2021 19th International Symposium on Modeling and Optimization in Mobile, Ad hoc, and Wireless Networks (WiOpt)

When To Pull Data for Minimum Age Penalty

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***Abstract*—A communication receiver that wants to pull data from a remote sensor by exploiting wireless energy transfer is considered. The receiver has a long-term average energy budget for this operation, and its goal is to keep the time average of a general age penalty function as small as possible. The channel from the source to the receiver is a two-state (ON/OFF) communication link whose state is IID or Markovian, and known instantaneously by the receiver. Modeling the problem as a constrained Markov decision problem, we obtain a randomized threshold-based decision policy that achieves the minimum possible average age penalty. We determine the optimal time average Age of Information and age violation probabilities by exploiting the optimality of the derived policy.**

I. Introduction

Wireless energy transfer (WET) using time-varying electric, magnetic, or electromagnetic fields, has been considered viable for various communication systems [1]–[3]. This technology is especially attractive in the scenario of collecting data from a sensor that does not have a significant energy source of its own. This implies a mode of energy harvesting, where the source of energy is directly controlled by the node who will pull the data from the sensor. The optimal planning of transmissions in such a scenario has been considered in the literature (see, e.g., [4], and references therein). In this paper, we formulate the problem from an Age of Information (AoI) optimization perspective.

Optimal transmission scheduling [5] is the problem of modifying rate and power in time according to energy availability, data demand, and channel variations, to transfer data as efficiently as possible, i.e., maximize the throughput with the given amount of energy, satisfy certain delay constraints, etc. Transmission scheduling to optimize AoI is relatively new.

Age of Information (AoI) was introduced in [6] and [7], in order to quantify the freshness of information in status-update systems and is defined as the time elapsed since the generation time of the most recent status update packet successfully received at the destination. In [8], general age penalty functions were defined to represent the level of dissatisfaction with information staleness. Optimization of non-linear age penalty has been studied under communication constraints such as random delays, time-varying channels and errors, in various studies including [9]–[16].

In recent years, there has been a growing interest in the combined analysis of information freshness and energy harvesting. In [17], the problem of when to generate updates under detailed energy harvesting constraints (energy causality constraints) was formulated and solved. Each transmission consumes unit energy, and the goal of the transmitter is to spread its transmissions as evenly as possible in time, while respecting the energy causality constraints, to minimize average age. In [18], a stationary transmission policy was considered for a source that harvests energy at a constant rate λ, and has an infinite battery so that the energy causality constraints are not binding: there is always energy available when the source decides to transmit, as long as it does not use energy at higher rate than λ. In this model, packets are subject to iid delays Y in the channel. A single-server policy is maintained, hence a new packet can only start transmission once the transmission of the current packet is completed. Loosely speaking, then, the long term average rate of transmissions cannot exceed $1/E[Y]$. Consequently, if the expected delay, $E[Y]$, exceeds $1/\lambda$, then the transmitter will have to use energy at a rate lower than the rate it is harvested at. Otherwise, the transmitter has a choice to transmit at rate up to λ. The *β-optimal* policy proposed in [18] computes the policy that ensures that the arriving energy is used at rate λ whenever feasible. However, it notices the curious phenomenon that with this policy, the resulting age is non-monotone in the energy harvest rate λ. This indicates that the policy is not optimal in general. The optimal policy was shown in [8] to be one that possibly inserts a non-zero waiting time, $Z(Y)$, depending on the value of delay, even though $E[Y + Z(Y)] > 1/\lambda$. In other words, for many delay distributions, it is not optimal in terms of average age to transmit at the largest allowed update rate. The result was also generalized to Markovian delay processes and general age penalties in [8].

In this paper, we consider a model where the receiver pulls data from a sensor by sending energy to be harvested by a transmitter connected to that sensor (see Fig. 1). This model allows the receiver to optimize the amount of energy it will deliver to the transmitter by taking the channel state information into account and thus, to control the long-term average age of information (AoI). Our aim is to derive the average age penalty in closed form and minimize it. By formulating minimization as a constrained Markov decision problem, we obtain an optimal decision policy which has minimum energy consumption and keeps the flow retrieved from the sensor as fresh as possible.

The problem we study is closely related to the above literature [8], [18] in the sense that it includes a finite average energy constraint, and a delay process that is caused by the channel state being on or off: when the transmitter makes a decision to transmit, the update is immediately received,

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followed by a random number of "off" slots during which there is no opportunity to pull data and the age increases. On the other hand, it differs from the models in [8], [18] in the sense that successful transmissions reset the age down to a deterministic constant. This implies zero-wait being optimal whenever feasible, hence the interesting case to be analyzed for this problem is the regime where zero-wait is not feasible.

