Multi-Cell Coordination in K-tier Heterogeneous Downlink Cellular Networks: Dynamic Clustering and Feedback Allocation

Jeonghun Park*, Namyoon Lee†, and Robert W. Heath Jr.‡
*Corporate Research and Development, Qualcomm Inc.,
San Diego, CA 92122, USA.
jeonghun@utexas.edu
†Electrical Engineering Department, POSTECH,
77 Cheongam-Ro, Nam-Gu, Pohang, Gyeongbuk 37673, South Korea.
nylee@postech.ac.kr
‡Wireless Networking and Communication Group, The University of Texas at Austin,
Austin, TX 78712, USA.
rheath@utexas.edu

***Abstract*—We characterize the ergodic spectral efficiency of a cooperative type of K-tier heterogeneous networks (HetNets) with limited feedback. Specifically, a base station (BS) coordination set is formed by using dynamic clustering across the tiers, wherein the intra-cluster interference is mitigated by using multi-cell zero-forcing based on limited feedback. Modeling the network based on stochastic geometry, we derive analytical expressions for the ergodic spectral efficiency as a function of the system parameters. Leveraging the obtained expression, we formulate a feedback allocation problem and obtain a solution to improve the ergodic spectral efficiency. Simulations show the spectral efficiency improvement by using the proposed feedback allocation. One major finding in the obtained solution is that allocating more feedback to stronger intra-cluster BSs is efficient.**

I. Introduction

In heterogeneous networks (HetNets), different types of base stations (BSs) are densely deployed to aggressively reuse the spectrum. One bottleneck in achieving the full gains of HetNets is interference. Compared to conventional single-tier cellular networks, a HetNet has various interference sources including intra-tier BSs and also cross-tier BSs. As a result, small network tiers such as femto (whose transmit power is small) can be vulnerable to the interference. One effective approach to control interference is multi-cell coordination. The idea is to create BS coordination sets that share channel state information at transmitters (CSIT), so that the BSs in the same set remove the mutual interference. Unfortunately, in practice, measuring CSI of multiple links increases associated overheads. Taking the overheads into account, it is only feasible to form a finite-size BS coordination set where the out-of-cluster interference is unmanageable [1]. In addition, accurate CSIT in the same coordination set cannot be obtained due to quantization distortion using limited feedback, resulting in that the intra-cluster interference cannot be removed perfectly. For this reason, it is important to investigate the performance of BS coordination with these practical constraints to reveal the real benefits of BS coordination applied in HetNets.

Random network models based on stochastic geometry has been widely used because of their analytical tractability. Beyond traditional non-cooperative cellular networks, there is prior work that investigated the performance of multi-cell coordination using random network models. In [1], a user forms a BS coordination set with the closest BSs, i.e., dynamic clustering. Once a coordination set is formed, the intra-cluster interference is mitigated by using coordinated beamforming. Using this way, the out-of-cluster interference is efficiently managed. In [2], [3], the performance of joint-transmission was analyzed in a single-tier and a HetNet, respectively and revealed that the relative BS geometry has a significant effect on the performance of joint transmission. In [4], a location-aware cross-tier cooperation scheme was proposed, where macro and small BSs can cooperate to avoid the mutual interference if the user receives large amount of interference from small cells. In [5], the performance of BS coordination in spectrum-shared mmWave cellular networks was analyzed and it was found that BS coordination is useful in mmWave when sharing the spectrum with high power and dense network operators. A common assumption in [1]–[5] is that they did not consider a practical constraint in BS coordination, e.g., limited CSIT. Resolving this, [6], [7] assumed that the CSIT in the same coordination set is obtained via limited feedback, and analyzed the signal-to-interference plus noise ratio (SINR) performance as a function of the amount of feedback. In [6], [7], however, a single-tier cellular network was assumed.

In this paper, we characterize the ergodic spectral efficiency of a cooperative type of K-tier downlink HetNet with limited feedback. The locations of each tier's BS are modeled as mutually independent Poisson point processes (PPPs). The BSs form a coordination set by using dynamic clustering, and mitigate the intra-cluster interference by using multi-cell

ZF based on the limited feedback. Dynamic clustering is applied across the tiers in the HetNet, so that a coordination set can include different tiers' BSs. Unfortunately, analyzing the performance of the considered BS coordination is not straightforward since the performance of the cluster can be different depending on the tiers of the BSs included in the cluster. For example, assuming that a cluster has L BSs in a K-tier HetNet, there can be K^L possibilities of the cluster's configuration. For this reason, we should consider all the cases to completely characterize the performance of the L-size cluster. To resolve this analytical complexity, we derive a lemma showing that the intensity measure of received signal power in a HetNet can be transformed to the intensity measure of signal power in a statistically equivalent single-tier network by rescaling each tier's density. By exploiting this lemma, we obtain the SIR complimentary cumulative distribution function (CCDF) and the ergodic spectral efficiency as a function of the relative system parameters such as the cluster size, the transmit power, the biasing factor, the relative signal power of the intra-cluster BSs, and the used feedback. Assuming that each intra-cluster BS uses the same number of antennas, we formulate and solve an optimization problem to allocate the feedback. Numerical results show the spectral efficiency gains obtained by using the proposed feedback allocations compared to the equal feedback partition, where the feedback is equally allocated to each intra-cluster BS.

II. System Model

In this section, we introduce the system model assumed in the paper.

