UAV Placement Games for Optimal Wireless Service Provision

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Abstract—Unmanned Aerial Vehicle (UAV) networks have emerged as a promising technique to rapidly provide wireless services to a group of mobile users simultaneously in the threedimensional (3D) geographical space, where a flying UAV facility can be deployed closely based on users' 3D location reports. The paper aims to address a challenging issue that each user is selfish and prefers the UAV to be located as close to himself as possible, by misreporting his location and changing the optimal UAV location. We study the social planner's problem to determine the final deployment location of a UAV facility in a 3D space, by ensuring all selfish users' truthfulness in reporting their locations. To minimize the social service cost in this UAV placement game, we design a strategyproof mechanism with approximation ratio 2, when comparing to the social optimum. On the other hand, as the UAV to be deployed may interfere with another group of incumbent users in the same space, we also study the obnoxious UAV placement game to maximally keep their social utility, where each incumbent user may misreport his location to keep the UAV away from him. We propose a strategyproof mechanism with approximation ratio 5. Besides the worst-case analysis, we further analyze the empirical performances of the proposed mechanisms and show that they converge to the social optimum as the number of users becomes large. Finally, we extend to the dual-preference UAV placement game by considering the coexistence of the two groups of users, where users can misreport both their locations and preference types. We successfully propose a strategyproof mechanism with approximation ratio 8.

I. Introduction

The use of unmanned aerial vehicles (UAVs) as flying cell cites is a promising technique to dynamically solve the coverage problem of existing wireless networks [8]. Traditional base stations are deployed at fixed locations on the ground for a long term by catering to the average traffic load in the two-dimensional area, while flying UAVs' deployment does not have such constraint in space or time. Owing to their agility and mobility, UAVs can be quickly deployed as alternatives to meet time-varying traffic load. Wireless carriers such as AT&T started to use UAVs to opportunistically boost wireless coverage for crowds in big concerts and sports, where people continuously post their selfies and videos online [10]. Moreover, UAVs can be rapidly deployed in events of disasters to enable air-to-ground communications if the territorial base stations fail to work. For example, Verizon successfully launched an exercise in mid 2017 to deploy UAVs to Cape May, New Jersey, and provide local users with LTE connectivity [11]. Upon UAV deployment, many of these UAV placement schemes need to know users' locations beforehand for closely servicing them.

To fully reap the benefits of UAV-enabled wireless services, one must decide the final UAV location to keep serving a group of target users in a 3D geographical space. As the UAV number is small as compared to the target user size, final UAV placement needs to balance all target users' different locations. Such problems have been recently investigated in the literature by assuming that the UAV knows the real locations or at least the distribution of mobile users (e.g., [1], [15], [17]). For example, [1] aimed to maximize the UAV's wireless coverage on the ground, by considering the air-to-ground signal propagation. [15] improved the energyefficiency of UAV communication with ground users by designing the UAV's trajectory. [17] studied how to minimize the deployment delay of UAVs till providing the full wireless coverage in the worse scenario. Differently, we aim to study the optimal UAV placement without knowing any user's location information beforehand, by requesting users' direct information revelation.

In practice, it can be difficult to quickly collect users' true location data, and traditional user positioning techniques require multiple base stations' continuous help or users' GPS reporting [3], [5]. When requiring UAV helps, ground network infrastructure are often congested and may even fail to work in events of disasters, which makes it difficult for accurately tracking users' locations [6]. It is desirable for the social planer to directly ask users to report their own locations upon UAV deployment. In this case, however, the key challenge for the optimal UAV placement is that, users are selfish (preferring the closest UAV location to themselves) and may not report their true locations to help the UAV placement for best serving all users. Consider an illustrative uplink example that we deploy a UAV to a point in a line interval for serving user 1 at location $x_1 = 0$ and user 2 at location $x_2 = 2$ simultaneously. Each user prefers the final UAV location to be as close to his own location as possible to obtain higher signal-to-noise ratio or save the transmission power. If the two users report truthfully, the UAV chooses to locate at the mean of the users' locations (i.e., x = 1). However, if user 2 misreports his location from $x_2 = 2$ to $x_2' = 4$, then mean UAV location changes to x=2 which is the closest to user 2. In this paper, we aim to study the strategyproof (truthful) mechanish design, where users should be motivated to report their locations truthfully.

Besides the UAV placement game, we also study the obnoxious UAV placement game. As the new UAV facility may interfere with another group of incumbent (adverse) users in the same space, we want to best control the interference and maximally keep these users' social utility when locating the UAV. In this game, we also require all such users to report their locations for determining the UAV location, where a user may misreport his location to mislead the final UAV location to be further away from his true location to reduce interference from the UAV signal.

Finally, as both UAV facility users (who prefer to be close to the UAV) and adverse users (who prefer to be far away from the UAV) may coexist at the same time, we want to reach the good balance between service quality and interference control when locating the UAV, and require strategyproof mechanism design to ensure all involved users' truthfulness in reporting locations and even preference types.

In our paper, we study a family of strategyproof mechanisms for the three kinds of UAV placement games in the 3D space. Our key novelty and main contributions are summarized as follows.

