A Hierarchical Green Mean-Field Power Control with eMBB-mMTC Coexistence in Ultradense 5G

(Invited Paper)

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***Abstract*—Small cell densification is recognized as one of the most significant characteristics in the fifth-generation of communication systems (5G) and beyond. A substantial capacity boost can be achieved at a low cost by supplementing macro networks with numerous small cells to create ultra-dense heterogeneous networks, which can serve as the foundation for the next generation of services. In this paper, we investigate a model that accounts for the location and channel quality of an enhanced Mobile Broadband (eMBB) user as well as the locations, density, and energy levels of a large number of Internet of Things (IoT) devices. More specifically, the eMBB user is randomly distributed in the coverage area of the MBS, and given its channel gain, it adjusts its transmit power to achieve an acceptable Quality of Service (QoS). In contrast, the IoT devices are gathered around SBS and regulate their transmission power in accordance with their energy budget to minimize energy-efficient utility function. Due to the coupling, the Stackelberg-Nash differential game is initially used to model the power allocation problem, with the eMBB user playing the role of the leader and the IoT devices playing the role of the followers. Then, we use the mean-field approximation to construct a hierarchical mean-field game from which we can recover a set of equations that may be solved iteratively to provide the optimal power allocation strategies. Simulation results illustrate the optimal power allocation strategies and show the effectiveness of the proposed approach.**

***Index Terms*—5G and Beyond; Internet of Things (IoT); Power Allocation; Stackelberg Game; Mean-Field Equilibrium.**

I. Introduction

In a recent report, Cisco predicts that the 5G network will have to deal with up to 13.1 billion mobile users by 2023. Furthermore, the number of Internet-enabled devices that should be handled after integrating the Internet of Things (IoT) into 5G will have increased from 18.4 billion in 2018 to 29.3 billion by 2023 [1]. Since this increasing trend doesn't seem to be slowing down, the massive access challenge is one of the most important targets for the sixth-generation of communication systems (6G). This massive access challenge has raised the need to rethink the PHY and the MAC layers in order to intelligently share the available spectrum between enhanced Mobile Broadband (eMBB) and massive Machine Type Communications (mMTC). Indeed, network densification is recognized as one of the most significant characteristics in 5G and future generations [2], [3]. Note that ultra-dense networks (UDNs) enable us to achieve a powerful capacity enhancement with a relatively low cost. In fact, Small Base Stations (SBSs) are densely deployed under the coverage of a traditional Macro Base Station (MBS) to share the frequency resources and enhance the spectrum efficiency. Indeed, UDN can be seen as a key technology for 6G since it enables us to handle high density, increase network capacity, improve the coverage and the quality of service (QoS) [4].

A. Related Work

In [9] , the authors proposed a resource allocation technique that maximizes the energy efficiency in UDNs. Authors of [5] proposed a three-stage joint clustering and resource allocation in user-centric UDNs in order to maximize the sum rate. In [8] , L. Sun et al. proposed a CSI compression mechanism to reduce the upload overhead and improve the spectral efficiency in cell-free UDNs. Authors of [6] and [7] proposed a cell-free mMIMO operating in mmWave for UDNs and investigated the effect of shadowing correlation and beamforming techniques.

Game theory is an interesting mathematical tool to model the interaction between the network's components and derive distributed algorithms. Particularly, resource allocation has been deeply investigated using game theory, especially for densely deployed networks [13], [15], [16], [17]. When the network gets denser, it is more interesting for a player to interact with the collective behavior of its opponents rather than being concerned with the specific individual strategy of each player in the game. Hence, Mean-Field Game (MFG) theory has increasingly gained attention in ultra-dense 5G networks. It is worth noting that MFG simplifies the resolution of game by reducing the mathematical complexity to a two-body complexity. In fact, MFG is mainly based on the coupled equations Hamilton-Jacobi-Bellman (HJB) and Fokker-Planck-Kolmogorov (FPK), instead of considering all the one-to-one interactions. The former equation characterizes the interactions between the players and the mean-field, enabling them to make decisions, whereas the latter equation rules the evolution of the mean field according to the players' decisions. Afterward, the mean field equilibrium (MFE) is obtained by iteratively solving these coupled equations. For example, authors of [10] proposed a Mean-Field Deep Deterministic Policy Gradient algorithm to solve the resource allocation problem for NOMA-MEC in an ultra-dense network. They illustrated through numerical results that the proposed algorithm efficiently reduces both the energy consumption and the delay of users. Zhang et al. have proposed

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in [11] a power control algorithm based on Mean Field Game to mitigate the interference in UDNs taking into account the SINR requirement, and proved that they were able to improve both energy efficiency and spectrum efficiency.

B. Our Contributions

In this paper, we consider a general model that takes into account the eMBB user's location and channel quality, and also the IoT devices' locations, density, and energy levels. More precisely, the eMBB user is randomly distributed in the coverage area of the MBS, and given its channel gain, it adjusts its transmit power to obtain a satisfactory Quality of Service (QoS). The IoT devices, on the other hand, are grouped around SBS using a general cluster process, and given their energy budget, they adapt their transmit power to optimize an energy-efficient utility function over a period of time T. The power allocation problem is initially treated as a Stackelberg-Nash differential game with the eMBB user acting as the leader and the IoT devices serving as the followers due to the coupling of interference. Then, using stochastic geometry analysis and mean-field approximation, we construct a hierarchical mean-field optimal control problem to offer distributed power allocation strategies for both the eMBB user and the IoT devices.