There have been studies on transmission scheduling under WET constraints: In [19], time average AoI is investigated in a WET system with a Rayleigh block fading channel, where the transmitter waits until its battery is completely filled and uses all the acquired energy for a single transmission. In [9], an energy harvesting transmitter with a finite battery is studied in continuous time and it is shown that a threshold policy optimizes the expected time average age penalty. Another closely related recent work is [20], which studies the long-term time average AoI under a constraint on the average number of transmissions at the source node and examines standard ARQ and hybrid ARQ (HARQ) protocols. Threshold policies for controlling age under various energy harvesting settings have been studied in recent literature (see [21], and references therein.)

The main contributions of this paper are:

- We model the general age penalty minimization problem over imperfect channels as a constrained Markov decision process (CMDP) and show that there is a a stationary policy that is optimal (Theorem 1).
- For IID channel states, we show that a threshold policy is optimal if the available energy is restricted, and we compute the optimal threshold policy in closed form (Theorems 2 and 3). Further, we derive the optimal values for the average AoI and age violation probability in closed form.
- We extend our analysis to the temporally correlated Markovian channels and reveal the optimal threshold policy (Theorem 4).
- We simulate the performance of the optimal threshold policy and compare it with a uniform transmission policy.

II. System Model and Problem Formulation

A point-to-point channel comprising a transmitter-receiver pair is considered. The channel can be in one of two states during any time slot: ON or OFF. The transmitter is passive, and the receiver will decide when to send energy to the transmitter, and pull data from it. The transmitter, which relies solely on the energy harvested from the receiver, is only responsible for transmitting data to the receiver on demand, and each transmission takes one time slot duration and requires one unit of energy. It is also assumed that the receiver has an infinite battery, hence the energy causality constraints are inactive as in [18]. The allowed long term average energy usage is constrained by λ units per time slot. Transmissions always fail while the channel state is OFF and succeed otherwise. The random transitions of the channel states are analyzed under two models: (i) IID in each time slot, and (ii) Markovian.

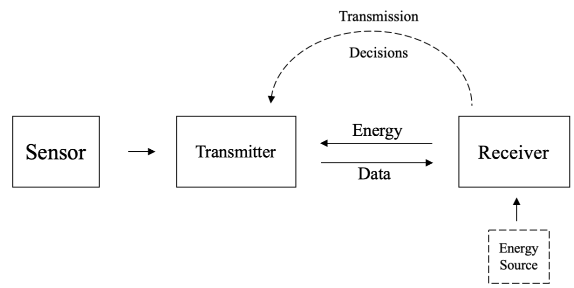

Figure 1. System model.

Whenever the receiver receives a new packet from the source, it resets the age to unity at the end of the slot. In the absence of a new reception, the age increases by 1 with every new slot. Consequently, the age at the end of time slot t, denoted by Δ_t, is known by the receiver. The state of the system at any time t can be described by the AoI Δ_t and the channel state C_t at that time.

In order to generically quantify the staleness of data packets under different conditions, we define a general age penalty function $g(\Delta)$ as a function of AoI. The function $g : \mathbb{Z}^+ \to \mathbb{R}$ is non-decreasing. In the rest of this paper, we analyze the time average age penalty function, as described in Problem 1. If the age penalty function is an identity function, the expected age penalty becomes the time average AoI and if $g(\Delta) = \mathbb{1}_{\Delta > \gamma}$, then expected age penalty becomes the age violation probability; corresponding to two commonly used metrics in the literature.

This leads to the constrained Markov decision process (CMDP) [22] formulation, defined by the 5-tuple: $(\mathcal{S}, \mathcal{A}, P, c, d)$ with the countable set of states $\mathcal{S} = \mathbb{Z}^+ \times \{\mathrm{ON}, \mathrm{OFF}\}$ and the finite action set $\mathcal{A} = \{0, 1\}$. $a_t = 1$ denotes that the transmission will be performed and $a_t = 0$ denotes that no transmission occurs. The state s_t consists of the age Δ_t and the channel state C_t at time t. P refers to the transition function, where $\mathrm{P}(s'|s,a) = Pr(s_{t+1} = s'|s_t = s, a_t = a)$ is the probability that action a in state s at time t will lead to state s at time $t+1$. The cost function $c : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$ is a non-decreasing function of the AoI at the destination, and is defined as $c(s,a) = g(\Delta_t)$, for any $s \in \mathcal{S}$, $a \in$ A, independently of action a. The transmission cost d is related with the energy constraint λ and is identical for each transmission, $d = 1$ if $a_t = 1$ and $d = 0$ otherwise. The age Δ_t evolves as:

$$\Delta_{t+1} = \begin{cases} 1, & \text{if } C_t = \text{ON and } a_t = 1 \\ \Delta_t + 1, & \text{otherwise} \end{cases} \quad (1)$$