- Novel UAV placement games under information asymmetry: To our best knowledge, our paper is the first to propose and analyze UAV placement games for optimal wireless service provision without knowing target users' locations. We completely study a UAV placement game for facility users, an obnoxious UAV placement game for adverse users, and a dual-preference UAV placement game for both groups of users, where users are selfish and may misreport their locations to mislead the UAV placement. We aim to propose strategyproof mechanisms for these three games to ensure truthful location reporting and optimize the social cost/utility.
- Mechanism design for the UAV placement game: In Section III, we propose two strategyproof mechanisms such that any user's misreporting of his locations can only increase his service cost. Especially, we design the weighted median strategyproof mechanism with approximation ratio 2, when comparing to the social optimal cost under ideally full information. Besides the worst-case analysis of the proposed strategyproof mechanisms, we also analyze the empirical performances of the mechanisms and show that they converge to the social optimum as the number of users becomes large.
- Mechanism design for the obnoxious UAV placement game: In Section IV, we consider the opposite problem of locating an obnoxious UAV facility in the 3D space. Each user attempts to stay far away from the UAV to reduce its signal interference, by misreporting his location. Our target is to design strategyproof mechanisms of UAV placement to maximally keep the social utility. Accordingly, we design a strategyproof mechanism with approximation ratio 5. Our empirical analysis further shows that proposed mechanism converges to the social optimum as the number of users becomes large.
- Mechanism design for the dual-preference UAV placement game: In Section V, we extend to the general

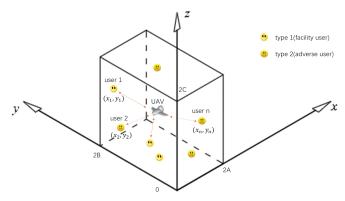


Fig. 1: System model about the UAV placement on the 3D space. There are generally two types of users: type 1 (facility users) and type 2 (adverse users). In the UAV placement game, all users are of type 1; in the obnoxious UAV placement game, all users are of type 2; and in the dual-preference UAV placement game, both types of users coexist.

case of the dual-preference UAV placement game by including both facility users and adverse users. Besides locations, we further allow users to misreport their preference types. We successfully design a mechanism with approximation ratio 8.

A. Related work

There are some studies on the generic facility location game and strategyproof mechanisms to prevent users from misreporting locations. Such mechanisms are just based on users' location reports and are easy to implement (without using complicated schemes such as pricing). For example, [12] studied median strategyproof mechanisms with provable approximation ratios on a one-dimensional line, which gives us some inspiration of building our Strategyproof mechanism in the UAV placement game. [13] provided characterizations of strategyproof mechanisms on special line, tree, and cycle networks. In the obnoxious facility location game, the mechanism design for the objective of maximizing total utility was first studied by [2]. It presented a 3-approximation group strategyproof deterministic mechanism. [7] provided the randomized strategyproof mechanism with the approximation ratio of 4 in general metric spaces. [18] investigated the properties of the facility location game with dual-preference.

Such works mostly focus on facility placement in onedimension, while the real UAV placement is in 3D. Further, our paper practically models that users have different wireless service sensitivities (weights) and preference types for the UAV and they can also cheat on such data (besides their locations). Such unique wireless features translate to a new problem objective and require new methods in designing the strategyproof mechanisms and proving approximation ratios.

II. SYSTEM MODEL

Let $N=\{1,2,\cdots,n\}$ be the set of users that are located in the 3D space I^3 . Without loss of generality, we suppose I^3 is a finite cuboid $[0,2A]\times[0,2B]\times[0,2C]$ containing all n users as shown in Fig. 1. The

real location of user $i \in N$ is $(x_i, y_i, z_i) \in I^3$. We denote $\mathbf{x} = (x_1, x_2, \cdots, x_n), \mathbf{y} = (y_1, y_2, \cdots, y_n)$ and $\mathbf{z} = (z_1, z_2, \cdots, z_n)$ as users' location profiles in the 3D space. Depending on the users' locations, the final UAV's location is denoted as point (x, y, z). The distance between user i and the UAV after deployment thus is $d((x_i, y_i, z_i), (x, y, z)) = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2}$.

We first introduce the UAV placement game, where each user (of type 1 in Fig. 1) prefers the UAV location to be close to his own location for saving his service cost. The service cost of a particular user i to associate with the UAV is modelled as the weighted square distance to the UAV location (x, y, z). This is motivated by a typical uplink UAV-enabled communication scenario, where a user i consumes power c_i as his service cost to transmit signal to the UAV access point. Following the widely used light of sight link model for UAV communications [9], [16], his signal attenuates over distance according to path loss exponent 2. To let the received signal strength at the UAV exceed a decodable requirement w_i , i.e., $c_i d((x_i, y_i, z_i), (x, y, z))^{-2} \ge w_i$, user i bears the minimum service cost $c_i = w_i d((x_i, y_i, z_i), (x, y, z))^2$ as in [14]. Here, weight $w_i > 0$ models user i's sensitivity in his specific traffic application (e.g., voice or video), and he prefers the UAV to be closely located for his cost saving. If a user has a larger weight, the UAV should be located closer to him. We denote w_i as user i's weight and $\mathbf{w} = (w_1, w_2, \cdots, w_n)$ as weight profile. Note that Each user i's weight w_i can be estimated by the UAV and is public information. Yet the UAV does not know the users' location profiles x, y and z. We denote $\Omega = \{x, y, z | w\}$ as the full user profile. The UAV's objective is to minimize the sum of weighted costs by choosing (x, y, z) based on users location reports.

In the UAV placement game, a mechanism outputs a UAV location (x,y,z) based on a given profile Ω and thus is a function $f:I^{3n}\to I^3$, i.e., $(x,y,z)=f(\mathbf{x},\mathbf{y},\mathbf{z})$. As explained, the cost of user i is his weighted square distance to the UAV. The cost of user i is given by,

$$c_i(f(\mathbf{x}, \mathbf{y}, \mathbf{z}), (x_i, y_i, z_i))$$

= $w_i((x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2).$ (1)

Let $\mathbf{x}_{-i} = (x_1, \cdots, x_{i-1}, x_{i+1}, \cdots, x_n)$, $\mathbf{y}_{-i} = (y_1, \cdots, y_{i-1}, y_{i+1}, \cdots, y_n)$ and $\mathbf{z}_{-i} = (z_1, \cdots, z_{i-1}, z_{i+1}, \cdots, z_n)$ denote the location profiles for all users except user i. The social cost of a mechanism f is defined as the sum of all users' costs, i.e.,

$$SC(f(\mathbf{x}, \mathbf{y}, \mathbf{z}), (\mathbf{x}, \mathbf{y}, \mathbf{z})) = \sum_{i=1}^{n} c_i(f(\mathbf{x}, \mathbf{y}, \mathbf{z}), (x_i, y_i, z_i)).$$
(2)

In the following, we formally define the strategyproofness for mechanism design in the UAV placement game.