The contributions of this paper can be summarized as follows:

- We construct a Stackelberg-Nash differential game model that incorporates the location and channel gain dynamic of the eMBB user as well as the density, locations, and energy budget dynamics of the IoT devices;
- We present a hierarchical mean-field power allocation scheme by leveraging stochastic geometry analysis and mean-field theory. By implementing a proper estimating procedure to know and update the channel statistics of the eMBB user, our approach enables each IoT device to determine its optimal power allocation strategy based entirely on the initial energy state distribution (mean-field) and its own energy budget. The eMBB user, on the other hand, adjusts its transmit power dependent on its channel gain;
- We propose an iterative algorithm to solve the hierarchical mean-field power allocation problem in an ultra-dense network;

The rest of the paper is organized as follows: The spatial model, the transmission model, and the Stackelberg-Nash differential game model are all presented in Section II. In Section III, the basic structure of the mean-field framework is laid out. The numerical and simulation findings are presented in Section IV. Finally, Section V concludes the paper.

II. System Model

In this paper, we analyze a MBS that uses a wide-bandwidth B_w to connect to an eMBB user and multiple SBSs that utilize L orthogonal channels, denoted by $\mathbf{L} = \{\ell_i, i = 1, \dots, L\}$, with narrow-bandwidths of length $\frac{B_w}{L}$ to provide connectivity for a large number of IoT devices that operate mostly in the uplink and belong to a certain closed subscriber group. As an illustration, consider a NB-IoT system with a total bandwidth of 180 kHz using the single-channel configuration (either the 3.75 or 15 kHz channel bandwidth) [13]. Additionally, we assume synchronous deployment in all small cells using the same channels. It means that the bandwidth utilized for communication is the same across the entire network.

A. Spatial Model

In the proposed framework, we consider a single MBS at location $\mathbf{m}$. The spatial domain of our analysis is denoted by $\mathcal{D}_m$, which is the coverage area of the MBS, i.e., the disk with center $\mathbf{m}$ and radius R_m. We also consider an ultra-dense small cell cellular network in which the SBSs are randomly distributed in the compact set $\mathcal{D}_m$ according to a homogeneous Poisson Point Process (PPP) denoted by $\phi_s = \{\mathbf{s}_i, i = 1, 2, \dots\}$ with intensity λ_s where $\mathbf{s}_i$ is the location of the $i-th$ SBS. Active IoT devices transmit simultaneously using a cluster-based setting, where the cluster centers are the SBSs. The number of active IoT devices associated with a specific SBS at the location $\mathbf{s} \in \phi_s$, defined as N_s, is a Poisson random variable with mean intensity λ_a.

Let $\mathcal{D}_s$ be the coverage area (disk of center $\mathbf{s}$ and radius R_s) of the SBS at the location $\mathbf{s}$. We denote by $\phi_{x,s} = \{\mathbf{x}_{i,s}, i = 1, 2, \dots\}$ the point process of active IoT devices associated to SBS at the location $\mathbf{s}$ whose locations are independently and identically distributed with a probability density function $f(\cdot|\mathbf{s})$. Moreover, by using the polar coordinate system, we write

$$f(\mathbf{x}|\mathbf{s}) = \frac{f_r(||\ \mathbf{x}-\mathbf{s}\ ||)}{2\pi\ ||\ \mathbf{x}-\mathbf{s}\ ||}, \quad \mathbf{x} \in \mathcal{D}_s/\{\mathbf{s}\}, \tag{1}$$

where $||\cdot||$ denotes the euclidean distance and $f_r(\cdot)$ is a probability density function on $]0, R_s]$.

In this work, the distance r between an arbitrary active IoT device at the location $\mathbf{x}$ and its serving SBS at the location $\mathbf{s}$, given as $r = ||\ \mathbf{x} - \mathbf{s}\ ||$, follows a general probability density function, such that $\frac{r}{R_s}$ follows a Beta(a, b) distribution, expressed as:

$$f_r(r) = \frac{1}{R_s} \frac{(\frac{r}{R_s})^{a-1}(1-\frac{r}{R_s})^{b-1}}{\int_0^1 u^{a-1}(1-u)^{b-1}du}, \quad 0 < r \le R_s. \tag{2}$$

This density allows us to implement different cases of IoT device distribution per small cell, based on the parameters a and b :

• If $a > 1$ and $b > 1$, the IoT devices are away from both the SBS and the small cell edges.

• If $a \ge 1$, $0 < b \le 1$ and $(a, b) \neq (1, 1)$, the IoT devices are away from the SBS and are around the small cell edges. In addition, if $(a, b) = (2, 1)$, then, in this case, our cluster process is a Matern cluster process.

• If $0 < a \le 1$, $b \ge 1$ and $(a, b) \neq (1, 1)$, the IoT devices are around the SBS and are away from the small cell edges.

• If $0 < a < 1$ and $0 < b < 1$, the IoT devices are around both the SBS and the small cell edges.

• If $a = 1$ and $b = 1$, the IoT devices are distributed uniformly in the circle of radius R_s.