The evolution of the channel states is examined over two different scenarios. In section III, we assume that the channel state becomes ON and OFF at each time slot in an independent and identically distributed (IID) fashion with their correspond-

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ing probability values P_{ON} and P_{OFF}.

$$\Pr(C_{t+1}=c)=\begin{cases} P_{ON}, & \text{if } c=\text{ON} \\ P_{OFF}, & \text{if } c=\text{OFF} \end{cases} \quad (2)$$

where $P_{ON}>0$ and C_{t+1} is independent of the age or the past realizations of the channel states.

In section IV, the results obtained for IID channel states are extended by considering time-correlated channel states which evolve as a Markovian process:

$$\Pr(C_{t+1}=c_1 \mid C_t=c_0)=\begin{cases} 1-p_{10}, & \text{if } (c_1,c_0)=(\text{ON,ON}) \\ p_{01}, & \text{if } (c_1,c_0)=(\text{ON,OFF}) \\ p_{10}, & \text{if } (c_1,c_0)=(\text{OFF,ON}) \\ 1-p_{01}, & \text{if } (c_1,c_0)=(\text{OFF,OFF}) \end{cases} \quad (3)$$

where $p_{ij}\in(0,1)$, with $i,j\in\{0,1\}$ indices standing for the channel state in former and latter time slots, respectively.

A stationary policy is a decision rule denoted by $\pi:\mathcal{S}\times\mathcal{A}\to[0,1]$ which maps the states s into actions a with some probability $\pi(a|s)$. We try to minimize the average age penalty under energy constraint λ, given the initial state $s_0=(1,\text{ON})$. In this manner, our focus is the age penalty function $g(\Delta_t)$. We can state the CMDP optimization problem as follows, where $E[\cdot]$ represents expectation with respect to the distribution of the age process induced by policy π and channel states C_t:

Problem 1.

$$\begin{aligned} \min_{\pi}\ \Delta^{\pi}(s_0) = & \lim_{T\to\infty}\frac{1}{T}E[\sum_{t=1}^{T} g(\Delta_t)|s_0], \\ \textit{s.t.} \quad & \lim_{T\to\infty}\frac{1}{T}E[\sum_{t=1}^{T} a_t^{\pi}|s_0] \leq \lambda. \end{aligned} \quad (4)$$

A policy π^* that is a solution of the above minimization problem is called optimal, and we are interested in finding optimal policies.

III. Minimizing the Age Penalty for IID Channel State

Constrained MDPs with countably infinite state-spaces as defined in Problem 1 are generally difficult to solve since a stationary optimal policy, or an optimal policy in general, are not guaranteed to exist [22]. Next, we show that an optimal stationary policy exists for Problem 1 and define the structure of the optimal policy.

Theorem 1. *There exists an optimal stationary policy for the CMDP in Problem 1 and it is randomized in at most a single point in the state-space $\mathcal{S}$.*

Proof. A sketch of the proof is given as follows: First, we show that Theorem 2.5, Proposition 3.2, and Lemma 3.9 of [23] hold for Problem 1 by showing that Assumptions 1-4 of [23] hold. Then, by Theorem 2.5 of [23], there exists an optimal stationary policy that is a mixture of two deterministic policies which differ in at most one state and there exists a randomization coefficient denoted by $p_\theta\in[0,1]$ such that π^* satisfies the constraint with equality. The detailed proof can be obtained by following the same steps as in [20]. □

As a result of Theorem 1, we restrict our attention to the stationary policies in the rest of the paper.

A. Steady-State Analysis

In this section, we investigate steady-state behavior of the Markov Chain constructed by the states s_t as defined in Section II, under a reasonable stationary policy. The MC has a unique steady state distribution if it is irreducible and positive recurrent [24, Ch. 6].

All states in the MC are reachable from the $\Delta=1$ state, because for any $k\geq 1$; if the channel state is OFF between t and $t+k$, then age increases by k with probability 1. Hence, $\Pr(\Delta_{t+k}=k+1 \mid \Delta_t=1)\geq P_{OFF}^k$. To show that the $\Delta=1$ state is positive recurrent, we consider the expected time between consecutive transmissions. If the expected time is infinite, then the average energy cost would be 0 and such a policy would obviously be inferior to the ones that satisfy the constraint in (4) with equality. If the expected time between the transmissions is finite, then expected return time to $\Delta=1$ state is finite and the MC is positive recurrent. Consequently, there are policies that lead to a steady-state distribution and we focus on such policies. The resulting MC is illustrated in Fig. 2. We use $\Pr(\Delta=k)$ to denote the steady-state probability of the age being equal to k.