Definition 1. A mechanism is strategyproof in the UAV placement game if no user can benefit from misreporting his location. Formally, given profile $\Omega = (\langle x_i, \mathbf{x}_{-i} \rangle, \langle y_i, \mathbf{y}_{-i} \rangle, \langle z_i, \mathbf{z}_{-i} \rangle | \mathbf{w}) \in I^{3n}$, and any misreported location $(x_i', y_i', z_i') \in I^3$ for any user $i \in N$, it holds that

$$c_i(f((x_i, y_i, z_i), (\mathbf{x}_{-i}, \mathbf{y}_{-i}, \mathbf{z}_{-i})), (x_i, y_i, z_i))$$

$$\leq c_i(f((x_i', y_i', z_i'), (\mathbf{x}_{-i}, \mathbf{y}_{-i}, \mathbf{z}_{-i})), (x_i, y_i, z_i)).$$

For the UAV placement game, we are interested in designing strategyproof mechanisms that perform well with respect to minimizing the social cost. Given a location profile Ω , let $OPT_1(\mathbf{x},\mathbf{y},\mathbf{z})$ be the optimal social cost. A strategyproof mechanism f has an approximation ratio $\gamma \geq 1$, if for any location profile $(\mathbf{x},\mathbf{y},\mathbf{z}) \in I^{3n}, \gamma OPT_1(\mathbf{x},\mathbf{y},\mathbf{z}) \geq SC(f,(\mathbf{x},\mathbf{y},\mathbf{z}))$.

Similarly, in the obnoxious UAV placement game, the UAV faces a different group of n adverse users (of type 2 in Fig. 1) and introduces downlink interference to them. They prefer to be far away from the UAV and their (positive) weights w_i 's here tell their different interference sensitivities in their traffic applications. We define adverse user i's utility u_i as $w_id((x_i,y_i,z_i),(x,y,z))^2$ under interference, which is the same as (1). u_i nonlinearly increases with the distance from the UAV. Opposite to the UAV placement game, the objective in this game is to maximize the sum of weighted utility, by designing strategyproof mechanisms f(x,y,z) for the UAV placement. The social utility of a mechanism f is defined as:

$$SU(f(\mathbf{x}, \mathbf{y}, \mathbf{z}), (\mathbf{x}, \mathbf{y}, \mathbf{z})) = \sum_{i=1}^{n} u_i(f(\mathbf{x}, \mathbf{y}, \mathbf{z}), (x_i, y_i, z_i)).$$
(3)

Next, we formally define the strategyproofness for the obnoxious UAV placement game.

Definition 2. A mechanism is strategyproof in the obnoxious UAV placement game if no adverse user can benefit from misreporting his location. Formally, given profile $\Omega = (\langle x_i, x_{-i} \rangle, \langle y_i, y_{-i} \langle y_i, z_{-i} \rangle | \mathbf{w}) \in I^{3n}$, and any misreported location $(x_i', y_i', z_i') \in I^3$ for user i, it holds that

$$u_i(f((x_i, y_i, z_i), (\mathbf{x}_{-i}, \mathbf{y}_{-i}, \mathbf{z}_{-i})), (x_i, y_i, z_i))$$

 $\geq u_i(f((x'_i, y'_i, z'_i), (\mathbf{x}_{-i}, \mathbf{y}_{-i}, \mathbf{z}_{-i})), (x_i, y_i, z_i)).$

For the obnoxious UAV placement game, we are interested in designing strategyproof mechanisms that perform well with respect to maximizing the social utility. Given a location profile Ω , let $OPT_2(\mathbf{x}, \mathbf{y}, \mathbf{z})$ be the optimal social utility. A strategyproof mechanism f has an approximation ratio $\gamma \geq 1$, if for any location profile $(\mathbf{x}, \mathbf{y}, \mathbf{z}) \in I^{3n}$, $OPT_2(\mathbf{x}, \mathbf{y}, \mathbf{z}) \leq \gamma SU(f, (\mathbf{x}, \mathbf{y}, \mathbf{z}))$.

We will introduce the model of the dual-preference UAV placement game in Section V, by combining the models of the two games defined above.

III. UAV PLACEMENT GAME

In this section, we design strategyproof mechanisms for the UAV placement game. According to (1) and (2), we have the following social cost objective

$$SC(f, (\mathbf{x}, \mathbf{y}, \mathbf{z})) = \sum_{i=1}^{n} w_i((x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2),$$

which is a convex function with respect to (x,y,z). By checking the first-order conditions, we obtain weighted mean $(x,y,z)=(\bar{x},\bar{y},\bar{z})$ as the optimal location, where

$$\bar{x} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}, \bar{y} = \frac{\sum_{i=1}^{n} w_i y_i}{\sum_{i=1}^{n} w_i} \text{ and } \bar{z} = \frac{\sum_{i=1}^{n} w_i z_i}{\sum_{i=1}^{n} w_i}.$$
 (4)

However, this weighted mean mechanism is not strategyproof as we explained in the illustrative example in Section I.

A. Design and Analysis of strategyproof mechanisms

In the following, we present two strategyproof mechanisms inspired by the median mechanism on a line.

Mechanism 1. Given a profile Ω , return median location $(x,y,z) = med(x,y,z) = (x_{med},y_{med},z_{med})$ as the UAV location, where x_{med} is the median of \mathbf{x} , y_{med} is the median of y and z_{med} is the median of z.

We change the optimal mean location to median location in this mechanism. Before we show it is strategyproof and evaluate its worst-case performance, we show Lemma 1 first.

Lemma 1. Given a set of numbers $\{a_1, \ldots, a_n\}$, we have $\sum_{i=1}^{n} (a_i - m)^2 \leq 2 \sum_{i=1}^{n} (a_i - \mu)^2$, where m is median of $\{a_1, \ldots, a_n\}$ and μ is mean of $\{a_1, \ldots, a_n\}$.