It is worth mentioning that the resulting cluster process $\phi_x =$

$\bigcup_{\mathbf{s}\in\phi_s}\phi_{x,s} = \{\mathbf{x}_i, i = 1, 2, \dots\}$ is a stationary point process with intensity $\lambda_s\lambda_a$. Moreover, the number of active IoT devices (resp. the number of active IoT that attempts an uplink transmissions using a specific channel $\ell_i \in \mathbf{L}$) in $\mathcal{D}_m$, denoted as $\mathcal{N}_a$ (resp. $\overline{\mathcal{N}_a}$), is a Poisson random variable with mean intensity $\lambda_s\lambda_a\pi R_m^2$ (resp. $\frac{1}{L}\lambda_s\lambda_a\pi R_m^2$).
In the other hand, we consider a single eMBB user at random location $\mathbf{y}$ distributed in $\mathcal{D}_m$ with a probability density function given as

$$g(\mathbf{y}|\mathbf{m}) = \frac{g_r(||\ \mathbf{y}-\mathbf{m}\ ||)}{2\pi\ ||\ \mathbf{y}-\mathbf{m}\ ||}, \quad \mathbf{y}\in\mathcal{D}_m/\{\mathbf{m}\}, \tag{3}$$

where $g_r(\cdot)$ is a probability density function on $]0, R_m]$ derived as a special case of (2) by selecting $a = 2$ and $b = 1$, given by

$$g_r(r) = \frac{2r}{R_m^2}, \quad 0 < r \leq R_m. \tag{4}$$

The distance between a SBS at the location $\mathbf{s}$ and the eMBB user is denoted by $r_u =||\ \mathbf{s}-\mathbf{y}\ ||$. Conditioning on the distance between the MBS at the location $\mathbf{m}$ and the SBS at the location $\mathbf{s}$, given as $r_m =||\ \mathbf{m}-\mathbf{s}\ ||$, the probability density function of r_u is given by [14]:

$$f_{r_u}(r|r_m) = \begin{cases} \dfrac{\psi_1(r)}{\pi R_m^2}, & 0 \leq r \leq R_m - r_m, \\ \dfrac{\psi_2(r)}{\pi R_m^2}, & R_m - r_m < r \leq r, R_m + r_m, \end{cases} \tag{5}$$

where

$$\psi_1(r) = 2\pi r \tag{6}$$

$$\psi_2(r) = 2r\cos^{-1}\left(\frac{r_m^2 - R_m^2 + r^2}{2r_m r}\right), \tag{7}$$

B. Transmission Model

The outcome of a transmission is assessed through the received time varying Signal to Interference-plus-Noise Ratio (SINR).

IoT device i (follower):
Without loss of generality, the experienced SINR of the IoT device i at time t, whose serving SBS is at location $\mathbf{s}$, writes:

$$\gamma_i(t,p_i(t), \mathbf{p}_{-i}(t), p_u(t)) = \frac{p_i(t)h_{i,i}(t)\ ||\ \mathbf{x}_i - \mathbf{s}\ ||^{-\alpha}}{\sigma\frac{B_w}{L} + I_{i,u}(t,p_u(t)) + I_i(t,\mathbf{p}_{-i}(t))}, \tag{8}$$

where in the above expression, $p_i(t) \in [0, P_{max}]$ is the transmit power of active IoT device i, $p_u(t) \in [0, P_u]$ is the transmit power of eMBB user , $\mathbf{p}_{-i}$ denotes the transmit power vector of the active IoT devices using the same channel without i, $h_{i,j}(t)$ is a parameter representing the multipath fading between the device j and base station serving the device i, σ is the noise power spectral density, $I_{i,u}(t, p_u(t))$ denotes the interference caused by eMBB user given by

$$I_{i,u}(t, p_u(t)) = p_u(t)h_{i,u}(t)\ ||\ \mathbf{s}-\mathbf{y}\ ||^{-\alpha}, \tag{9}$$

and $I_i(t, \mathbf{p}_{-i}(t))$ denotes the interference caused by the active IoT devices using the same channel for transmission expressed as:

$$I_i(t, \mathbf{p}_{-i}(t)) = \sum_{j=1, j\neq i}^{|\overline{\mathcal{N}_a}|} p_j(t)h_{i,j}(t)\ ||\ \mathbf{x}_j - \mathbf{s}\ ||^{-\alpha}\ . \tag{10}$$

eMBB user u (Leader):
The experienced SINR of eMBB user at time t writes:

$$\gamma_u(t, p_u(t), \mathbf{p}(t)) = \frac{p_u(t)H_u(t)}{\sigma B_w + I(t, \mathbf{p}(t))}, \tag{11}$$

where $H_u(t)$ is a channel gain between the eMBB user and the MBS, $\mathbf{p}(t)$ denotes the transmit power vector of the active IoT devices, and $I(t, \mathbf{p}(t))$ denotes the interference caused by all the active IoT devices expressed as:

$$I(t, \mathbf{p}(t)) = \sum_{j=1}^{|\mathcal{N}_a|} p_j(t)h_{u,j}(t)\ ||\ \mathbf{x}_j - \mathbf{m}\ ||^{-\alpha}\ . \tag{12}$$

Finally, we suppose that the channel between all the transmitters and all the receivers experiences an independent Rayleigh fading h, exponentially distributed with unity mean, and a path-loss exponent $\alpha > 2$.

C. Stackelberg-Nash Differential Game Model

We consider a dynamic Stackelberg-Nash differential game involving a leader and a large number of followers making decisions over a finite horizon T. To learn more about the feedback Stackelberg equilibrium of a stochastic differential game, we advise the reader to [12]

• **Player sets:**
$\mathcal{N} = \{u, \mathcal{N}_a\}$ where u denotes the eMBB user (the leader) and $\mathcal{N}_a = \{1, 2, \dots, |\mathcal{N}_a|\}$ denotes the set of active IoT devices (the followers).