B. Structure of the Optimal Stationary Policy

In Problem 1, if there was no energy constraint ($\lambda\to\infty$), the transmitter would be able to take advantage of all transmission opportunities. Note that, if $\lambda\geq P_{ON}$, such an unconstrained policy is feasible (due to the infinite battery assumption, the transmitter will never have to idle at a transmission opportunity.) Any policy that misses a transmission opportunity can only do worse, because in any sample path of the channel state process consisting of random realizations of ON and OFF slots, the age graph of a policy that exploits all the ON slots will be dominated by any other feasible age plot. That is, as the zero wait policy brings the age down to unity at all ON slots, its age will be below or equal to that of any other feasible age graph attainable on the same sample path. Therefore, unlike in [8], [18], in the case that $\lambda\geq P_{ON}$, the optimal policy is a zero-wait policy.

Having made this observation, for the rest of the paper, we focus on the nontrivial case $\lambda<P_{ON}$. In the following, we show the optimality of a threshold policy that fully utilizes the energy constraint for remaining cases:

Theorem 2. *Let Θ be an integer for which there exists a stationary policy, π^*, such that*

(i) $\Pr(a=1 \mid \Delta<\Theta, C=\text{ON})=0$
(ii) $\Pr(a=1 \mid \Delta>\Theta, C=\text{ON})=1$
(iii) $\Pr(a=1 \mid C=OFF)=0$
(iv) $\Pr(a=1)=\lambda$

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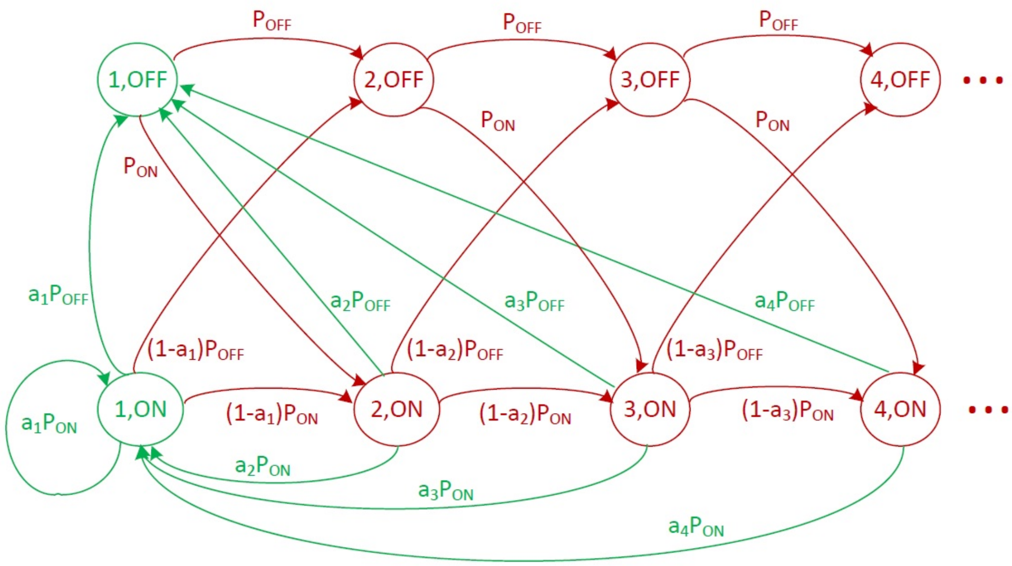

Figure 2. Markov chain representation of the joint age and channel state, where a_j stands for the probability of transmitting when age is equal to j and the channel is ON. Green arrows represent transitions from successful tranmsissions and red arrows indicate otherwise.

when π^* is employed. Then, π^* is optimal for Problem 1 such that for any stationary policy π;

$$\Delta^{\pi}(s_0) \geq \sum_{k=1}^{\infty} g(k)h(k) \tag{5}$$

where $h(k)$ is defined as:

$$h(k) = \begin{cases} \lambda, & k \leq \Theta \\ (1-\lambda\Theta)P_{ON}P_{OFF}^{k-\Theta-1}, & k \geq \Theta+1 \end{cases} \tag{6}$$

Furthermore, (5) *is tight when $\pi \equiv \pi^*$.*

In this theorem, the optimal age penalty and the properties of the optimal policy are stated. The first two conditions on π^* suggest a threshold policy, while the latter conditions ensure that the policy does not waste energy.

