is possible, thus we have regime (III) where the *some upgrade* will be the only one possible; this is the case if $\mathsf{V}_\mathsf{p}^{(\mathsf{L})} < V_p < \mathsf{V}_\mathsf{p}^{(\mathsf{M})}$. If on the other hand $\min \mathcal{C}(\phi) = C_0$, then $C_0 < C < C_1$ implies one equilibrium of each type is possible (i.e. regime (V)), and this is the case if $\mathsf{V}_\mathsf{p}^{(\mathsf{M})} < V_p < \mathsf{V}_\mathsf{p}^{(\mathsf{H})}$.

If on the other hand $\max\{C_0, C_1\} < C < \max \mathcal{C}(\phi)$, then the unimodal nature of the function results in two solutions existing to $\mathcal{C}(\phi) = C$, thus two *some upgrade* equilibria are possible. However, because $C > C_0$ also holds in this case, the *none upgrade* equilibrium is also possible per Lemma 1, and these conditions therefore satisfy regime (IV). ■

In Figure 2, we provide visual demonstration of how the choices of parameters impact the behavior of $\mathcal{C}(\phi)$ and thus influence which mixed equilibrium regimes are possible; in contrast to regimes (I) and (II) which are always possible if C is set, respectfully, sufficiently low or high. The incumbent parameters are kept the same in all three cases, with the values used derived from measurement students of naval radars of the type that typically operate on the CBRS band [21].

Similarly, the choices of the parameters for the customers are derived from measurements of traffic in commercial cognitive radios [22], [23], and measurements of users in cellular networks on short time scales [24]. However, said measurements suggest a distribution such that $K = 1.3$. As $K > 2$ is a requirement for equilibrium regimes (III) or (IV) to also be possible, we consider cases where $K = 3.9$; by extension, we consider a customer traffic rate of $\rho = 0.1$ to satisfy the condition that ρ be below the resulting threshold for our parameters $\rho^T = 0.236$.

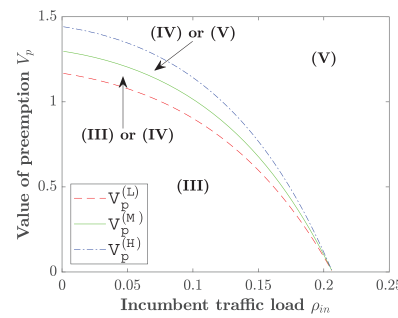


Fig. 3: Plot of the boundaries between possible equilibrium regimes when $\min \mathcal{C}(\phi) < C < \max \mathcal{C}(\phi)$, as the incumbent traffic load ρ_{in} and value on preemption V_p vary, with the remaining parameters as in Figures 2(b) and 2(c). We find that depending on the values of the parameters, we could have a equilibrium regime (III), (IV), or (V) occur. For any regime other than (V) to be possible however, we require the customers to have relatively low sensitivity to preemption (i.e. $V_p < 1.44$), or incumbent traffic load $\rho_{in} < 0.207$. Further, (IV) will never occur on its own, but is a possibility alongside either (III) or (V), depending on the value of C.

In the low variance $K = 1.3$ case, $\mathcal{C}(\phi)$ will always be monotone increasing. As seen in Figure 2(a), when $C_0 < C < C_1$, there is clearly exactly one solution to $\mathcal{C}(\phi) = C$, yielding the single *some upgrade* equilibrium. Thus, the only possible regime featuring a mixed equilibrium is a regime (V) - that the pure equilibria are also possible equilibria in this range follows directly from the earlier Lemma 1.

In the high variance $K = 3.9$ case, there are other possible equilibrium regions; which ones are possible will depend on the value of preemption V_p. As our choices of parameters yield $\mathsf{V}_\mathsf{p}^{(\mathsf{L})} = 1.078$, $\mathsf{V}_\mathsf{p}^{(\mathsf{M})} = 1.204$, and $\mathsf{V}_\mathsf{p}^{(\mathsf{H})} = 1.34$, we consider scenarios where $V_p = 1.15$ and $V_p = 1.25$. The first of these are plotted in Figure 2(b). In this case, if $C_1 < C < C_0$ the function is locally monotone decreasing, leading to an equilibrium regime (III). If however $C_0 < C < \max \mathcal{C}(\phi)$ there are two solutions and therefore two *some upgrade* equilibria possible. But again per Lemma 1, $C > C_0$ results in a *none upgrade* equilibrium also being possible, solidifying that this is a region where regime (IV) is in effect.

When $V_p = 1.25$, as is the case in Figure 2(c), the function is again unimodal but now with $C_0 < C_1$. Therefore, we have a similar situation to the previous figure, but instead of a region where regime (III) is in effect, we have a region where regime (V) is in effect when $C_0 < C < C_1$.