A. Network and Cell Association Model

We consider a K-tier downlink HetNet. Focusing on the k-th tier for $k \in \mathcal{K} = \{1, 2, \cdots K\}$, BSs equipped with N_k antennas are spatially distributed according to a homogeneous PPP, $\Phi_k = \left\{\mathbf{d}_i^k \in \mathbb{R}^2, i \in \mathbb{N}\right\}$ with density λ_k. All the BSs in the k-th tier use the same transmit power P_k and biasing factor S_k. Equivalently, the k-th tier network may be represented as a marked PPP, $\Phi_k^{\mathrm{M}} = \{\mathbf{d}_i^k, P_k, S_k, N_k, i \in \mathbb{N}\}$ with density λ_k where P_k, S_k, and N_k are the same marks for all the points in Φ_k. Without loss of generality, we assume that $\left\|\mathbf{d}_i^k\right\| \leq \left\|\mathbf{d}_j^k\right\|$ if $i < j$; thereby $\mathbf{d}_1^k$ indicates the nearest BS location to the origin in the k-th tier. Spatial locations of BSs in different tiers are assumed to be mutually independent. Using the superposition property of independent PPPs, we compactly represent the K-tier HetNet as an unified marked PPP $\tilde{\Phi}^{\mathrm{M}} = \sum_{k \in \mathcal{K}} \Phi_k^{\mathrm{M}}$. We write $\tilde{\Phi}^{\mathrm{M}} = \{\mathbf{d}_i, \pi(i), P_{\pi(i)}, S_{\pi(i)}, N_{\pi(i)}, i \in \mathbb{N}\}$, where $\pi(i) \in \mathcal{K}$ is an index function indicating the tier of the corresponding point $\mathbf{d}_i$. Assuming that $\|\mathbf{d}_i\| \leq \left\|\mathbf{d}_j\right\|$ if $i < j$, $\mathbf{d}_i$ means the i-th nearest BS location to the origin among all the tiers and $\pi(i)$ indicates that the tier of that BS. For example, assuming that the nearest BS to the origin is in the k-th tier, i.e., $\mathbf{d}_1 = \mathbf{d}_1^k$, then $\pi(1) = k$.

Single-antenna users are distributed according to a homogeneous PPP, $\Phi_{\mathrm{U}} = \{\mathbf{u}_i, i \in \mathbb{N}\}$, which has density $\lambda_{\mathrm{U}} \gg \lambda_k$ for $k \in \mathcal{K}$. Since the user density is far larger than the BS density, we assume that there is no empty cell with high probability, so that all the cells are occupied. We note that in HetNets, the BSs can be densely deployed so that empty cells can exist, which is a topic for future work. We focus on the typical user located on $\mathbf{u}_1 = \mathbf{0}$ per Slivnyak's theorem [8].

We consider an open access policy wherein a user is able to communicate with all the BSs in any tier k for $k \in \mathcal{K}$. For cell association, the typical user measures the biased average received power and associates with the BS whose the measured power is maximum. For instance, the user associates with the BS located at $\mathbf{d}_1^k$ if $k = \arg\max_{k' \in \mathcal{K}} P_{k'} S_{k'} \left\|\mathbf{d}_1^{k'}\right\|^{-\beta}$, where β is the path-loss exponent. Since we are interested in a HetNet sharing the spectrum among all the tiers, we assume that the path-loss exponent is same in all the tiers.

We note that biasing factor S_k is mainly used for offloading in HetNets [9], [10]. For example, as S_k increases, the number of users associated with the k-th tier BS also increases, which relieves the number of users associated with the other tiers. This allows other tiers to allocate more resources per one user. Typically, a small network tier such as femto tends to have large biasing factor to save the resources of the macro tier. Jointly considering feedback design and offloading will be interesting future work.

B. Clustering Model

Dynamic BS coordination is used to form a BS cluster. With the cluster size L, the typical user connects to the L BSs that provides L strongest biased average received power. Denoting the BS coordination set $C = \{i_1, ..., i_L\}$, we have

$$P_{\pi(i_1)} S_{\pi(i_1)} \left\|\mathbf{d}_{i_1}\right\|^{-\beta} \geq P_{\pi(i_2)} S_{\pi(i_2)} \left\|\mathbf{d}_{i_2}\right\|^{-\beta} \geq \ldots \geq P_{\pi(i_L)} S_{\pi(i_L)} \left\|\mathbf{d}_{i_L}\right\|^{-\beta}, \quad (1)$$

where $P_{\pi(i_L)} S_{\pi(i_L)} \left\|\mathbf{d}_{i_L}\right\|^{-\beta} \geq P_{\pi(j)} S_{\pi(j)} \left\|\mathbf{d}_j\right\|^{-\beta}$ for all $j \in \mathbb{N} \backslash C$. According to the association rule, the typical user associates with the BS located at $\mathbf{d}_{i_1}$ and receives the desired signal from it.

Using the described dynamic clustering, a cooperative region is mathematically defined by using the notion of the L-th order weighted Voronoi region, which is an extended version of the typical Voronoi region. For example, the weighted Voronoi region corresponding to the coordination set $C = \{i_1, ..., i_L\}$ is defined as

$$\begin{aligned} &\mathcal{V}_L^{\mathrm{w}}(\mathbf{d}_{i_1}, ..., \mathbf{d}_{i_L}) \\ =&\left\{\mathbf{d} \in \mathbb{R}^2 \mid \cap_{\ell=1}^{L} \left\{(P_{\pi(i_\ell)} S_{\pi(i_\ell)})^{-\frac{1}{\beta}} \left\|\mathbf{d} - \mathbf{d}_{i_\ell}\right\| \right.\right. \\ &\left.\left. < (P_{\pi(j)} S_{\pi(j)})^{-\frac{1}{\beta}} \left\|\mathbf{d} - \mathbf{d}_j\right\| \right\}, j \notin \{i_1, ..., i_L\}\right\}. \end{aligned} \quad (2)$$