Proof. It is true that $|m-\mu| \leq \sigma$ from Minimization Property of the Median [4], where σ is the standard deviation of set $\{a_1,\ldots,a_n\}$. We have

$$|m - \mu| \le \sigma$$

$$\Leftrightarrow n(m - \mu)^2 \le \sum_{i=1}^n (a_i - \mu)^2$$

$$\Leftrightarrow \sum_{i=1}^n (a_i - \mu)^2 + \sum_{i=1}^n (\mu - m)^2 \le 2 \sum_{i=1}^n (a_i - \mu)^2$$

$$\Leftrightarrow \sum_{i=1}^n (a_i - \mu + \mu - m)^2 \le 2 \sum_{i=1}^n (a_i - \mu)^2,$$

since $2(\mu - m) \sum_{i=1}^{n} (a_i - \mu) = 0$.

Theorem 1. Define $w_{max} = \max\{w_1, \dots, w_n\}$ and $w_{min} =$ $\min\{w_1,\ldots,w_n\}$. Mechanism 1 is a strategyproof mechanism with approximation ratio $2\frac{w_{max}}{w_{min}}$ as compard to the social optimum.

Proof. First we prove Mechanism 1 is a strategyproof mechanism. Assume $x_1 \leq x_2, \ldots, \leq x_n$ without loss of generality, and x-location of UAV x_{med} is x_j (ie, $x = x_{med} =$ x_i). If user i ($i \le j$) chooses to misreport his x location, we have two situations: (i) The misreported x-value is smaller than the original x-value x_i and the x-value of the new UAV location (ie, x) will not change; (ii) The misreported x-value is greater than the original x-value x_i and the x-value of the new UAV location (ie, x) will not smaller than x_i . However, $(x_i - x)^2$ will not decrease which means his cost $w_i((x_i-x)^2+(y_i-y)^2+(z_i-z)^2)$ will not decrease. Therefore user i cannot decrease his cost by misreporting his x_i and similarly he cannot decrease his cost by misreporting his y_i and z_i in the other independent directions. Similar arguments hold symmetrically for i > j.

Next, we will prove γ . Observing similarities in x, y, zdomains, we can divide the optimal cost and the social cost of Mechanism 1 into three additive parts. That is, $OPT_1 = OPT_{1,x} + OPT_{1,y} + OPT_{1,z}$, where $OPT_{1,x} = \sum_{i=1}^n w_i (x_i - \bar{x})^2$, $OPT_{1,y} = \sum_{i=1}^n w_i (y_i - \bar{y})^2$ and

$$OPT_{1,z} = \sum_{i=1}^{n} w_i (z_i - \bar{z})^2$$
; $SC = SC_x + SC_y + SC_z$, where $SC_x = \sum_{i=1}^{n} w_i (x_i - x_{med})^2$, $SC_y = \sum_{i=1}^{n} w_i (y_i - y_{med})^2$ and $SC_z = \sum_{i=1}^{n} w_i (z_i - z_{med})^2$.

Due to symmetry, we only need to prove that in the x-domain of the 3D space $SOPT_{r_i} > SC_y$ i.e.

domain of the 3D space, $\gamma OPT_{1,x} \geq SC_x$, i.e.,

$$\gamma \sum_{i=1}^{n} w_i (x_i - \bar{x})^2 \ge \sum_{i=1}^{n} w_i (x_i - x_{med})^2.$$
 (5)

The summation on the left-hand-side of (5) satisfies

$$\sum_{i=1}^{n} w_i (x_i - \bar{x})^2 \ge w_{min} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$\ge w_{min} \sum_{i=1}^{n} (x_i - \frac{\sum_{i=1}^{n} x_i}{n})^2.$$

Lemma 1 provides that

$$2\sum_{i=1}^{n} \left(x_i - \frac{\sum_{i=1}^{n} x_i}{n}\right)^2 \ge \sum_{i=1}^{n} \left(x_i - x_{med}\right)^2.$$

Additionally, the right-hand-side of (5) satisfies

$$w_{max} \sum_{i=1}^{n} (x_i - x_{med})^2 \ge \sum_{i=1}^{n} w_i (x_i - x_{med})^2.$$

By combining above inequalities we have

$$2\frac{w_{max}}{w_{min}}\sum_{i=1}^{n}w_{i}(x_{i}-\bar{x})^{2} \geq \sum_{i=1}^{n}w_{i}(x_{i}-x_{med})^{2}.$$

Similarly, in the y-domain and z-domain, we can derive the same γ value. Therefore, $\gamma=2\frac{w_{max}}{w_{min}}$.

Mechanism 1 counts each user equally and does not consider users' weights. If users have diverse weights such that $\frac{w_{max}}{w}$ is large, the approximation ratio γ is large. It should be noted that Mechanism 1 also has its merit: since the social planner doesn't need to gather the information of weights from users, it is strategyproof even if we allow users to misreport their weights. Next we propose a better mechanism to achieve a much smaller approximation ratio.

Mechanism 2. Consider x-domain first, we reorder $\{x_1, x_2, \dots, x_n\}$ as $\{x_{j_1}, x_{j_2}, \dots, x_{j_n}\}$, with $x_{j_1} \leq x_{j_2} \leq x_{j_2}$ $(x_{j_1}, x_{j_2}, \cdots, x_{j_n})$. Where $(x_{j_1}, x_{j_2}, \cdots, x_{j_n})$, where integer q satisfies $\sum_{i \leq q} w_{j_i} \geq \sum_{i > q} w_{j_i}$ and $\sum_{i < q} w_{j_i} < \sum_{i \geq q} w_{j_i}$. In y-domain and z-domain, y and z follow the same strategy. Given a profile Ω , return weighted median $wmed(\mathbf{x}, \mathbf{y}, \mathbf{z}) =$ $(x_{wmed}, y_{wmed}, z_{wmed})$ as the UAV location.

Theorem 2. Mechanism 2 is a strategyproof mechanism with approximation ratio 2.