While we have examples where equilibrium regimes (III) and (IV) are possible, for this to be the case in general, the customers must not be highly sensitive to preemption, as represented by V_p in addition to the conditions on traffic load and service variance. As seen in the definitions in Equations (4)-(7), the thresholds on the value of preemption and the traffic load depend on the behavior of the incumbents. Therefore, the bounds are relative. In Figure 3, we consider how varying the incumbent traffic load ρ_{in} impacts the values of the boundaries between possible equilibria regions (assuming $\min \mathcal{C}(\phi) < C < \max \mathcal{C}(\phi)$) The remaining parameters are as in the high customer variance case from the previous example.

We find that in general, regime (V) is most likely to prevail. Specifically, we observe that once ρ_{in} becomes larger than 0.207, the threshold ρ^T falls below 0.1. Thus, even when the combined traffic load is approximately 30% of total capacity, regime (V) is the most likely equilibrium outcome. Similarly, whenever $V_p > 1.44$, regime (V) always prevails regardless of the current value of the incumbent traffic load. Recalling that the per-time-unit cost of delay V_d is normalized, a V_p value of 1.44 represents a cost which is 44 percent greater than that of the delay. However, while for practical purposes it appears unlikely to have a regime (III) or (IV) outcome, these possibilities must still be taken into account by the provider due to concerns over equilibrium stability as discussed below.

B. Equilibrium Stability

As noted in the introduction of this section, providers are interested in the *stability* of equilibria as well as their existence, as stable equilibria correspond to guaranteed revenue streams. Indeed, Theorem 1 states that there are conditions where three equilibria are possible for a given cost C. Thus, an equilibrium strategy need not be a *unique* best response to C. We adopt the definition of stability in the Evolutionary Stable Strategy (ESS) sense to capture stability behavior over time as customers enter a system with fixed parameters [25]:

Definition 1: An equilibrium strategy x is stable in the *Evolutionary Stable Strategy (ESS)* sense if no alternate equilibrium strategy x' is a better response against itself than x.

This definition is particularly relevant in a dynamic or evolutionary game scenario, where the game is repeated over multiple rounds. Even if equilibrium strategy x is the initial strategy chosen, if some subset of players choose the alternate best response strategy x' in subsequent rounds, if x is not ESS but x' is, then the latter will eventually become the equilibrium state adopted by the population as a whole [4, p.5]. While the concept of ESS equilibria originated with evolutionary biology, the concept has been widely studied in mathematics and economics to consider dynamic systems more generally [26]. Given Definition 1, we assert the following:

Theorem 2: Pure equilibria, i.e. *all upgrade* or *none upgrade* are always ESS stable. *Some upgrade* equilibria are ESS stable if and only if it is the sole equilibrium state.

Thus, while a situation where the *all upgrade* equilibrium leads to the maximal revenues to the provider will also correspond to those revenues being guaranteed because it is a stable equilibrium. However, if a *some upgrade* equilibrium corresponds to the maximum revenue, it is not obvious if that equilibrium is stable. We examine this further in Section V.

V. Revenue Analysis

We turn now to the question of the provider's decision to set C. The provider's goal is to maximize their revenues; while the system is unobservable the provider also has knowledge of the queue statistics. Therefore, the provider is able to exploit their knowledge of the nature of $\mathcal{C}(\phi)$ to make their decision. If the cost is set at some $C = \mathcal{C}(\phi)$, the expected revenue is C multiplied by the number of customers opting to choose PAL tier service. However, users arrive according to Poisson processes, thus there is not a predetermined fixed number of customers who will make the decision. Therefore, we instead consider maximization of revenue per time unit. As the arrival rate of priority customers is $\lambda\phi$, then the expected revenue per time unit $\mathcal{R}(\phi)$ equals $\lambda\phi\mathcal{C}(\phi)$:

$$\frac{\rho^2\phi\Big(K_{in}\mu\rho_{in} + 2\mu_{in}\phi\gamma + K\mu_{in}(\alpha - \phi\gamma)\Big)}{2\mu_{in}\alpha\beta\gamma} + V_p\rho\lambda\phi^2. \quad (9)$$

Due to this relationship between $\mathcal{C}(\phi)$ and $\mathcal{R}(\phi)$, we may both assert the cost which leads to the maximum revenue for fixed parameters, and whether the associated equilibrium state ϕ^* is ESS per Theorems 1 and 2. If ϕ^* is not ESS, there is the possibility that the customers will divert to an alternate equilibrium strategy, in particular the *none upgrade* strategy which is always possible in the situation where *some upgrade* equilibria are not ESS. However, despite the multiple possible equilibria regimes asserted in Theorem 1, we assert that the maximum revenue is always associated to a stable equilibrium:

Theorem 3: For any valid and fixed user traffic statistics, there exists a stable Nash Equilibrium resulting in revenue arbitrarily close to the maximum.