The users located in $\mathcal{V}_L^{\mathrm{w}}(\mathbf{d}_{i_1}, ..., \mathbf{d}_{i_L})$ are connected to the coordination set C. Naturally, the typical user is also located in $\mathcal{V}_L^{\mathrm{w}}(\mathbf{d}_{i_1}, ..., \mathbf{d}_{i_L})$, i.e., $\mathbf{o} \in \mathcal{V}_L^{\mathrm{w}}(\mathbf{d}_{i_1}, ..., \mathbf{d}_{i_L})$. By allocating the orthogonal time-frequency resources to adjoint Voronoi regions, a conflict between any two different clusters can be prevented so that each cluster can serve the connected users simultaneously. Optimizing the resources allocated to each Voronoi region is a challenging yet important problem, and will be interesting future work. We note that in a simple case $K = 1$ and $L = 2$, this problem can be solved by using cooperative base station coloring [11].

C. Feedback Model

We explain the feedback process focusing on the typical user. This process is applied to other users equivalently. Let us assume that the typical user feeds back the channel information to a BS located at $\mathbf{d}_i$. First, the typical user estimates the channel coefficient vector $\mathbf{h}_{1,i} \in \mathbb{C}^{N_{\pi(i)}}$, indicating a channel coefficient vector from the BS at $\mathbf{d}_i$ to the typical user. To focus on the effect of limited feedback, we assume that the channel estimation is perfect. Once the typical user learns the channel coefficient vector $\mathbf{h}_{1,i}$, it quantizes the channel direction information $\tilde{\mathbf{h}}_{1,i} = \mathbf{h}_{1,i}/\|\mathbf{h}_{1,i}\|$ by using a predefined quantization codebook $\mathcal{Q}$. The codebook $\mathcal{Q}$ is shared with the BS at $\mathbf{d}_i$ and the typical user. Assuming that B bits are used for quantizing $\tilde{\mathbf{h}}_{1,i}$, the codebook $\mathcal{Q}$ is constructed as $\mathcal{Q} = \{\mathbf{w}_1, ..., \mathbf{w}_{2^B}\}$, where each codeword $\mathbf{w}_j$ is a $N_{\pi(i)}$-dimensional unit norm vector, i.e., $\|\mathbf{w}_j\| = 1$ for $j \in \{1, ..., 2^B\}$. Then, the codeword that has maximum inner product with $\tilde{\mathbf{h}}_{1,i}$ is selected, namely

$$j_{\max} = \underset{j=1,...,2^B}{\arg\max} \left|(\tilde{\mathbf{h}}_{1,i})^* \mathbf{w}_j\right|. \quad (3)$$

The chosen index $j_{\max}$ is sent to the BS at $\mathbf{d}_i$ and the BS recovers the quantized channel direction information from this index. We denote the quantized channel direction as $\hat{\mathbf{h}}_{1,i} = \mathbf{w}_{j_{\max}}$.

For analytical tractability, we adopt the quantization cell approximation [12]–[14] instead of using a specific limited feedback strategy. This approximates each quantization cell as a Voronoi region of a spherical cap [15]. In this technique, assuming that B-bits feedback is used, the area of the quantization cell is 2^{-B} and this leads to an expression of the CDF of quantization error

$$F_{\sin^2\theta_i}(x) = \begin{cases} 2^B x^{N_{\pi(i)}-1}, & 0 \le x \le \delta \\ 1, & \delta \le x \end{cases}, \quad (4)$$

where $\sin^2\theta_i = 1 - \left|(\tilde{\mathbf{h}}_{1,i})^* \hat{\mathbf{h}}_{1,i}\right|^2$ and $\delta = 2^{-\frac{B}{N_{\pi(i)}-1}}$. In isotropic channel distribution, this approximation technique provides an upper bound of the quantization performance, while the gap to a lower bound provided by random vector quantization is reasonably small [14].

D. Performance Metrics

Since cellular systems are usually interference limited [16], we focus on the SIR. The beamforming vector is designed as multi-cell ZF to mitigate the intra-cluster interference. Specifically, the beamforming vector used in the BS at $\mathbf{d}_{i_1}$, denoted as $\mathbf{v}_{i_1}$, satisfies

$$(\hat{\mathbf{h}}_{\ell,i_1})^* \mathbf{v}_{i_1} = 0, \ \|\mathbf{v}_{i_1}\| = 1, \ \ell \in \mathcal{C}\backslash 1, \quad (5)$$

where $\hat{\mathbf{h}}_{\ell,i_1}$ is the quantized channel coefficient vector from the BS at $\mathbf{d}_{i_1}$ to the user ℓ associated with the BS $\mathbf{d}_{i_\ell}$. The solution of (5) always exists if $L \le \min_{i_\ell \in \mathcal{C}} N_{\pi(i_\ell)}$. We denote that the feedback used for i_ℓ-th BS in the coordination set as B_{i_ℓ}. Since the feedback information is only used for managing the intra-cluster interference, the typical user does not send the feedback to its associated BS, i.e., $B_{i_1} = 0$. The total feedback used in one coordination set is $B_{\text{total}} = \sum_{\ell=2}^{L} B_{i_\ell}$. For analytical simplicity, we assume that all the BSs in the same coordination set $\mathcal{C}$ use only L antennas for multi-cell ZF, so that effectively the typical user has L-dimensional channel to each intra-cluster BS.