Proof. First we can follow the similar analysis of strategyproofness proof in Theorem 1 to prove that Mechanism 2 is strategyproof.

Now we prove the approximation ratio. By following the same process in the proof of Theorem 1, we divide optimal cost and social cost into three parts. Due to symmetry, we only need to consider x-domain, and only need to prove

$$\sum_{i=1}^{n} w_i (x_i - x_{wmed})^2 \le 2 \sum_{i=1}^{n} w_i (x_i - \bar{x})^2.$$
 (6)

¹If n is even, we choose the $\frac{n}{2}$ th-smallest value of **x** profile as x_{med} . This location strategy is the same for location profiles y and z.

Without loss of generality, we rescale each w_i uniformly as an integer in this proof. By partitioning user i into a number w_i of small users with unit weight 1, we obtain a new sequenced set of profile \mathbf{x} :

$$\{\underbrace{x_{j_1},\ldots,x_{j_1}}_{w_{j_1}},\underbrace{x_{j_2},\ldots,x_{j_2}}_{w_{j_2}},\ldots,\underbrace{x_{j_n},\ldots,x_{j_n}}_{w_{j_n}}\}. \tag{7}$$

Then we rewrite (6) as

$$\sum_{i=1}^{n} \underbrace{(x_{j_i} - x_{wmed})^2 + \dots + (x_{j_i} - x_{wmed})^2}_{w_{j_i}}$$

$$\leq 2\sum_{i=1}^{n} \underbrace{(x_{j_i} - \bar{x})^2 + \dots + (x_{j_i} - \bar{x})^2}_{w_{j_i}}.$$
 (8)

Note that x_{wmed} is the median in set (7) and \bar{x} is the mean in (7). According to Lemma 1, we can prove (8). Thus, (6) holds in x-domain and we can similarly obtain the same $\gamma = 2$ in y-domain and z-domain. Hence, $2OPT_1(\mathbf{x}, \mathbf{y}, \mathbf{z}) \geq SC(wmed(\mathbf{x}, \mathbf{y}, \mathbf{z}), (\mathbf{x}, \mathbf{y}, \mathbf{z}))$.

Comparing Mechanisms 1 and 2, we can see that Mechanism 2 achieves better worst-case performance. We will show in subsection III-B that these two mechanisms perform analogously in average sense.

B. Empirical Analysis of Mechanisms 1 and 2

So far we have only analyzed the approximation ratios for the two mechanisms in the worst case. In this subsection, we present empirical analysis to further evaluate the average performances of the mechanisms. In Mechanism 1, we choose median location as the UAV location and we define the social cost ratio by comparing to the optimum:

$$Ratio.1 = \frac{SC(med(\mathbf{x}, \mathbf{y}, \mathbf{z}), (\mathbf{x}, \mathbf{y}, \mathbf{z}))}{OPT_1(\mathbf{x}, \mathbf{y}, \mathbf{z})}.$$

In Mechanism 2, we choose weighted median location as the UAV location and we define the empirical social cost ratio:

$$Ratio.2 = \frac{SC(wmed(\mathbf{x}, \mathbf{y}, \mathbf{z}), (\mathbf{x}, \mathbf{y}, \mathbf{z}))}{OPT_1(\mathbf{x}, \mathbf{y}, \mathbf{z})}.$$

Note that Ratio.1 and Ratio.2 are random variables, depending on distributions of \mathbf{x} , \mathbf{y} and \mathbf{z} , while approximation ratio γ characterizes the maximum of each ratio in the worst-case.

We do simulations for Mechanisms 1 and 2 when n is finite. For simplicity, we assume $I^3 = [0,1]^3$, where each user's location follows the continuous uniform distribution in I^3 and every w_i follows the continuous uniform distribution in [0,1]. Fig. 2 shows the means of the two random ratios in Mechanisms 1 and 2 decrease to 1 as the number of users increases. Similar to the prior worst-case analysis, Fig. 2 also shows that in the average-case, Mechanism 2 outperforms Mechanism 1, as the mean of Ratio.2 is smaller than the mean of Ratio.1 given any fixed number of users. Yet such advantage is no longer obvious once n > 10. Interestingly, from Fig. 2 we can observe that Ratio.1 with odd number n (n = 2k - 1) with natural number n of user size is smaller

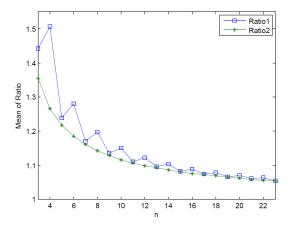


Fig. 2: Means of Ratio.1 and Ratio.2 versus user number n

than Ratio.1 with neighboring even number n (n=2k). This is because if n is odd, $med(\mathbf{x}, \mathbf{y}, \mathbf{z})$ can be relatively closer to $(\bar{x}, \bar{y}, \bar{z})$, as compared to the case that n is even.

As the number n of users goes to infinity, Ratio.1 in Mechanism 1 and Ratio.2 in Mechanism 2 converge in probability towards 1, given all x_i 's, all y_i 's, all z_i 's and all w_i 's are independent and identically distributed, respectively, and all x_i 's, all y_i 's and all z_i 's follow continuous symmetric distributions (including normal, uniform and logistic distributions), respectively.

IV. OBNOXIOUS UAV PLACEMENT GAME

In this section, we design a strategyproof mechanism for the obnoxious UAV placement game, where all n users view the UAV obnoxious due to the introduced interference and want to be far away from the UAV. We want to maximally keep their social utility.