• **State:**
- The leader : The state of the eMBB user at time t is described by her experienced channel gain $H_u(t) \in [H_{min}, H_{max}]$ evolving according to the following dynamic:

$$dH_u(t) = \frac{1}{2}(\tau - H_u(t)) + \sqrt{2\nu}d\mathbb{W}(t), \quad H_u(0) = H_0, \tag{13}$$

where H_0 is a random initial condition with prescribed probability density function $m_{u,0}$, τ and ν are constants coefficients linked to the channel statistics, and the infinitesimal of the Wiener process $\mathbb{W}(t)$ is introduced as the uncertainty term. In practice, proper estimating procedures can be implemented if those coefficients (τ and ν) must be known and updated [15].
- The followers : The state of an active IoT device i at time t is described by its remaining energy budget, given by $E_i(t) \in [0, E_{i,0}]$, evolving according to the following differential equation:

$$dE_i(t) = -p_i(t)\, dt, \quad E_i(0) = E_{i,0}, \tag{14}$$

where $E_{i,0}$ is the energy budget fixed by the active IoT device i to spend during the transmission. The dynamics (14) implies that the variation in the energy budget during dt is proportional to the transmission power.

• **Actions:**

- The leader : Transmit power of the eMBB user $p_u(t) \in [0, P_u]$.

- The followers : Transmit power of the active IoT devices $p_i(t) \in [0, P_{max}]$ for all $i \in \mathcal{N}_a$.

A power allocation strategy of the active IoT device i (resp. eMBB user) will be denoted by p_i (resp. p_u).

• **Utility function:**

- The leader : The eMBB user is assumed to have a large energy budget and aims at achieving a satisfactory Quality of Service (QoS). Namely, the eMBB user must have at any time t:

$$\gamma_u(t, p_u(t), \mathbf{p}(t)) \geq \gamma_u^{th}. \tag{15}$$

To satisfy this SINR requirement, we consider a satisfactory average utility function expressed as:

$$U_u(p_u, \mathbf{p}) = \mathbb{E}\left[\int_0^T F_u(t, p_u(t), \mathbf{p}(t))\, dt\right], \tag{16}$$

where

$$F_u := (\gamma_u - \gamma_u^{th})^2. \tag{17}$$

- The followers : The goal of each IoT device is to adapts its action according to its energy level while maximizing its throughput. Thus, the average utility function of an active IoT device i is given by:

$$U_i(p_i, \mathbf{p}_{-i}, p_u) = \mathbb{E}\left[\int_0^T F_i(t, p_i(t), \mathbf{p}_{-i}(t), p_u(t))\, dt\right], \tag{18}$$

where

$$F_i := -\log_2(1 + \gamma_i) + c\, p_i. \tag{19}$$

where $c \geq 0$ is the pricing coefficient per transmit power unit. Such a utility function is especially relevant when the IoT devices seek to meet a certain trade-off between achieving the highest possible throughput and expending as little power as necessary.

• **Stackelberg-Nash equilibrium:**

We consider a continuous-time feedback Stackelberg-Nash game, where at time t, the eMBB user (leader) takes an action first based on its state and states of all active IoT devices. Then, after observing the leader's action at time t, the active IoT devices (followers) make their instantaneous actions. More precisely, the solution procedure requires the leader to first anticipate the followers' best response to its announced policy. The anticipation is derived from analyzing the followers' Nash equilibrium given the leader's optimal power allocation strategy. We then substitute the followers' strategies into the leader's problem and solve for the leader's optimal decisions. Therefore, A set of strategies $\left(p_u^*, \mathbf{p}^* = (p_1^*, p_2^*, \cdots, p_{|\mathcal{N}_a|}^*)\right)$ is called a feedback Stackelberg-Nash equilibrium if the following holds:

1- The eMBB user, anticipating the IoT devices optimal power allocation strategies $\mathbf{p}*$, obtain its optimal power allocation strategy p_u^*, by solving the following optimal control problem:

$$\inf_{p_u} U_u(p_u, \mathbf{p}^*). \tag{20}$$

2- Given the optimal power allocation strategy p_u^* of the eMBB user, a power allocation strategy profile $\mathbf{p}^* = (p_1^*, p_2^*, \ldots, p_{|\mathcal{N}_a|}^*)$ is a feedback Nash equilibrium of the IoT devices' dynamic differential game if and only if $\forall i, p_i^*$ is a solution of the following optimal control problem:

$$\inf_{p_i} U_i(p_i, \mathbf{p}_{-i}^*, p_u^*). \tag{21}$$

To obtain the optimal power allocation strategies, the standard solution concept consists of analyzing the Stackelberg-Nash equilibrium. However, the complexity of this approach increases with the number of active IoT devices. Furthermore, it necessitates that each player (leader or follower) be fully aware of the states and actions of all other players, resulting in a tremendous volume of information flow. This is not feasible and impractical for an ultra-dense network. Nevertheless, since the effect of active IoT devices on a single player's average utility function is only via interference, it is intuitive that, as the number of followers increases, a single follower has a negligible effect. Thus, we suggest using a mean-field limit for this game to convert these multiple interactions (interference) into a single aggregate interaction known as mean-field interference.

III. Mean-Field Stackelberg-Nash Power Control

A. *Mean-field Stackelberg-Nash Regime*

The general setting of a mean-field Stackelberg-Nash regime is based on the following assumptions:

- The existence of large number of active IoT devices (followers).

- Interchangeability: the permutation of the state among the followers would not affect the optimal power allocation strategies. To guarantee this property, we assume that each follower i only knows its individual state and implements a homogeneous transmit power $p_i(t) = p(t, E_i(t))$.

- The eMBB user (leader) only knows its individual state and implements a homogeneous transmit power $p_u(t) = p_u(t, H_u(t))$.

- Finite mean-field interference.

Let $[0, E_{max}]$ be the energy domain of our analysis. We define the empirical energy state distribution of the followers in $\mathcal{D}_m$ at time t in $[0, T]$ as:

$$M(t, e) = \frac{1}{|\mathcal{N}_a|} \sum_{i=1}^{|\mathcal{N}_a|} \delta_e(E_i(t)), \quad \forall e \in [0, E_{max}], \tag{22}$$

where δ_e is the Dirac measure.