Due to the inaccurate channel feedback, the intra-cluster interference is not perfectly nullified. Considering the remaining intra-cluster interference, the instantaneous SIR is

$$\text{SIR} = \frac{P_{\pi(i_1)} \|\mathbf{d}_{i_1}\|^{-\beta} \left|(\mathbf{h}_{1,i_1})^* \mathbf{v}_{i_1}\right|^2}{I_{\text{In}} + I_{\text{Out}}}, \quad (6)$$

where $I_{\text{In}} = \sum_{i_\ell \in \mathcal{C}\backslash i_1} P_{\pi(i_\ell)} \|\mathbf{d}_{i_\ell}\|^{-\beta} \left|(\mathbf{h}_{1,i_\ell})^* \mathbf{v}_{i_\ell}\right|^2$, $I_{\text{Out}} = \sum_{j \in \mathbb{N}\backslash \mathcal{C}} P_{\pi(j)} \|\mathbf{d}_j\|^{-\beta} \left|(\mathbf{h}_{1,j})^* \mathbf{v}_j\right|^2$, each of which indicates the remaining intra-cluster interference and the out-of-cluster interference, respectively. Each entry of the channel coefficient vector $\mathbf{h}_{1,i} \in \mathbb{C}^L$ is drawn from independent and identically distributed (IID) complex Gaussian random variables $\mathcal{CN}(0,1)$ indicating Rayleigh fading We assume that the typical user is associated with a BS in the m-th tier, i.e., $\pi(i_1) = m$. Then the CCDF of the conditioned SIR is defined as

$$F^{\text{c}}_{\text{SIR}_{|m}}\left(\beta, \bar{\lambda}_K, \bar{N}_K, \bar{B}_L, \bar{P}_K, \bar{S}_K; \gamma\right) = \mathbb{P}\left[\text{SIR}_{|m} \ge \gamma\right], \quad (7)$$

where a set of the feedback $\bar{B}_L = \{B_{i_2}, ..., B_{i_L}\}$. The ergodic spectral efficiency is defined as

$$R_{|m}\left(\beta, \bar{\lambda}_K, \bar{N}_K, \bar{B}_L, \bar{P}_K, \bar{S}_K\right) = \mathbb{E}\left[\log_2\left(1 + \text{SIR}_{|m}\right)\right]. \quad (8)$$

III. Performance Analysis

In this section, we analyze the performance of BS coordination in a HetNet as a function of relative system parameters. In the performance characterization, a challenging part is obtaining the distribution of the distance to the BS at $\mathbf{d}_{i_L}$ (the PDF of $\|\mathbf{d}_{i_L}\|$). This indicates the distance of the BS located furthest from the typical user in the coordination set $\mathcal{C}$. It is important because it determines a boundary between the intra-cluster interference and the out-of-cluster interference, which is necessary for the feedback allocation. The main source of the difficulty is that each tier uses a different transmit power and biasing factor, so that the intensity measure of aggregated signal power of each tier has different features. Due to this heterogeneity, ordering the BSs according to their biased power across the tiers is complicated. To resolve this, we first derive the following lemma that transforms a K-tier HetNet to a statistically equivalent single-tier network.

Lemma 1 (Transformation lemma)**.** *Consider the ℓ-th tier network for $\ell \in \mathcal{K}$ denoted as $\Phi^{\text{M}}_\ell = \{\mathbf{d}^\ell_i, P_\ell, S_\ell, i \in \mathbb{N}\}$ with density λ_ℓ. The intensity measure of biased signal power of Φ^{M}_ℓ received by the typical user, i.e., $P_\ell S_\ell \|\mathbf{d}^\ell_i\|^{-\beta}$, is statistically* *equivalent to that of* $\Phi^{\mathrm{M}}_{\ell\to k} = \{\mathbf{d}_i^{\ell\to k}, P_k, S_k, i \in \mathbb{N}\}$ *with density* $\tilde{\lambda}_\ell$*, provided that the density* $\tilde{\lambda}_\ell$ *is scaled to*

$$\tilde{\lambda}_\ell = \lambda_\ell \left(\frac{P_\ell S_\ell}{P_k S_k}\right)^{\frac{2}{\beta}}. \tag{9}$$

Proof. By the displacement theorem [8], the intensity measure of biased signal power of Φ^{M}_ℓ experienced by the typical user is

$$\begin{aligned}\Lambda_\ell((0,t]) &= \mathbb{E}\left[\sum_{\mathbf{d}_i^\ell \in \Phi^{\mathrm{M}}_\ell} \mathbf{1}\left(\frac{\|\mathbf{d}_i^\ell\|^\beta}{P_\ell S_\ell} < t\right)\right] \\ &\overset{(a)}{=} 2\pi\lambda_\ell \int_0^{(P_\ell S_\ell t)^{\frac{1}{\beta}}} r\mathrm{d}r \\ &= \pi\lambda_\ell \left(P_\ell S_\ell\right)^{2/\beta} t^{2/\beta}\end{aligned} \tag{10}$$

where (a) follows Campbell's theorem [8]. Similarly, the intensity measure of biased signal power of $\Phi^{\mathrm{M}}_{\ell\to k}$ is

$$\Lambda_{\ell\to k}((0,t]) = \pi\tilde{\lambda}_\ell \left(P_k S_k\right)^{2/\beta} t^{2/\beta}. \tag{11}$$

For this reason, if $\tilde{\lambda}_\ell = \lambda_\ell \left(\frac{P_\ell S_\ell}{P_k S_k}\right)^{\frac{2}{\beta}}$, the two biased signal power becomes equivalent. This completes the proof. □

The implication of Lemma 1 is that by rescaling each density as $\tilde{\lambda}_\ell = \lambda_\ell \left(\frac{P_\ell S_\ell}{P_k S_k}\right)^{\frac{2}{\beta}}$ for $\ell \in \mathcal{K}$, a K-tier HetNet can be transformed to a statistically equivalent network where the transmit power and the biasing factor are same as P_k and S_k. Leveraging this, we obtain the PDF of $\|\mathbf{d}_{i_L}\|$ in the following lemma.