A. Design and Analysis of strategyproof mechanism

We first analyze the optimal UAV location under full information. We divide the 3D space into x-domain, y-domain and z-domain. Similarly, we split social utility as $SU = SU_x + SU_y + SU_z$. We first consider x-domain. By using \bar{x} in (4), we rewrite the social utility (3) in x-domain as

$$SU_x(f, \mathbf{x}) = \sum_{i=1}^n w_i ((x_i - \bar{x}) + (\bar{x} - x))^2$$
$$= \sum_{i=1}^n w_i ((x_i - \bar{x})^2 + (\bar{x} - x)^2) + 0.$$

Similarly, we can obtain SU_y , SU_z and finally SU as

$$SU(f, (\mathbf{x}, \mathbf{y}, \mathbf{z})) = ((x - \bar{x})^2 + (y - \bar{y})^2 + (z - \bar{z})^2) \sum_{i=1}^{n} w_i$$
$$+ \sum_{i=1}^{n} w_i ((x_i - \bar{x})^2 + (y_i - \bar{y})^2 + (z_i - \bar{z})^2).$$

We can see that $SU(f, (\mathbf{x}, \mathbf{y}, \mathbf{z}))$ is linear with the square of the distance between the UAV location (x, y, z) and the weighted mean $(\bar{x}, \bar{y}, \bar{z})$. As (x, y, z) is bounded in the cuboid $I^3 = [0, 2A] \times [0, 2B] \times [0, 2C]$, the optimal UAV location for

maximizing the social utility is one of eight vertices in cuboid I^3 which is furthest from point $(\bar{x}, \bar{y}, \bar{z})$. By considering x-domain, y-domain and z-domain separately, we obtain the optimal UAV location as

$$x_{opt} = \begin{cases} 0 & \text{if } \bar{x} \ge A, \\ 2A & \text{if } \bar{x} < A; \end{cases} y_{opt} = \begin{cases} 0 & \text{if } \bar{y} \ge B, \\ 2B & \text{if } \bar{y} < B; \end{cases}$$
and
$$z_{opt} = \begin{cases} 0 & \text{if } \bar{z} \ge C, \\ 2C & \text{if } \bar{z} < C; \end{cases}$$
(9)

This optimal solution is not strategyproof (consider this illustrative example: there are user 1 at $x_1=0.2$ and user 2 at $x_2=0.6$ in domain I=[0,1]. User 2 can misreport his location to $x_2'=1$ to keep the UAV away from him at x=0). Next we design a strategyproof mechanism.

Mechanism 3. Set UAV location f = (x, y, z), where

$$x = \begin{cases} 0 & \text{if } \sum_{x_i \in [0,A)} w_i \leq \sum_{x_i \in [A,2A]} w_i; \\ 2A & \text{otherwise}, \end{cases}$$

$$y = \begin{cases} 0 & \text{if } \sum_{y_i \in [0,B)} w_i \leq \sum_{y_i \in [B,2B]} w_i; \\ 2B & \text{otherwise}, \end{cases}$$

and

$$z = \begin{cases} 0 & \text{if } \sum_{z_i \in [0,C)} w_i \leq \sum_{z_i \in [C,2C]} w_i; \\ 2C & \text{otherwise.} \end{cases}$$

In Mechanism 3, we compare the total user weights in [0,A) and [A,2A] of the x-domain, and place the obnoxious UAV to the corner with the smaller total weight. Similarly, we place the UAV in y-domain and z-domain for the weighted majority's benefit.

Theorem 3. *Mechanism 3 is a strategyproof mechanism with approximation ratio* 5 *in the obnoxious UAV placement game.*

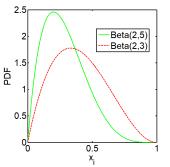
Proof. We only consider x-domain location first to prove strategyproofness, as y-domain and z-domain can be analyzed similarly. Assume, without loss of generality, that $\sum_{x_i \in [0,A)} w_i \leq \sum_{x_i \in [A,2A]} w_i$. Thus UAV's x-location of Mechanism 3 is x=0. We can see that any user in [0,A) prefers x=2A and any user in [A,2A] prefers x=0. Any user in [A,2A] is not willing to misreport his x-domain location; while any user in [0,A) can not change the fact that $\sum_{x_i \in [0,A)} w_i \leq \sum_{x_i \in [A,2A]} w_i$ by misreporting his x-domain location. Thus Mechanism 4 is strategyproof. Next we prove approximation ratio γ .

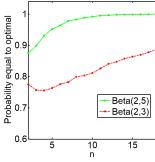
We also define $OPT_2 = OPT_{2,x} + OPT_{2,y} + OPT_{2,z}$ and we analyze x-domain first. Without loss of generality, assume that $\bar{x} \geq A$ and the other case that $\bar{x} < A$ can be analyzed similarly. In this case,

$$A \le \frac{\sum_{i=1}^n w_i x_i}{\sum_{i=1}^n w_i} \le 2A.$$

The optimal x-location of the UAV is 0. Mechanism 3 has the largest approximation ratio γ in x-domain when choosing x=2A under condition:

$$\sum_{x_i \in [0,A)} w_i > \sum_{x_i \in [A,2A]} w_i. \tag{10}$$





(a) Distribution of users' locations x_i 's in x-domain

(b) Probabilities of Mechanism 3 equal to optimal location

Fig. 3: Skewness of asymmetry distribution of users' locations versus convergence rate to optimal placement

The optimal social utility at $x_{opt} = 0$ in x-domain is

$$OPT_{2,x}(\mathbf{x}) = \sum_{x_i \in [0,A)} w_i x_i^2 + \sum_{x_i \in [A,2A]} w_i x_i^2.$$
 (11)

Under Mechanism 3, the social utility at x = 2A in x-domain is

$$SU_x(f, \mathbf{x}) = \sum_{x_i \in [0, A)} w_i (2A - x_i)^2 + \sum_{x_i \in [A, 2A]} w_i (2A - x_i)^2.$$
(12)

To determine the maximum approximation ratio γ , we want to increase the optimal social utility in (11) and reduce the social utility of Mechanism 3 in (12). We purposely design $x_i = A$ for all $x_i \in [0,A)$ and $x_i = 2A$ for all $x_i \in [A,2A]$. By substituting these new x_i 's into (11) and (12), we obtain the largest optimal social utility in x-domain,

$$OPT_{2,x}(\mathbf{x}) = \sum_{x_i \in [0,A)} w_i A^2 + \sum_{x_i \in [A,2A]} w_i (2A)^2, \quad (13)$$

and the smallest social utility under Mechanism 3 in x-domain,

$$SU_x(f, \mathbf{x}) = \sum_{x_i \in [0, A)} w_i A^2 + 0.$$
 (14)

Due to (10), by comparing (13) and (14), we have

$$OPT_{2,x}(\mathbf{x}) \le \sum_{x_i \in [0,A)} w_i A^2 + \sum_{x_i \in [0,A)} w_i (2A)^2 = 5SU_x(f,\mathbf{x}).$$

We follow the same process in y-domain and z-domain, and obtain the same γ . Hence, we conclude $\frac{OPT_2(\mathbf{x},\mathbf{y})}{SU(f,(\mathbf{x},\mathbf{y}))} \leq 5$ and $\gamma = 5$.