The basic idea behind the mean-field regime is to approximate a finite population with an infinite one, where the empirical energy state distribution $M(t, e)$ almost surely converges to a probability density function denoted as $m(t, e)$. We will refer to the energy state distribution of the followers $\left(m(t, .)\right)_{t \geq 0}$ as the mean-field. Additionally, as the followers become essentially indistinguishable, we can focus on a generic follower by dropping his index i where his individual dynamic is written as:

$$dE(t) = -p(t, E)\, dt, \quad E(0) = E_0. \tag{23}$$

Thus, the evolution of the mean-field $(m_t)_{t\geq 0}$over time $t$in $[0,T]$ is described by a first-order partial differential equation, known as Fokker-Planck Kolmogorov (FPK) equation, given by [13]:

$$\begin{cases}\partial_t m(t,e) - \partial_e(pm)(t,e) = 0,\\ m(0,.) = m_0.\end{cases} \tag{24}$$

Furthermore, we designate by $m_u(t,h)$ the probability density function of the leader's channel state variable H_u evolving over time t in $[0,T]$ as follows [15]:

$$\begin{cases}\partial_t m_u(t,h) - \frac{1}{2}\partial_h\big(m_u(\tau-h)\big)(t,h) = \nu\partial_{h^2} m_u(t,h),\\ m_u(0,.) = m_{u,0}.\end{cases} \tag{25}$$

Now, we turn our attention to determining the interference, SINR, and utility function, which are solely dependent on the players' (leader and generic follower) individual transmit powers, the probability density function m_u, and the mean-field.
The new parameters of the game are defined as:
1- Mean-field interference: By following a stochastic geometry-based approach, the mean-field interference caused by the followers at the MBS is given by:

$$\begin{aligned} I_{mf}(t) &= \mathbb{E}\big[I(t,\mathbf{p}(t))\big]\\ &= 2\pi\lambda_s\lambda_a\left[\frac{1}{2} + \frac{1-R_m^{2-\alpha}}{\alpha-2}\right]P(t),\end{aligned} \tag{26}$$

where $P(t) = \int_0^{E_{max}} p(t,e)m(t,e)\,de$.
For a better understanding, we direct the reader to [13], [16], [17].
Additionally, the mean-field interference caused by the followers using the same channel for transmission at a generic SBS is given by:

$$\begin{aligned} \overline{I}_{mf}(t) &= \mathbb{E}\big[I_i(t,\mathbf{p}_{-i}(t))\big]\\ &= \frac{2\pi}{L}\lambda_s\lambda_a\left[\frac{R_{min}^{2-\alpha} - R_m^{2-\alpha}}{\alpha-2}\right]P(t),\end{aligned} \tag{27}$$

where $R_{min} = \max\left(R_{safe}, \frac{1}{2\sqrt{\lambda_s}} - R_s\right)$ and R_{safe} represents the minimal distance between a SBS and the nearest interfering IoT device.
Finally, based on (5), the mean-field interference caused by the leader at a generic SBS writes:

$$\begin{aligned} I_u(t) &= \mathbb{E}\big[I_{i,u}(t,\mathbf{p}(t))\big]\\ &= P_u(t)\frac{1}{R_m}\int_0^{R_m}\left[\int_{[r_2-R_m]^+}^{r_2+R_m} r_1^{-\alpha} f_{r_u}(r_1|r_2)dr_1\right]dr_2,\end{aligned} \tag{28}$$

where $P_u(t) = \int_{H_{min}}^{H_{max}} p_u(t,h)m_u(t,h)dh$ and $[r_2-R_m]^+ = \max(0, r_2-R_m)$.

2- Mean-field SINR: we define the mean-field SINR for the leader as:

$$\gamma_{u,mf}\big(p_u(t,H_u), I_{mf}(t)\big) = \frac{p_u(t,H_u)H_u}{\sigma B_w + I_{mf}(t)}. \tag{29}$$

On the other hand, by using (2), the mean-field SINR for a generic follower is given by:

$$\gamma_{mf}\big(p(t,E), I_u(t), \overline{I}_{mf}(t)\big) = \frac{p(t,E)(\frac{a}{a+b}R_s)^{-\alpha}}{\sigma\frac{B_w}{L} + I_u(t) + \overline{I}_{mf}(t)}. \tag{30}$$

3- Mean-field utility function: The mean-field utility function of the leader is generalized as follows:

$$U_{u,mf}(p_u, m_u, I_{mf}) = \int_{H_{min}}^{H_{max}}\int_0^T F_{u,mf}\big(p_u(t,h), I_{mf}(t)\big)m_u(t,h)\,dt\,dh, \tag{31}$$

where

$$F_{u,mf} := (\gamma_{u,mf} - \gamma_u^{th})^2. \tag{32}$$

Moreover, The mean-field utility function for a generic follower is is generalized as follows:

$$U_{mf}(p,m,I_u,\overline{I}_{mf}) = \int_0^{E_{max}}\int_0^T F_{mf}\big(p(t,e), I_u, \overline{I}_{mf}(t)\big)m(t,e)\,dt\,de, \tag{33}$$

where

$$F_{mf} := -\log_2(1+\gamma_{mf}) + c\,p. \tag{34}$$

B. Mean-Field Stackelberg-Nash Optimal Control

Optimal control of the leader :
The optimal control problem of the leader is derived based on (20) and consists in finding the optimal power allocation strategy p_u^* satisfying:

$$\inf_{p_u} U_{u,mf}(p_u, m_u, I_{mf}^*). \tag{35}$$

where I_{mf}^* is the mean-field interference caused by the followers at the equilibrium, by assuming that the followers use their optimal power allocation strategies, and m_u is a solution of

$$\begin{cases}\partial_t m_u(t,h) - \frac{1}{2}\partial_h\big(m_u(\tau-h)\big)(t,h) = \nu\partial_{h^2} m_u(t,h),\\ m_u(0,.) = m_{u,0}.\end{cases} \tag{36}$$