Lemma 2. *Assume that the furthest BS of the coordination set* C *belongs to the* k*-th tier, i.e.,* $\pi(i_L) = k$*. Then the PDF of the distance* $\|\mathbf{d}_{i_L}\|$ *is*

$$f_{\|\mathbf{d}_{i_L}\|}(r) = \frac{2\left(\pi \sum_{i=1}^K \lambda_i \left(\frac{P_i S_i}{P_k S_k}\right)^{2/\beta} r^2\right)^L}{r\Gamma(L)} \exp\left(-\pi \sum_{i=1}^K \lambda_i \left(\frac{P_i S_i}{P_k S_k}\right)^{2/\beta} r^2\right). \tag{12}$$

Proof. We first transform a K-tier HetNet to a single-tier network whose transmit power and biasing factor are equal to P_k and S_k. By exploiting Lemma 1, we rescale the density as $\lambda_\ell \left(\left(P_\ell S_\ell\right)/\left(P_k S_k\right)\right)^{2/\beta}$. By doing this, we transform the ℓ-th tier network to $\Phi^{\mathrm{M}}_{\ell\to k} = \{\mathbf{d}_i^{\ell\to k}, P_k, S_k, i \in \mathbb{N}\}$ with density $\lambda_\ell \left(\left(P_\ell S_\ell\right)/\left(P_k S_k\right)\right)^{2/\beta}$. Note that the original ℓ-th tier network Φ^{M}_ℓ and the transformed ℓ-th tier network $\Phi^{\mathrm{M}}_{\ell\to k}$ are statistically equivalent as shown in Lemma 1. Then, by the superposition theorem [8], the aggregated network $\sum_{\ell\in\mathcal{K}} \Phi^{\mathrm{M}}_{\ell\to k}$ is a homogeneous network with transmit power P_k, biasing factor S_k, and density $\sum_{i=1}^K \lambda_i \left(\frac{P_i S_i}{P_k S_k}\right)^{2/\beta}$. Since $\sum_{\ell\in\mathcal{K}} \Phi^{\mathrm{M}}_{\ell\to k}$ is a homogeneous network, we can use the conventional PDF of the distance presented in [17]. In a homogeneous PPP with density λ, the PDF of the L-th closest point to the origin is

$$f(r) = \frac{2(\lambda\pi r^2)^L}{r\Gamma(L)} e^{-\lambda\pi r^2}. \tag{13}$$

Plugging $\sum_{i=1}^K \lambda_i \left(\frac{P_i S_i}{P_k S_k}\right)^{2/\beta}$ into λ completes the proof. □

Next, we define the intra-cluster BS geometry parameter $\delta_{1,\ell}$, $\ell \in \{2, ..., L\}$ to characterize the relative intra-cluster interference power. We define the geometric parameter $\delta_{1,\ell}$ as the ratio between the path-loss of the home BS and the ℓ-th closest BS for $\ell \in \{2, ..., L\}$, i.e., $\delta_{1,\ell} = \left(P_{\pi(i_\ell)} \|\mathbf{d}_{i_\ell}\|^{-\beta}\right) / \left(P_{\pi(i_1)} \|\mathbf{d}_{i_1}\|^{-\beta}\right)$. We note that the geometric parameter $\delta_{1,\ell}$ is originally introduced in [1], and is generalized for HetNets in our work. As explained in [1], $\delta_{1,\ell}$ measures the relative intra-cluster interference power coming from $\mathbf{d}_{i_\ell}$, so that a large value of $\delta_{1,\ell}$ means large amount of intra-cluster interference. When each biasing factor is same, i.e., $S_1 = ... = S_K$, $\delta_{1,\ell_1} > \delta_{1,\ell_2}$ if $\ell_1 < \ell_2$ by the definition. For general biasing factors, however, this is not necessarily guaranteed. We denote a set of the geometric parameters as $\bar{\delta}_{1,L} = \{\delta_{1,2}, ..., \delta_{1,L}\}$, and analyze the performance under the assumption that the relative intra-cluster interference power is fixed, while out-of-cluster interference is random as in [1].

By using Lemma 2 and the intra-cluster BS geometry, we derive the following theorem that presents the SIR CCDF.

Theorem 1. *Assume that* $\bar{\delta}_{1,L}$*,* $\bar{B}_L$ *is given, and also* $\pi(i_1) = m$*,* $\pi(i_L) = k$*. Then, the conditioned SIR CCDF of a K-tier HetNet is*

$$\begin{aligned}&F^{\mathrm{c}}_{\mathrm{SIR}|m}\left(\beta, \bar{\lambda}_K, \bar{N}_K, \bar{B}_L, \bar{P}_K, \bar{S}_K, \bar{\delta}_{1,L}; \gamma\right) = \\ &\prod_{i_\ell \in C\setminus i_1} \left(\frac{1}{1 + \gamma\delta_{1,\ell} 2^{-\frac{B_{i_\ell}}{L-1}}}\right) \cdot \\ &\left(\frac{\sum_{i=1}^K \lambda_i \left(\frac{P_i S_i}{P_k S_k}\right)^{2/\beta}}{\sum_{i=1}^K \lambda_i \left(\frac{P_i S_i}{P_k S_k}\right)^{2/\beta} \left[1 + \mathcal{D}\left(\gamma\delta_{1,L} \cdot \left(\frac{S_k}{S_i}\right), \beta\right)\right]}\right)^L,\end{aligned} \tag{14}$$

where

$$\mathcal{D}(x,y) = \frac{2x}{y-2} {}_2F_1\left(1, 1 - \frac{2}{y}, 2 - \frac{2}{y}, -x\right), \tag{15}$$

with ${}_2F_1(\cdot,\cdot,\cdot,\cdot)$ *is the Gaussian hypergeometric function.*

Proof. We omit this due to space limitation. Please see the proof of Theorem 2 in [18]. □

For the perfect CSIT case, the conditioned SIR CCDF is obtained by $B_{i_\ell} \to \infty$ for $i_\ell \in C$.