B. Empirical Analysis of Mechanism 3

In this subsection, we present empirical analysis to evaluate the average performances of Mechanism 3. We provide empirical simulations in Fig. 3 for Mechanisms 3 when n is finite. For simplicity, we assume $I^3 = [0,1]^3$, each user's location follows asymmetric Beta distribution in I^3 and every w_i follows the continuous uniform distribution in [0,1]. We have two groups of simulations for comparisons. We can see from Fig. 3(a), distribution Beta(2,5) has larger skewness than Beta(2,3), and provides faster convergence rate for Mechanism

3 towards the social optimum, as observed from Fig. 3(b). Intuitively, a larger higher skewness of users' distribution tells a higher probability of UAV location of Mechanism 3 equal to the optimal UAV location.

As the number of users n goes to infinity, the probability that UAV location (x,y,z) under Mechanism 3 equals the socially optimal location goes to 1, given all x_i 's, all y_i 's, all z_i 's and all w_i 's are independent and identically distributed, respectively, and all x_i 's, all y_i 's and all z_i 's follow continuous asymmetric distributions (including Beta distribution and skew normal distribution), respectively.

V. DUAL-PREFERENCE UAV PLACEMENT GAME

In this section, we design the strategyproof mechanism in the dual-preference UAV placement game. For the ease of exposition , we assume all users' weights as 1, i.e., $w_i=1$, for any $i\in N$, and our results can also be extended to the weighted case, where users' weights can follow any general distributions.

As shown in Fig. 1, each user has his own preference type and we define user i's type as θ_i which is either 1 or 2. A user i with $\theta_i=1$ (facility user) prefers to be close to the UAV and a user i with $\theta_i=2$ (adverse user) prefers to be far away from the UAV. We denote $\Theta=\{\theta_1,\ldots,\theta_n\}$ as the profile of all n users' preferences. Still, the UAV need to gather information of users' preference types and users' locations to determine (x,y,z). Given the location of the UAV (x,y,z), we define a user i's utility as

$$u_i((x, y, z), (x_i, y_i, z_i, \theta_i))$$

$$= \begin{cases} (2A)^2 - (x_i - x)^2 + (2B)^2 - (y_i - y)^2 + (2C)^2 \\ -(z_i - z)^2, & \text{if } \theta_i = 1, \\ (x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2, & \text{if } \theta_i = 2. \end{cases}$$

Note that we have used the service cost for a type 1 user (facility user) in Section III, which is equivalent to the user's utility $-(x_i-x)^2-(y_i-y)^2-(y_i-y)^2$. To make our definition of approximation ratio meaningful, we require nonnegative utilities and purposely add $(2A)^2+(2B)^2+(2C)^2$ to the utility. This technique is widely used (e.g., [18]) and does not change our main results.

Definition 3. A mechanism is strategyproof in the dual-preference UAV placement game if no user can benefit from misreporting his location and preference type. Formally, given location profile $\Omega = (\langle x_i, \mathbf{x}_{-i} \rangle, \langle y_i, \mathbf{y}_{-i} \rangle, \langle z_i, \mathbf{z}_{-i} \rangle) \in I^{3n}$, preference profile Θ , and any misreported location $(x_i', y_i', z_i') \in I^3$ and preference type θ_i' for user $i \in N$, it holds that

$$u_i(f((x_i, y_i, z_i, \theta_i), (\mathbf{x}_{-i}, \mathbf{y}_{-i}, \mathbf{z}_{-i}, \Theta_{-i})), (x_i, y_i, z_i, \theta_i)) \ge u_i(f((x_i', y_i', z_i', \theta_i'), (\mathbf{x}_{-i}, \mathbf{y}_{-i}, \mathbf{z}_{-i}, \Theta_{-i})), (x_i, y_i, z_i, \theta_i)).$$

Given a location profile Ω , let $OPT_3(\mathbf{x}, \mathbf{y}, \mathbf{z}, \Theta)$ be the optimal social utility in this game. A strategyproof mechanism f has an approximation ratio $\gamma \geq 1$, if for any location profile $(\mathbf{x}, \mathbf{y}, \mathbf{z}, \Theta)$ and Θ , $OPT_3(\mathbf{x}, \mathbf{y}, \mathbf{z}, \Theta) \leq \gamma SU(f, (\mathbf{x}, \mathbf{y}, \mathbf{z}, \Theta))$.

We can see that the social utility function is quadratic and it is not difficult to derive the optimal UAV location by checking the first-order condition. However, the optimal location is not a strategyproof mechanism and we need to design a stragetyproof mechanism.

Mechanism 4. Consider x-domain first and define two user sets

$$R = \{i : \theta_i = 1, x_i \ge A\} \cup \{i : \theta_i = 2, x_i \le A\}$$

and

$$L = \{i : \theta_i = 1, x_i < A\} \cup \{i : \theta_i = 2, x_i > A\}.$$

The x-location of the UAV is x=2A if $|R| \ge |L|$ and x=0 otherwise. The y-location and z-location of the UAV follow the same placement rule in their domains.

Actually, Mechanism 4 for the dual-preference UAV placement game derives from Mechanism 3 for the obnoxious UAV placement game. This new mechanism further considers that users may cheat on their preference types (besides locations).