The optimization problem (35) under the partial differential equation constraint (36) has a closed-form solution expressed as:

$$p_u^*(t,h) = \frac{\sigma B_w + I_{mf}^*(t)}{h}\gamma_u^{th}. \tag{37}$$

Optimal control of a generic follower :
The mean-field optimal control problem of a generic follower is derived based on (21) and consists in finding p^* and m^* satisfying:

$$\inf_p U_{mf}(p, m, I_u^*, \overline{I}_{mf}^*), \tag{38}$$

Algorithm 1: Follower's MF-Nash Power Control

Initialization:

1 Generate random vector p_0;
2 Estimate τ and ν channel statistics of the leader ;
3 Find m_u using (36) with initial condition $m_{u,0}$;

Learning pattern:

4 Find m using (39) with initial condition m_0 ;
5 Estimate interference $\overline{I}_{mf}$ using (27);
6 Estimate leader interference I_u using (28) and (37) ;
7 Find μ using (42) with final condition (43);
8 Update transmit power p using (41);
9 Repeat until convergence : go to step 4;

where I_u^* and $\overline{I}_{mf}^*$ are the mean-field interferences at the equilibrium, by assuming that the leader and the interfering followers use their optimal power allocation strategies, and m is a solution of

$$\begin{cases} \partial_t m(t,e) - \partial_e (pm)(t,e) = 0, \\ m(0,.) = m_0. \end{cases} \tag{39}$$

Since there is no closed-form solution to the optimization problem (38) under the partial differential equation constraint (39), the mean-field optimal control problem of the followers is solved iteratively until the convergence point is reached.

First, we derive the first-order optimality conditions using the adjoint method. We refer the interested reader to [18] for a rigorous derivation of these first-order optimality conditions. Let's start by defining the Lagrangian of the minimization problem (38) under the constraint (39) as

$$\mathcal{L}(p,m,\mu) = \int_0^{E_{max}} \int_0^{T} \Big(F_{mf}(p, I_u^*, \overline{I}_{mf}^*) m - \mu\big(\partial_t m - \partial_e (pm)\big) \Big)\, dt\, de, \tag{40}$$

where we omit the dependency on t and e and $\big(\mu(t,e)\big)_{\forall t,e}$ represents the Lagrange multipliers.
Let (p^*, m^*, μ^*) be the optimal solution. By using the integration by parts, the optimality conditions are expressed as (39) together with:

$$\partial_p F_{mf}(p^*, I_u^*, \overline{I}_{mf}^*) - \partial_e \mu^* = 0, \tag{41}$$

$$\partial_t \mu^* - p^* \partial_e \mu^* + F_{mf}(p^*, I_u^*, \overline{I}_{mf}^*) = 0, \tag{42}$$

$$\mu^*(T,.) = 0. \tag{43}$$

Note that the equation (42) reflects the adjoint equation of (39), popularly known as the Hamilton-Jacobi-Bellman (HJB) equation in mean-field game theory.
Then, we use a successive sweep method consisting of generating a sequence of nominal solutions $p_0, p_1, \ldots, p_k, \ldots$ that converges to the optimal power allocation strategy of the followers p^*. This iterative approach is summarized in **Algorithm 1** in our context. We refer the reader to [19] for a review of several aspects of numerical approaches for mean-field optimal control problems.

Parameter	Values	Description
R_m	2 km	Coverage area of MBS
R_s	20 m	Coverage area of SBS
λ_s	10^{-3}, 10^{-4} SBS / m^2	SBS density
λ_a	30	Average number of active IoT devices per SBS
a	2	Parameters of the beta distribution
b	4	
B_w	180 kHz	Total bandwidth
L	48	Number of channels
c	60	Pricing coefficient
P_{max}	0.025 W	Maximal transmit power of the IoT device
P_u	0.2 W	Maximal transmit power of the eMBB user
T	4 s	Time horizon
E_{max}	100 mJ	Maximum energy budget
H_{min}	10^{-7}	Channel state domain
H_{max}	10^{-2}	
γ_u^{th}	100	SINR threshold
σ	-174 dBm/Hz	Thermal noise density
α	4	Path-loss exponent

TABLE I: Simulation parameters

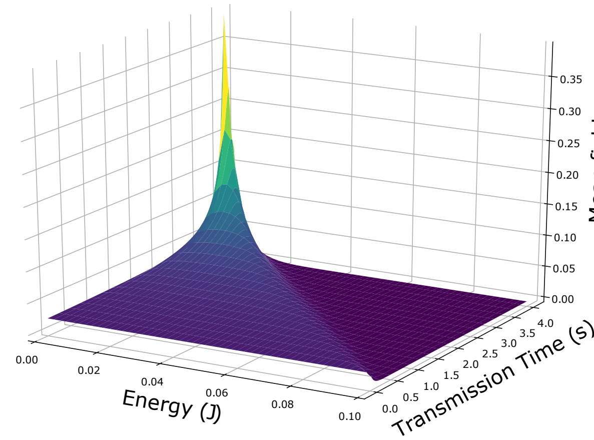

Fig. 1: Mean-field at the equilibrium with $\lambda_s = 10^{-4}$ SBS/m^2.

IV. Numerical Simulation

In this section, we provide numerical results on the performance of the proposed scheme. The key simulation parameters are shown in Table I. The SBS density is specified for each figure.