We summarize the conditions presented in Theorem 1. The SIR CCDF is derived under the conditions that (i) $\pi(i_1)$ is fixed as m, (ii) $\pi(i_L)$ is fixed as k, and (iii) $\bar{\delta}_{1,L}$ is fixed, so that the relative intra-cluster interference power is given. Now the ergodic spectral efficiency is derived as an integral form in Corollary 1.

Corollary 1. *Assume that $\bar{\delta}_{1,L}$, $\bar{B}_L$ is given and $\pi(i_L) = k$. Then, the ergodic spectral efficiency of a K-tier HetNet is*

$$R_{|m}\left(\beta, \bar{\lambda}_K, \bar{N}_K, \bar{B}_L, \bar{P}_K, \bar{S}_K, \bar{\delta}_{1,L}\right) = \log_2(e)\int_0^\infty \frac{1}{1+z}\prod_{i_\ell \in C\setminus i_1}\left(\frac{1}{1+z\delta_{1,\ell}2^{-\frac{B_{i_\ell}}{L-1}}}\right)\cdot \left(\frac{\sum_{i=1}^K \lambda_i\left(\frac{P_iS_i}{P_kS_k}\right)^{2/\beta}}{\sum_{i=1}^K \lambda_i\left(\frac{P_iS_i}{P_kS_k}\right)^{2/\beta}\left[1+\mathcal{D}\left(z\delta_{1,L}\cdot\left(\frac{S_k}{S_i}\right),\beta\right)\right]}\right)^L \mathrm{d}z, \quad (16)$$

where $\mathcal{D}(x, y)$ is defined as (15).

Proof. See the proof of Corollary 2 in [18]. □

IV. Feedback Allocation

In this section, we determine B_{i_ℓ} for $\ell \in \{2, ..., L\}$ to maximize the ergodic spectral efficiency (16).

Proposition 1. *The feedback partition that maximizes the ergodic spectral efficiency $R_{|m}\left(\beta, \bar{\lambda}_K, \bar{N}_K, \bar{B}_L, \bar{P}_K, \bar{S}_K, \bar{\delta}_{1,L}\right)$ is*

$$B^\star_{i_\ell} = \frac{B_{\text{total}}}{L-1} + (L-1)\log_2\left(\frac{\delta_{1,\ell}}{\left(\prod_{\ell=2}^L \delta_{1,\ell}\right)^{\frac{1}{L-1}}}\right). \quad (17)$$

Proof. We first formulate the optimization problem for maximizing the SIR CCDF (14). Since the Laplace transform of the out-of-cluster interference is independent to the feedback, we can treat this as a constant and omit it in the problem. Then the problem is

$$\underset{B_{i_\ell}\in\mathbb{Z}^+, \ell\in\{2,\dots,L\}}{\text{maximize}} \prod_{i_\ell\in C\setminus i_1}\left(\frac{1}{1+\gamma\delta_{1,\ell}2^{-\frac{B_{i_\ell}}{L-1}}}\right),$$
$$\text{subject to } \sum_{\ell=2}^L B_{i_\ell} \le B_{\text{total}}. \quad (18)$$

Since (18) is integer programming which is hard to solve, we first relax the feasible field of B_ℓ to $\mathbb{R}^+$ and apply the floor function to the solution later. We rewrite (18) as

$$\underset{B_{i_\ell}\in\mathbb{R}^+, \ell\in\{2,\dots,L\}}{\text{minimize}} \sum_{\ell=2}^L \ln\left(1+\gamma\delta_{1,\ell}2^{-\frac{B_{i_\ell}}{L-1}}\right),$$
$$\text{subject to } \sum_{\ell=2}^L B_{i_\ell} \le B_{\text{total}}. \quad (19)$$

Since the function $f(B) = \ln(1 + C2^{-\frac{B}{L-1}})$ is monotonically increasing function and convex for any positive C, we apply a convex optimization technique to solve (19). At first, the corresponding Lagrangian function of the objective function in (19) is

$$L(\bar{B}_L, \mu) = \sum_{\ell=2}^L \ln\left(1+\gamma\delta_{1,\ell}2^{-\frac{B_{i_\ell}}{L-1}}\right) + \mu\left(\sum_{\ell=2}^L B_{i_\ell} - B_{\text{total}}\right), \quad (20)$$

where μ denotes the Lagrangian multiplier. Solving the KKT conditions for (20) leads to

$$B^\star_{i_\ell} = \frac{B_{\text{total}}}{L-1} + (L-1)\log_2\left(\frac{\delta_{1,\ell}}{\left(\prod_{\ell=2}^L \delta_{1,\ell}\right)^{\frac{1}{L-1}}}\right). \quad (21)$$

Since the obtained feedback partition (21) is not a function of a specific threshold γ, this is optimal for any threshold, which means it is optimal for maximizing the ergodic spectral efficiency. This completes the proof. □