Proof: To show this, we note that the relationship between $\mathcal{C}(\phi)$ and $\mathcal{R}(\phi)$ results in $\mathcal{R}(\phi)$ being a continuous function, therefore we may evaluate the derivative to determine the behavior of $\mathcal{R}(\phi)$:

$$\begin{aligned}\mathcal{R}'(\phi) = \frac{\rho^2}{2\mu_{in}\alpha\beta^2\gamma}\Big(&K_{in}\mu\rho_{in}\alpha + 2\mu_{in}\phi\gamma(\alpha+\beta) + \\ &K\mu_{in}(\alpha^2 + \phi^2\rho\gamma - 2\phi\alpha\gamma)\Big) + 2V_p\rho\lambda\phi.\end{aligned} \quad (10)$$

Evaluating $\mathcal{R}'(\phi)$, we find that the function is either monotone increasing, or unimodal with a unique maximum at some $\phi^{opt} \in (0,1)$. For this to be the case, the following must be satisfied for any $K_{in} \geq 1$, $\mu_{in} > 0$, and $\mu > 0$:

$$\begin{aligned}&K > 4\\ &\rho_{in} < \frac{\mu_{in}(K-4)}{K_{in}\mu + \mu_{in}(K-4)}\\ &\rho < \frac{3\alpha}{2} - \frac{1}{2}\sqrt{\frac{\alpha(5K-2)\mu_{in}\alpha + 4K_{in}\mu\rho_{in}}{(K-2)\mu_{in}}}\\ &V_p \leq \frac{K\mu_{in}(\rho^2 - 3\rho\alpha + \alpha^2) - K_{in}\mu\alpha\rho_{in} - 2\mu_{in}\gamma(\alpha+\beta)}{4\mu\mu_{in}\alpha\gamma^3}\end{aligned} \quad (11)$$

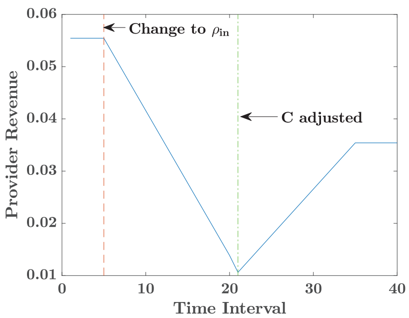


Fig. 4: Example of a phase transition impacting the provider revenue over the course of a dynamic game occurring over 40 time intervals. The customers have parameters $K = 3.9$, $\rho = 0.1$, $\mu = 0.628$, $V_p = 1.15$, while the incumbents have service distributed with $\mu_{in} = 0.033$ and $K_{in} = 1.855$, and initial traffic load $\rho_{in} = 0.1$. Initially the provider charges $C = 0.883$ to maximize their revenue. However, after 5 time intervals, the incumbent traffic drops to $\rho_{in} = 0.05$. The cost C is now larger than customers are willing to pay, thus customers progressively opt to remain in the GAA tier and revenues decrease. After another 15 time intervals, the provider reacts by readjusting the cost to the new revenue maximizing value of $C = 0.564$, which happens to the minimum value of the corresponding $\mathcal{C}(\phi)$ in this case. The provider eventually sees their revenues rebound, however the total collected in the maximizing case is lower than what was possible initially.

Comparing the conditions from Equation (11) to those for a lone *some upgrade* equilibrium being possible per Theorem 1, we find that ϕ^{opt} will indeed be the sole equilibrium possible whenever the conditions are satisfied. This equilibrium is stable per Theorem 2 and therefore the revenue is guaranteed.

If on the other hand $\mathcal{R}(\phi)$ is monotone increasing, then revenues are maximized when $C = C_1$. As this is a boundary case between equilibrium regions, in practical terms we assume the cost is instead set to $C = C_1 - \epsilon$ for some arbitrary epsilon. Per Theorem 1, depending on the value of the queuing parameters either then the *all upgrade* equilibrium is the only one possible, or there is a *some upgrade* ϕ^* associated with this C. ϕ^* will be an unstable equilibrium, but as it is arbitrarily close to the *all upgrade* equilibrium, customers will have greater incentive to switch to the *all upgrade* strategy than the alternative *none upgrade* one. And as the *all upgrade* strategy is ESS, the revenue is guaranteed less some ϵ. ■

This guarantee assumes that the traffic statistics are known exactly by the provider. The results potentially differ if the provider is still learning the statistics, or otherwise has some estimation error. These considerations could be investigated in future work.