IoT devices (followers) :
The evolution of the energy state distribution (mean-field) at the equilibrium is shown in Fig. 1. For this simulation, we take into account a uniform initial energy state distribution m_0. As time passes, fewer IoT devices have higher energy budgets. Additionally, the frequency of IoT devices with zero energy has increased (nearly 30% of the IoT devices consume their whole energy allowance while transmitting). However, due to the utility function, precisely the pricing coefficient C, the IoT devices with a larger energy budget at the beginning of the transmission save energy at the end of the transmission.

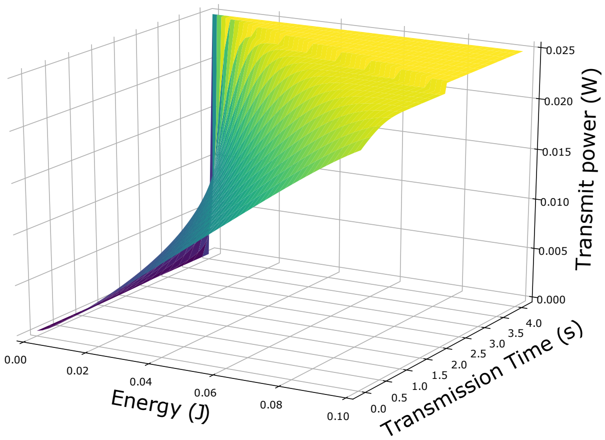

Fig. 2: Optimal power allocation strategy of IoT devices with $\lambda_s = 10^{-4}$ SBS/m^2.

The Fig. 2 shows the optimal power allocation strategy as a function of time and energy. Based on its remaining energy budget at each time instant, an IoT device controls the amount of power it transmits. Moreover, Fig. 3 shows a cross-section of the optimal power allocation strategy for different energy budgets and SBS densities. The IoT devices having a higher energy budget start their transmission with a high transmit power especially in an ultra-dense environment ($\lambda_s = 10^{-3}$ SBS / m^2). In the meanwhile, the IoT devices that are starting the game with lower energy budgets abstain from transmitting at higher transmit power and even reduce their transmit power in a dense environment ($\lambda_s = 10^{-4}$ SBS / m^2) in order to allow the IoT devices with greater energy budgets to boost their power.

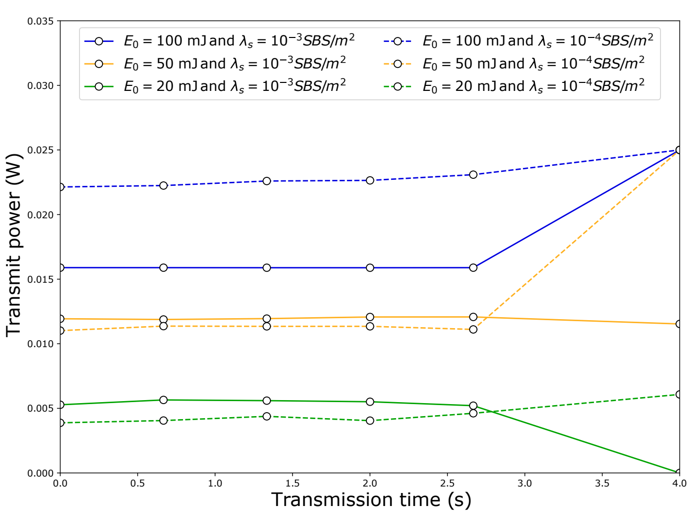

Fig. 3: Cross-section of the optimal strategy of the IoT devices for various energy budgets and SBS densities.

eMBB user (leader) :

The Figs. 4 and 5 show the optimal power allocation strategy of the eMBB user as a function of time and channel gain for two SBS densities. The eMBB user can regulate its transmit power based on its channel gain at each instant. Moreover, when the channel quality deteriorates, the eMBB user increases its transmit power to maintain a reasonable QoS. It can also be noted that, in an ultra-dense environment ($\lambda_s = 10^{-3}$ SBS / m^2), the eMBB user with a good channel gain starts transmission with a lesser transmit power and gradually increases it as time goes. This is because IoT devices with larger energy budgets increase their transmit power towards the end of the transmission.

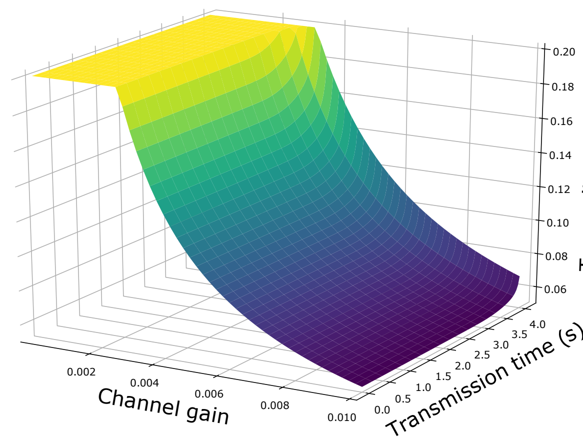

Fig. 4: Optimal power allocation strategy of eMBB user with $\lambda_s = 10^{-4}$ SBS/m^2.

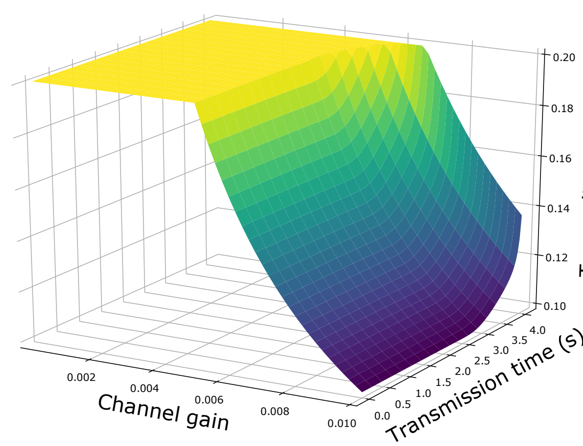

Fig. 5: Optimal power allocation strategy of eMBB user with $\lambda_s = 10^{-3}$ SBS/m^2.