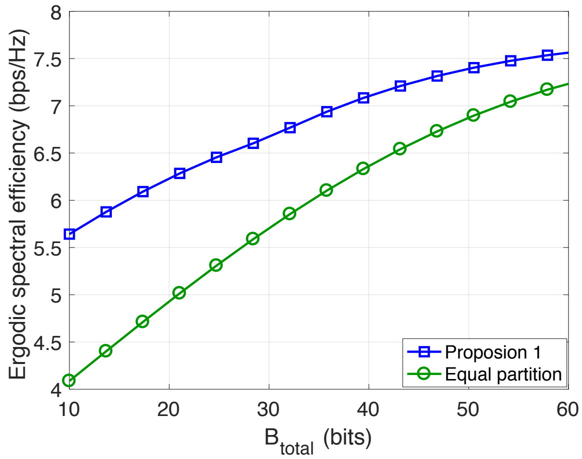
Remark 1. Since the feedback has an positive integer value in practice, we have to perform further processes to the solution $B^\star_{i_\ell}$. We introduce two possible methods. First, we can use the round function $\lfloor B^\star_{i_\ell}\rceil$. With the round function, however, it is possible that $\sum_{\ell=2}^L\lfloor B^\star_{i_\ell}\rceil \ge B_{\text{total}}$, therefore the manual feedback adjustment is necessary after applying the round function. Second, we can iteratively add a feedback bit to each intra-cluster BS. For example, starting with the floored solution $\lfloor B^\star_{i_\ell}\rfloor$, we iteratively find which BS is the best choice for adding a remaining feedback bit by computing the sum ergodic spectral efficiency. Subsequently, we add a feedback bit to the selected tier. We repeat this until the used feedback equals to B_{total}. Due to space limitation we do not explore these methods in this paper.

Remark 2. Proposition 1 implies that the feedback is allocated proportional to the intra-cluster interference power, i.e., $B_{i_\ell} \propto \delta_{1,\ell}$. Note that this is similar to the previous results [6], in which adaptive feedback allocation is proposed in a homogeneous cooperative network for minimizing the rate gap to perfect CSIT case. One noticeable point is that Proposition 1 only depends on relative intra-cluster BS power while it does not change depending on instantaneous SIR.

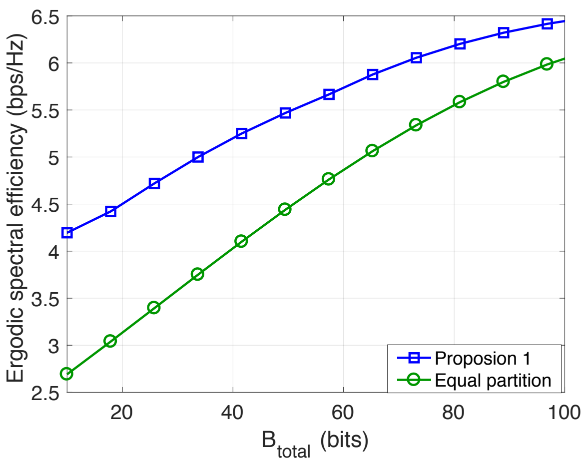
Remark 3. Since we only use a part of the antennas, our result may indicate a lower bound on the spectral efficiency that can be achieved by using the full antennas. Using the full antennas, the BSs in the coordination set can mitigate the intra-cluster interference and also increase the desired signal power by coordinated beamforming. In this case, $B_1 > 0$ unlike the case that we consider in this paper. For this reason, in the full antenna case, feedback allocation should be different from Proposition 1. This case is covered in [18] in detail. As observed in [18], the full antenna increases the spectral efficiency compared to the reduced antenna case.

V. Numerical Results

In this section, we demonstrate the proposed feedback allocation with numerical simulations. Specifically, we compare the ergodic spectral efficiency of Proposition 1 and the baseline method in Fig. 1, whose caption includes the simulation setting. The baseline method is the equal partition, where the feedback is equally partitioned to each of intra-cluster BS, i.e., $B_{i_2} = .. = B_{i_L} = B_{\text{total}}/(L-1)$. In Fig. 1-(a), we have 38.2% spectral efficiency gain by using Proposition 1 at $B_{\text{total}} = 10$,



(a)



(b)

Fig. 1. The ergodic spectral efficiency comparison in a cooperative HetNet. A 3-tier HetNet is assumed. The simulation parameters are as follows: In (a), $\bar{N}_K = \{L, L, L\}$, $L = 4$, $\bar{\lambda}_K = \{1\lambda_{\text{ref}}, 10\lambda_{\text{ref}}, 20\lambda_{\text{ref}}\}$ where $\lambda_{\text{ref}} = 10^{-4}/\pi$, $\bar{P}_K = \{20, 15, 10\}$dBm, $\bar{S}_K = \{0, 3, 5\}$dB, $\bar{\delta}_{1,L} = \{0.1, 0.01, 0.001\}$, $\pi(i_1) = 1$, $\pi(i_L) = 2$, and $\beta = 4$. In (b), the other parameters are same except that $L = 5$ and $\bar{\delta}_{1,L} = \{0.2, 0.04, 0.008, 0.0016\}$.

and in Fig. 1-(b), we have 56.2% gain at $B_{\text{total}} = 10$. We observe that Proposition 1 provides more gains when 1) L increases or 2) B_{total} decreases. This is because, when L increases or B_{total} decreases, the equal partition allocates smaller amount of feedback to the strong BSs whose $\delta_{1,\ell}$ is large. Then, due to lack of sufficient feedback, the interference from those strong BSs is not mitigated well, resulting in significant spectral efficient loss. On the contrary, by using Proposition 1, the appropriate amount of feedback is allocated to each BS proportional to $\delta_{1,\ell}$, so that considerable spectral efficiency gain is obtained even when L increases or B_{total} decreases.