Theorem 4. Mechanism 4 is a stragegyproof mechanism with approximation ratio 8.

Proof. We only consider x-location, as y-location and z-location can be analyzed similarly. Assume, without loss of generality, that |R| < |L|. Thus UAV location of Mechanism 4 is x=0. We can see that any user in R prefers x=2A and any user in L prefers x=0. Any user in L is not willing to misreport his x-domain location and preference type; any user in R can not change the fact that |R| < |L| by misreporting his x-domain location and preference type. Thus Mechanism 4 is strategyproof. Next we prove approximation ratio γ .

Let $R_1=\{i:\theta_i=1,x_i\geq A\},\ R_2=\{i:\theta_i=2,x_i\leq A\}$ and $L_1=\{i:\theta_i=1,x_i< A\},\ L_2=\{i:\theta_i=2,x_i>A\}.$ Social utility in Mechanism 4 is $\sum_{i\in L_2\cup R_2}x_i^2+\sum_{i\in L_1\cup R_1}(4A^2-x_i^2).$ We should prove for any optimal location x and user location x_i , the approximation ratio γ

$$\frac{\sum_{i \in L_2 \cup R_2} (x_i - x)^2 + \sum_{i \in L_1 \cup R_1} (4A^2 - (x_i - x)^2)}{\sum_{i \in L_2 \cup R_2} x_i^2 + \sum_{i \in L_1 \cup R_1} (4A^2 - x_i^2)}$$
(15)

is at most 8. We split the proof into two parts.

First we consider the case that $x \in [0, A]$ for the optimal UAV location. By checking the value ranges of x_i and x, the numerator of (15) is,

$$\sum_{i \in L_2 \cup R_2} (x_i - x)^2 + \sum_{i \in L_1 \cup R_1} (4A^2 - (x_i - x)^2)
\leq \sum_{i \in L_2} (2A)^2 + \sum_{i \in R_2} A^2 + \sum_{i \in L_1} (4A^2 - 0) + \sum_{i \in R_1} (4A^2 - 0)
= 4|L_2|A^2 + |R_2|A^2 + 4|L_1|A^2 + 4|R_1|A^2
= (4|L_2|A^2 + 4|L_1|A^2) + (|R_2|A^2 + 4|R_1|A^2)
\leq (4|L_2|A^2 + 4|L_1|A^2) + (4|R_2|A^2 + 4|R_1|A^2)
\leq (4|L_2|A^2 + 4|L_1|A^2) + (4|L_2|A^2 + 4|L_1|A^2)
= 8|L_2|A^2 + 8|L_1|A^2.$$
(16)

By checking the value ranges of x_i and x, we have the denominator of (15),

$$\sum_{i \in L_2 \cup R_2} x_i^2 + \sum_{i \in L_1 \cup R_1} (4A^2 - x_i^2)$$

$$\geq \sum_{i \in L_2} A^2 + \sum_{i \in R_2} 0^2 + \sum_{i \in L_1} (4A^2 - A^2) + \sum_{i \in R_1} (4A^2 - 4A^2)$$

$$= |L_2|A^2 + 3|L_1|A^2. \tag{17}$$

Combining the results of (16) and (17), (15) gives that

$$\gamma \le \frac{8|L_2|A^2 + 8|L_1|A^2}{|L_2|A^2 + 3|L_1|A^2} \le 8.$$

Then we consider the case that $x \in (A, 2A]$. By checking the value ranges of x_i and x, we have the numerator of (15),

$$\sum_{i \in L_2 \cup R_2} (x_i - x)^2 + \sum_{i \in L_1 \cup R_1} (4A^2 - (x_i - x)^2)$$

$$\leq \sum_{i \in L_2} A^2 + \sum_{i \in R_2} (2A)^2 + \sum_{i \in L_1} (4A^2 - 0) + \sum_{i \in R_1} (4A^2 - 0)$$

$$= |L_2|A^2 + 4|R_2|A^2 + 4|L_1|A^2 + 4|R_1|A^2$$

$$= (|L_2|A^2 + 4|L_1|A^2) + (4|R_2|A^2 + 4|R_1|A^2)$$

$$\leq (|L_2|A^2 + 4|L_1|A^2) + (4|L_2|A^2 + 4|L_1|A^2)$$

$$= 5|L_2|A^2 + 8|L_1|A^2.$$
(18)

Combining the results of (18) and (17), (15) gives that

$$\gamma \le \frac{5|L_2|A^2 + 8|L_1|A^2}{|L_2|A^2 + 3|L_1|A^2} \le 5.$$

Therefore, for any $x \in [0, 2A]$, approximation ratio γ is 8 under the condition that |R| < |L|. Similarly, for the case that $|R| \ge |L|$ in Mechanism 4, we can prove approximation ratio is also 8.

As both groups of users with dual-preference are involved, Mechanism 4 does not have convergence result with user number n as in Fig. 2 or Fig. 3. As we have possibly both types of users in the dual-preference UAV placement game, it is more difficult to achieve an approximation ratio smaller than 2 in the UAV placement game and 5 in the obnoxious UAV placement game.

VI. CONCLUSIONS

We study the social planner's problem to determine the final deployment location of a UAV on a 3D space, by ensuring selfish users' truthfulness in reporting their locations. To minimize the social cost in the UAV placement game, we design the strategyproof mechanism with approximation ratio 2, as compared to the social optimum under ideally full information. We also study the obnoxious UAV placement game to maximize the social utility of such interfered users and propose a strategyproof mechanism with approximation ratio 5. Besides the worst-case analysis, we show that the empirical performances of the proposed mechanisms improve with the number of users. We study the dual-preference UAV placement game for the coexistence of the two groups of users, and propose a strategyproof mechanism with approximation ratio 8.

In the future we will consider the mechanism design of mutiple facilties location games. Some of our proposed mechanisms can be similarly applied. Take the two-UAVs placement game in a line interval as an example, we can extend our weighted median Mechanism 2 to respectively locate the two UAVs at the first weighted quartile and third weighted quartile of users' location profile.

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