A. Phase Transition Behavior

Thus, as shown above the provider is guaranteed their maximum revenue if they set C accordingly. However, this applies provided the queuing parameters remain constant. Clearly if the parameters do change, a change in strategy on the part of the provider will be necessary. And this includes a situation where the incumbent parameters change but the customer ones do not. However, a change in incumbent behavior can in fact prompt a phase change between equilibrium regions.

Consider a game that evolves over multiple epochs. The customers have queue parameters of $K = 3.9$, $\rho = 0.1$, $\mu = 0.628$, with a cost of preemption $V_p = 1.15$. The incumbents initially have a traffic load of $\rho_{in} = 0.1$, with service distributed such that $\mu_{in} = 0.033$ and $K_{in} = 1.855$ as in the earlier examples. The resulting $\mathcal{C}(\phi)$ is monotone increasing, thus the revenue is maximized when all customers pay the maximum amount possible, $C_1 = 0.883$. Suppose however that after 5 time intervals the incumbent traffic load drops to $\rho_{in} = 0.05$; this is now the example from Figure 2(b). Thus, the resulting $\mathcal{C}(\phi)$ is now unimodal. The revenue is still maximized when the cost equals $C_1 = 0.564$, however this is now the minimum value of the function. Moreover, the original cost is now in a region where regime (II) applies and *none upgrade* is the only possible equilibrium; as opposed to C_1 being a boundary between regions where regimes (I) and (III) prevail, and thus there is only a single stable equilibrium.

Suppose that 5% of customers re-evaluate their strategy each time interval. We plot the resulting revenue in Figure 4. Once the incumbent behavior changes, the revenue steadily decreases as customers realize they are now in a *none upgrade* region and thus opt out of upgrading from GAA to PAL tier service. Eventually, after a further 15 time intervals, the provider realizes what is happening and adjusts C to the new $C_1 = 0.564$ value. Customers begin rejoining the PAL tier as they now realize that they are better off when they all join the PAL tier in this situation, with the revenue eventually stabilizing, albeit at a lower level than was possible previously.

Thus, in order to ensure their revenue streams are unimpeded by all customers opting out of the PAL tier, providers must be alert to the traffic patterns of incumbents and customers and adjust accordingly. This is particularly true for incumbents since as just described, changes in the incumbent behavior can force phase transitions. Such changes could be due to natural variation of incumbent use or due to Primary User Emulation Attacks, wherein hostile users manage to impersonate the incumbents to gain control of the band while bypassing the provider entirely. From the provider's perspective, it is thus necessary to be constantly monitoring usage patterns in order to be resilient against such changes, alongside implementing a form of dynamic pricing so that C may be updated on short notice in the event of a sudden change.

VI. Conclusions

In this work, we developed a joint queuing-theoretic and game-theoretic model governing the interactions between the priority tiers of users and providers operating on the CBRS. In particular, we analyzed the impact of the cost of preemption V_p and the level of incumbent tier traffic ρ_{in} on the customer priority purchasing decision. We found that there are five possible equilibrium regimes that can arise based on the customer's sensitivity to preemption and the traffic load and service distribution of each user class. In particular, should V_p and ρ_{in} be sufficiently low, then it is possible to encounter equilibrium regimes where there are stable mixed equilibria, and others where multiple mixed equilibria are possible.

For most practical queue parameters, the equilibrium regime which prevails will be one where there are three possible equilibria states: a stable equilibrium where all customers join the PAL tier, a stable equilibrium where all customers join the GAA tier, and an unstable mixed equilibrium. Indeed, even if the combined traffic of all users is roughly 30% of total capacity, this is the regime that prevails regardless of how sensitive the customers are to preemption. The same holds true if the cost of preemption is 44% higher than the per-unit cost of the delay of service.

Regardless of which regime prevails however, the provider's maximum possible revenue will be guaranteed for fixed parameters, as we show that regardless of the values of the parameters, this quantity will always be associated with a stable equilibrium state. Should the incumbent behavior change, the provider is still assured of the maximum possible revenue so long as they react accordingly to adjust the fee to upgrade to the PAL tier. However, the incumbent behavior can induce a phase change between equilibrium regions, resulting in a drastic reduction in revenue compared to what was achievable before the behavior change.

Future work includes modeling how deadlines for service completion impact the notion of the cost of preemption. In addition, one could consider a PAL provider with access to multiple channels and the impact of this extra capacity on the customers' willingness to pay access to the PAL tier.

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