VI. Conclusions

In this paper, we studied BS coordination in HetNets and proposed feedback allocation methods to improve the spectral efficiency. We considered that the BSs form a coordination set using dynamic clustering applied across the tiers. Using stochastic geometry, we characterized the SIR CCDF and the ergodic spectral efficiency mainly as functions of the feedback and other relevant system parameters. To do this, we derived a lemma that transforms a HetNet to a statistically equivalent single-tier network where characterizing the performance of BS coordination is much simple. Leveraging the obtained expressions, we formulated a feedback allocate problem and proposed a solution. The simulation results showed that the proposed feedback partitions bring gains in the spectral efficiency compared to the equal partition. One observation in the proposed feedback allocation is that using more feedback to strong BSs is efficient for improving the spectral efficiency.

Acknowledgement

This work was supported in part by the National Science Foundation under Grant NSF-CCF-1319556, in part by the Institute for Information & communications Technology Promotion (IITP) under grant funded by the MSIT of the Korea government (No.2018(2016- 0-00123), Development of Integer-Forcing MIMO Transceivers for 5G & Beyond Mobile Communication Systems.

References

[1] N. Lee, D. Morales-Jimenez, A. Lozano, and R. W. Heath, "Spectral efficiency of dynamic coordinated beamforming: A stochastic geometry approach," *IEEE Trans. Wireless Comm.*, vol. 14, no. 1, pp. 230–241, Jan. 2015.

[2] J. Park, N. Lee, and R. W. Heath, "Performance analysis of pair-wise dynamic multi-user joint transmission," in *Proc. IEEE Int. Conf. on Comm.*, Jun. 2015, pp. 3981–3986.

[3] G. Nigam, P. Minero, and M. Haenggi, "Coordinated multipoint joint transmission in heterogeneous networks," *IEEE Trans. Comm.*, vol. 62, no. 11, pp. 4134–4146, Nov. 2014.

[4] A. H. Sakr and E. Hossain, "Location-aware cross-tier coordinated multipoint transmission in two-tier cellular networks," *IEEE Trans. Wireless Comm.*, vol. 13, no. 11, pp. 6311–6325, Nov. 2014.

[5] J. Park, J. G. Andrews, and R. W. Heath, "Inter-operator base station coordination in spectrum-shared millimeter wave cellular networks," *to appear in IEEE Trans. Cognitive Comm. and Networking*, 2018.

[6] S. Akoum and R. W. Heath, "Interference coordination: Random clustering and adaptive limited feedback," *IEEE Trans. Sig. Proc.*, vol. 61, no. 7, pp. 1822–1834, Apr. 2013.

[7] C. Li, J. Zhang, M. Haenggi, and K. B. Letaief, "User-centric intercell interference nulling for downlink small cell networks," *IEEE Trans. Comm.*, vol. 63, no. 4, pp. 1419–1431, Apr. 2015.

[8] F. Baccelli and B. Blaszczyszyn, *Stochastic Geometry and Wireless Networks, Volume I - Theory*, ser. Foundations and Trends in Networking. Now Publishers, 2009, vol. 3.

[9] S. Singh and J. G. Andrews, "Joint resource partitioning and offloading in heterogeneous cellular networks," *IEEE Trans. Wireless Comm.*, vol. 13, no. 2, pp. 888–901, Feb. 2014.

[10] H. S. Jo, Y. J. Sang, P. Xia, and J. G. Andrews, "Heterogeneous cellular networks with flexible cell association: A comprehensive downlink SINR analysis," *IEEE Trans. Wireless Comm.*, vol. 11, no. 10, pp. 3484–3495, Oct. 2012.

[11] J. Park, N. Lee, and R. W. Heath, "Cooperative base station coloring for pair-wise multi-cell coordination," *IEEE Trans. Comm.*, vol. 64, no. 1, pp. 402–415, Jan. 2016.

[12] K. K. Mukkavilli, A. Sabharwal, E. Erkip, and B. Aazhang, "On beamforming with finite rate feedback in multiple-antenna systems," *IEEE Trans. Info. Th.*, vol. 49, no. 10, pp. 2562–2579, Oct. 2003.

[13] S. Zhou, Z. Wang, and G. B. Giannakis, "Quantifying the power loss when transmit beamforming relies on finite-rate feedback," *IEEE Transactions on Wireless Communications*, vol. 4, no. 4, pp. 1948–1957, Jul. 2005.

[14] J. Park, N. Lee, J. G. Andrews, and R. W. Heath, “On the optimal feedback rate in interference-limited multi-antenna cellular systems,” *IEEE Trans. Wireless Comm.*, vol. 15, no. 8, pp. 5748–5762, Aug. 2016.
[15] A. Gersho, “Asymptotically optimal block quantization,” *IEEE Trans. Info. Th.*, vol. 25, no. 4, pp. 373–380, Jul. 1979.
[16] H. S. Dhillon, R. K. Ganti, F. Baccelli, and J. G. Andrews, “Modeling and analysis of K-tier downlink heterogeneous cellular networks,” *IEEE Jour. Select. Areas in Comm.*, vol. 30, no. 3, pp. 550–560, Apr. 2012.
[17] M. Haenggi, “On distances in uniformly random networks,” *IEEE Trans. Info. Th.*, vol. 51, no. 10, pp. 3584–3586, Oct. 2005.
[18] J. Park, N. Lee, and R. W. Heath, “Feedback design for multi-antenna K-tier heterogeneous downlink cellular networks,” *to appear in IEEE Trans. Wireless Comm.*, 2018.