Before laying out the proof of the theorem, we investigate the steady state distribution of the AoI. At any time t, the relation between $\Pr(\Delta_t = k)$ and $\Pr(\Delta_{t+1} = k+1)$ can be derived using (1):

$$\Pr(\Delta_{t+1} = k+1) = \Pr(\Delta_t = k)(1 - P_{ON}\Pr(a_t = 1 \mid s_t = (k, ON))) \tag{7}$$

As we restrict attention to stationary policies (without loss of optimality, by Thm. 1), the above equation can be rewritten as:

$$\frac{\Pr(\Delta = k+1)}{\Pr(\Delta = k)} = 1 - P_{ON}\Pr(a = 1 \mid s = (k, ON)) \tag{8}$$

From (8), $\Pr(\Delta = k+1) \leq \Pr(\Delta = k)$ and therefore the PMF of AoI at steady state is monotonic. The following Lemma uses the monotonicity to establish a lower bound on the age violation probability.

Lemma 1. *For any $\gamma \in \mathbb{Z}^+$,*

$$\Pr(\Delta \geq \gamma+1) \geq 1 - \lambda\gamma \tag{9}$$

with equality if and only if

(i) $\Pr(a = 1 \mid \Delta < \gamma, C = \text{ON}) = 0$
(ii) $\Pr(a = 1 \mid C = OFF) = 0$
(iii) $\Pr(a = 1) = \lambda$

Proof. The state of $\Delta = 1$ corresponds one-to-one to the successful transmissions by the transmitter and the probability of a transmission occurring on average must not be greater than λ. Hence,

$$\Pr(\Delta = 1) \leq \Pr(a = 1) \leq \lambda \tag{10}$$

Equality in (10) holds iff[1] a successful transmission happens with probability λ at steady state. In other words, the energy constraint shall be fully utilized and available energy shall not be wasted on transmitting while the channel is OFF, corresponding to the second and third properties. Due to the monotonicity, $\Pr(\Delta = k) \leq \lambda$ for any k as a result of (10). Finally,

$$\Pr(\Delta \geq \gamma+1) = 1 - \sum_{k=1}^{\gamma} Pr(\Delta = k) \geq 1 - \lambda\gamma \tag{11}$$

Equality in (11) holds iff $\Pr(\Delta = k) = \lambda$ for all $k < \gamma$. In order for this to happen, there must be no successful transmission while the age is smaller than γ due to (8), yielding the first condition. □

Lemma 1 describes a lower limit on the age violation probabilities for small violation thresholds. If γ is larger than

[1] if and only if

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$1/\lambda$, then $1-\lambda\gamma$ would be negative and the inequality in Lemma 1 would be loose. In order to support larger violation thresholds, we present Lemma 2.

Lemma 2. *For any $\gamma, m \in \mathbb{Z}^+$; if $\gamma > m$, then*

$$\Pr(\Delta \geq \gamma+1) \geq P_{OFF}^{\gamma-m}(1-\lambda m) \tag{12}$$

with equality if and only if

(i) $\Pr(a=1 \mid \Delta < m, C=\mathrm{ON}) = 0$
(ii) $\Pr(a=1 \mid \Delta > m, C=\mathrm{ON}) = 1$
(iii) $\Pr(a=1 \mid C=OFF) = 0$
(iv) $\Pr(a=1) = \lambda$

Proof. Let k be an arbitrary positive integer. If the channel state is OFF and AoI is k at time t, AoI at time $t+1$ is $k+1$ with probability 1. Therefore,

$$P_{OFF}\Pr(\Delta=k) \leq \Pr(\Delta=k+1) \tag{13}$$

Through induction, we can show that for any $r \in \mathbb{Z}^+$,

$$P_{OFF}^r\Pr(\Delta=k) \leq \Pr(\Delta=k+r) \tag{14}$$

holds. Using this property, following relation between $\Pr(\Delta \geq k+1)$ and $\Pr(\Delta=k)$ is obtained:

$$\begin{aligned}\Pr(\Delta \geq k+1) &= \sum_{r=1}^{\infty}\Pr(\Delta=k+r)\\ &\geq \sum_{r=1}^{\infty}P_{OFF}^r\Pr(\Delta=k) = \frac{P_{OFF}}{P_{ON}}\Pr(\Delta=k)\end{aligned} \tag{15}$$