V. Conclusion

In this paper, we present a distributed power allocation strategies with the eMBB user acting as the leader and the IoT devices serving as the followers. To satisfy both the QoS requirement of the eMBB user and the energy limitations of the IoT devices, we construct a hierarchical mean-field optimal control. Our approach enables each IoT device to determine its optimal strategy based just on the initial mean-field and its own energy budget. The eMBB user, on the other hand, adjusts its transmit power dependent on its channel gain. The numerical findings show the optimal power allocation strategies under various SBS densities.

REFERENCES

[1] Cisco. (Mar. 2020). Cisco Annual Internet Report (2018–2023) White Paper. [Online]. Available: https://www.cisco.com/c/en/us/solutions/collateral/executive-perspectives/annual-internet-report/white-paperc11- 741490.html

[2] Y. Benchaabene, N. Boujnah, and F. Zarai, "5G cellular: Survey on some challenging techniques," in Proc. IEEE Int. Conf. Parallel Distrib. Comput., Taipei, Taiwan, Dec. 2017, pp. 348–353.

[3] M. Shafi et al., "5G: A tutorial overview of standards, trial, challenges, deployment, and practice," IEEE J. Sel. Areas Commun., vol. 35, no. 6, pp. 1201–1221, Jun. 2017.

[4] M.A.AdedoyinandO.E.Falowo,"Combination of ultra-dense networks and other 5G enabling technologies: A survey," IEEE Access, vol. 8, pp. 22893–22932, 2020.

[5] Y. Lin,R. Zhang,L. Yang,and L. Hanzo,"Modularity-based user-centric clustering and resource allocation for ultra dense networks," IEEE Trans. Veh. Tech., vol. 67, no. 12, pp. 12457–12461, Dec. 2018.

[6] S.-E. Hong, "On the effect of shadowing correlation on hybrid precoding performance for cell-free mmWave massive MIMO UDN system," in Proc. Int. Conf. Inf. Commun. Technol. Converg. (ICTC), Oct. 2020, pp. 914–919.

[7] S.-E. Hong, "Comparison study of RF beamforming schemes in hybrid precoding for cell-free mmWave massive MIMO UDN system," in Proc. IEEE 92nd Veh. Technol. Conf. (VTC-Fall), Nov. 2020, pp. 1–5.

[8] L. Sun, J. Hou, and T. Shu, "Bandwidth-efficient precoding in cell-free massive MIMO networks with rician fading channels," in Proc. 18th Annu. IEEE Int. Conf. Sens., Commun., Netw. (SECON), Jul. 2021, pp. 1–9.

[9] L. Zhang, G. Zhang, X. Zhao, Y. Li, C. Huang, E. Sun, and W. Huang, "Resource allocation for energy efficient user association in user-centric ultra-dense networks integrating NOMA and beamforming," AEU Int. J. Electron. Commun., vol. 124, Sep. 2020, Art. no. 153270.

[10] L. Li et al., "Resource allocation for NOMA-MEC systems in ultradense networks: A learning aided mean-field game approach," IEEE Trans. Wireless Commun., vol. 20, no. 3, pp. 1487–1500, Mar. 2021.

[11] Y. Zhang, C. Yang, J. Li, and Z. Han, "Distributed interference-aware traffic offloading and power control in ultra-dense networks: Mean field game with dominating player," IEEE Trans. Veh. Technol., vol. 68, no. 9, pp. 8814–8826, Sep. 2019.

[12] Bensoussan, Alain; Sethi, Suresh; Chen, Shaokuan, "Feedback Stackelberg Solutions of Infinite-Horizon Stochastic Differential Games", SSRN Electronic Journal, 2012.

[13] S. Nadif, E. Sabir, H. Elbiaze and A. Haqiq, "Traffic-Aware Mean-Field Power Allocation for Ultra-Dense NB-IoT Networks," in IEEE Internet of Things Journal, 2022.

[14] M. Afshang, C. Saha and H. S. Dhillon, "Nearest-Neighbor and Contact Distance Distributions for Matérn Cluster Process," in IEEE Communications Letters, vol. 21, no. 12, pp. 2686-2689, Dec. 2017.

[15] Meriaux, F., Lasaulce, S. and Tembine, H. Stochastic Differential Games and Energy-Efficient Power Control. Dyn Games Appl 3, 3–23, 2013.

[16] S. Nadif, H. Elbiaze, E. Sabir and A. Haqiq, "Dynamic Satisfactory Power Allocation for Multi-Class Ultra Dense 5G and Beyond," 2020 IEEE Canadian Conference on Electrical and Computer Engineering (CCECE), 2020.

[17] P. Semasinghe and E. Hossain, "Downlink Power Control in Self-Organizing Dense Small Cells Underlaying Macrocells: A Mean Field Game," in IEEE Transactions on Mobile Computing, vol. 15, no. 2, pp. 350-363, 1 Feb. 2016.

[18] G. Albi, Y. Choi, M. Fornasier, D. Kalise, "Mean field control hierarchy", in ArXiv:1608.01728, 2017.

[19] M. Lauriere, "Numerical Methods for Mean Field Games and Mean Field Type Control", in ArXiv, abs/2106.06231, 2021.