The fact that $\Pr(\Delta \geq k) - \Pr(\Delta \geq k+1) = \Pr(\Delta=k)$ can be used to rewrite (15) as:

$$P_{OFF}\Pr(\Delta \geq k) \leq \Pr(\Delta \geq k+1) \tag{16}$$

Through induction,

$$P_{OFF}^r\Pr(\Delta \geq k) \leq \Pr(\Delta \geq k+r) \tag{17}$$

follows for any $r \in \mathbb{Z}^+$. From Lemma 1,

$$\Pr(\Delta \geq m+1) \geq 1-\lambda m \tag{18}$$

For $k=m+1$ and $r=\gamma-m$ in (17), we obtain:

$$\Pr(\Delta \geq \gamma+1) \geq P_{OFF}^{\gamma-m}\Pr(\Delta \geq m+1) \geq P_{OFF}^{\gamma-m}(1-\lambda m) \tag{19}$$

Equality holds in (13) iff a transmission happens with probability 1 at $s=(k,ON)$ state. Equality in (14)-(17) holds iff a transmission always takes place when the channel state is ON and the AoI is greater than or equal to k. Due to the choice of $k=m+1$, *(ii)* is required for an equality. Rest of the equality conditions follow from (18) and Lemma 1. □

Finally, we prove Theorem 2 using Lemmas 1 and 2.

Proof of Theorem 2. Expected age penalty can be written in terms of the steady state probabilities of Δ:

$$\begin{aligned}\lim_{T\to\infty}\frac{1}{T}E\left[\sum_{t=1}^{T}g(\Delta_t)\right] &= \sum_{k=1}^{\infty}g(k)\Pr(\Delta=k)\\ &= \sum_{k=1}^{\infty}g(k)(\Pr(\Delta \geq k) - \Pr(\Delta \geq k+1))\\ &= g(1) + \sum_{k=1}^{\infty}(g(k+1)-g(k))\Pr(\Delta \geq k+1)\\ &\overset{(a)}{\geq} g(1) + \sum_{k=1}^{\Theta}(g(k+1)-g(k))(1-\lambda k)\\ &\quad + \sum_{k=\Theta+1}^{\infty}(g(k+1)-g(k))(1-\lambda\Theta)P_{OFF}^{k-\Theta}\\ &= \lambda\sum_{k=1}^{\Theta}g(k) + (1-\lambda\Theta)P_{ON}\sum_{k=\Theta+1}^{\infty}g(k)P_{OFF}^{k-\Theta-1}\end{aligned} \tag{20}$$

where (a) follows from Lemma 1 and Lemma 2. □

Corollary 1. *If the function g is strictly increasing, then equality in* (5) *holds if and only if the conditions (i)-(iv) are satisfied. In this case, π^* becomes the optimal stationary policy and π^* is unique.*

C. Derivation of the Threshold

In the previous section, we showed what the transmission probabilities should be under an optimal policy, except that we did not find the value of Θ. We also did not reveal the transmission probability of π^* when the age is equal to Θ. In this section, we fully derive the policy π^* that satisfies the conditions of Theorem 2. This policy is expressed in Theorem 3.

Theorem 3. *The optimal policy π^* for Problem 1 under IID channel states is a randomized threshold policy, which can be written as:*

$$\pi^*(a_t=1 \mid s_t=(\Delta_t,C_t)) = \begin{cases} 1, & \Delta_t > \Theta \text{ and } C_t = ON\\ p_\Theta, & \Delta_t = \Theta \text{ and } C_t = ON\\ 0, & \Delta_t < \Theta \text{ or } C_t = OFF\end{cases} \tag{21}$$

where threshold Θ and randomization coefficient p_Θ are given as:

$$\Theta = \left\lfloor 1 + \frac{1}{\lambda} - \frac{1}{P_{ON}}\right\rfloor \tag{22}$$

$$p_\Theta = \Theta - \left(\frac{1}{\lambda} - \frac{1}{P_{ON}}\right) \tag{23}$$

Proof. In the proof, we shall derive the values of Θ and p_Θ. Note that there can be two different choices of thresholds depending on whether $p_\Theta \in [0,1)$ or $p_\Theta \in (0,1]$. We assume $p_\Theta \in (0,1]$ without loss of generality to ensure that there

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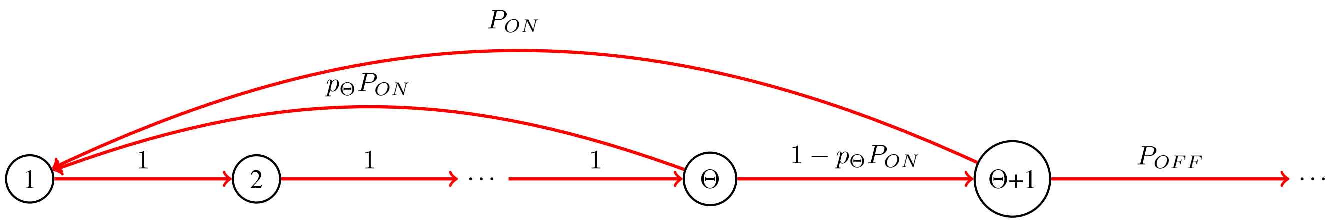

Figure 3. State diagram under the threshold policy

exists a unique threshold and a unique set of parameters as a result of our analysis.

In Fig. 3, the state diagram for the policy above is illustrated. Corresponding state transition probabilities and total probability equation are as follows:

$$\Pr(\Delta = k) = \Pr(\Delta = 1) \text{ if } k \leq \Theta \tag{24}$$

$$\Pr(\Delta = \Theta + 1) = \Pr(\Delta = \Theta)(1 - p_\Theta P_{ON}) \tag{25}$$

$$\Pr(\Delta = k) = \Pr(\Delta = k-1) P_{OFF} \text{ if } k \geq \Theta + 2 \tag{26}$$

$$\sum_{k=1}^{\infty} \Pr(\Delta = k) = 1 \tag{27}$$

Note that the probability of making a succesful transmission and returning to the state $\Delta = 1$ is zero while the age is less than Θ, leading to (24). We obtain a closed-form solution of $\Pr(\Delta = k)$ by solving these equations together, with the first element of the series being equal to:

$$\Pr(\Delta = 1) = \frac{1}{\Theta - p_\Theta + \frac{1}{P_{ON}}} \tag{28}$$

Due to the conditions *(iii)* and *(iv)* in Theorem 2, $\Pr(\Delta = 1) = \lambda$. Therefore,

$$\frac{1}{\Theta - p_\Theta + \frac{1}{P_{ON}}} = \lambda \tag{29}$$

Finally, we use the fact that Θ is an integer and $p_\Theta \in (0, 1]$ to derive the unknown parameters as in (22) and (23). □

Corollary 2 (Optimal Age Violation Probability)**.** *If g function is set as $g(\Delta) = u(\Delta - \gamma)$ where $u(\cdot)$ is the unit step function such that $u(x) = \mathbb{1}_{x>0}$, expected age penalty would be equal to the age violation probability with a violation threshold γ. The optimal threhold policy π^* derived in Theorem 3 minimizes the age violation probability and optimal age violation probability can be computed in closed form as in the following:*

$$\Pr(\Delta > \gamma) \geq \begin{cases} 1 - \lambda\gamma, & \gamma \leq \Theta \\ P_{OFF}^{\gamma-\Theta}(1 - \lambda\Theta), & \gamma \geq \Theta. \end{cases} \tag{30}$$

Corollary 3 (Optimal Time Average AoI)**.** *The threshold policy π^* derived in Theorem 3 minimizes the time average AoI, when g function is set as the identity function such that $g(\Delta) = \Delta$. Then, optimal time average AoI can be computed in closed form as in the following:*

$$\lim_{T\to\infty} \frac{1}{T} E\left[\sum_{t=1}^{T} \Delta_t\right] \geq \frac{1}{P_{ON}} + \lambda\Theta\left(\frac{1}{\lambda} - \frac{1}{P_{ON}} - \frac{\Theta - 1}{2}\right). \tag{31}$$

IV. Optimal Policy for Time Correlated Markovian Channels

In this section, we alter the system model and extend our analysis to the case of Markovian channels. In this case, the channel states evolve as in (3). The optimal policy for this model is given in the following:

Theorem 4. *The optimal policy π^* for Problem 1 under Markovian channel states is a randomized threshold policy, which can be written as:*

$$\pi(a_t = 1 \mid s_t = (\Delta_t, C_t)) = \begin{cases} 1, & \Delta_t > \Theta \text{ and } C_t = ON \\ p_\Theta, & \Delta_t = \Theta \text{ and } C_t = ON \\ 0, & \Delta_t < \Theta \text{ or } C_t = OFF \end{cases} \tag{32}$$

where the age threshold Θ is the greatest integer that satisfies the following inequality:

$$\theta - ca^\theta \leq b \tag{33}$$

where

$$a = 1 - p_{10} - p_{01}$$

$$b = \frac{1}{\lambda} - \frac{1}{p_{01}} + \frac{1}{p_{01} + p_{10}}$$

$$c = \frac{1}{p_{01}} - \frac{1}{p_{01} + p_{10}}$$

and the randomization coefficient p_Θ is:

$$p_\Theta = \frac{b - \theta + ca^\theta}{1 + ca^\theta - ca^{\theta+1}} \tag{34}$$

We defer the proof of this theorem to the full version of this paper [25]. Note that in the special case of $p_{01} + p_{10} = 1$, the channel states become IID and Theorem 4 becomes identical to Theorem 3.

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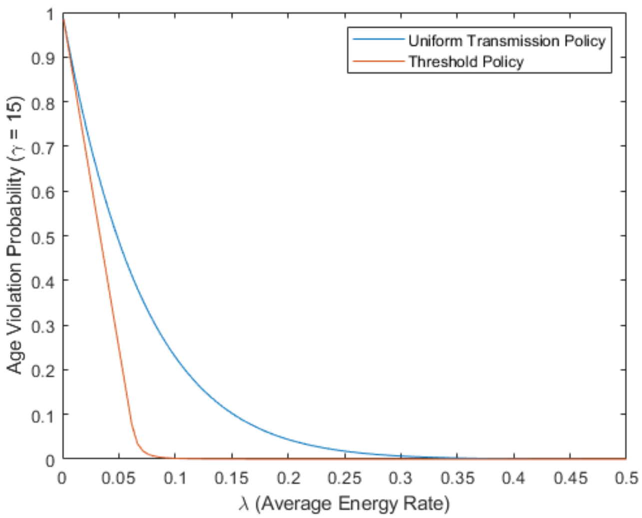

Figure 4. Age violation probability vs average energy rate

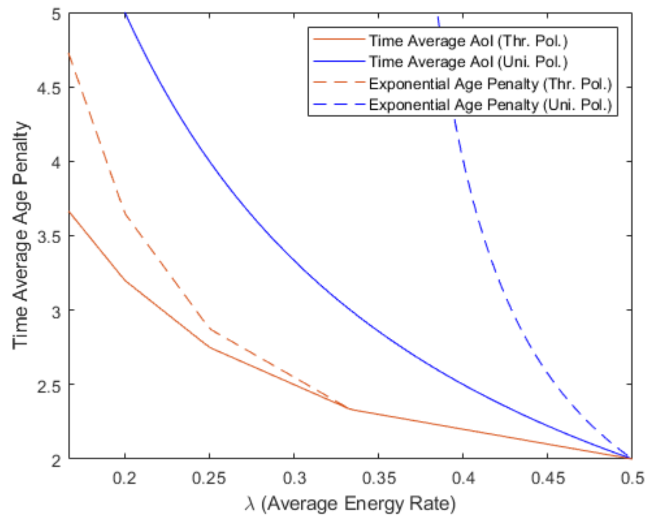

Figure 5. Time average age penalty under different penalty functions

V. Numerical Results

We compare the performance of our threshold policy to a uniform transmission policy that performs a transmission at any time with probability $\frac{\lambda}{P_{ON}}$ while the channel state is ON. The value of $\frac{\lambda}{P_{ON}}$ is chosen such that the time average energy constraint is fully utilized by both policies and a fair comparison can be made, however, two policies converge to a zero-wait policy as λ approaches P_{ON} with diminishing differences in terms of performance. Note that, we assume λ to be smaller than P_{ON}, as explained in Section III-B.

We run Monte Carlo simulations for 10^6 time slots with 100 iterations and for $P_{ON} = 0.5$. Fig. 4 depicts the age violation probability of the optimal threshold policy and uniform transmission policy for a violation threshold of 15 time slots. We observe that the age violation probability is reduced substantially compared to the uniform policy, especially when λ is between 0 and P_{ON} and far from both extremes. The results were verified to be consistent with the theoretical findings. In Fig. 5, the optimal time average age penalty is illustrated for linear ($g(\Delta) = \Delta$) and exponential ($g(\Delta) = 1.5^{\Delta-1}$) age penalty functions. We observe that optimal average age penalty changes linearly when the threshold Θ stays fixed within a limited range of λ, however, the plots resemble a geometrical decay over a long range of λ in which Θ changes with λ, as in Fig. 4.

VI. Conclusion

We designed a point-to-point information retrieval policy that minimizes a generalized age penalty, on a binary ON/OFF channel with a power constrained information pulling receiver. Modeling the problem as a CMDP, we showed that there exists a threshold policy that is optimal for the problem. We computed the threshold. The optimal time average Age of Information and age violation probabilities were found as corollaries to our main findings. We also unveiled an optimal policy for temporally correlated channels. Finally, we illustrated the performance impact of using this age-optimized policy, by comparing it to a benchmark uniform policy with the same energy expenditure. In the future, learning scenarios can be explored where the channel status information is not available at the nodes.

VII. Acknowledgements

This study is supported in part by TUBITAK grant 119C028, and in part by Turkcell Technology within the framework of 5G and Beyond Joint Graduate Support Programme coordinated by Information and Communication Technologies Authority.

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