Stochastic Geometry Model for Multi-Channel Fog Radio Access Networks

Mostafa Emara*, Hesham ElSawy†, Sameh Sorour‡, Samir Al-Ghadhban*, Mohamed-Slim Alouini† and Tareq Y. Al-Naffouri†

*Department of Electrical Engineering, King Fahd University of Petroleum and Minerals (KFUPM), Saudi Arabia
Emails: {mostafaemara, samir} @kfupm.edu.sa
†CEMSE Division, EE program, King Abdullah University of Science and Technology (KAUST), Saudi Arabia
Emails: {hesham.elsawy, slim.alouini, tareq.alnaffouri}@kaust.edu.sa
‡Department of Electrical and Computer Engineering, University of Idaho, USA
Email: samehsorour@uidaho.edu

***Abstract*—Cache-enabled base station (BS) densification, denoted as a fog radio access network (F-RAN), is foreseen as a key component of 5G cellular networks. F-RAN enables storing popular files at the network edge (i.e., BS caches), which empowers local communication and alleviates traffic congestions at the core/backhaul network. The hitting probability, which is the probability of successfully transmitting popular files request from the network edge, is a fundamental key performance indicator (KPI) for F-RAN. This paper develops a scheduling aware mathematical framework, based on stochastic geometry, to characterize the hitting probability of F-RAN in a multi-channel environment. To this end, we assess and compare the performance of two caching distribution schemes, namely, uniform caching and Zipf caching. The numerical results show that the commonly used single channel environment leads to pessimistic assessment for the hitting probability of F-RAN. Furthermore, the numerical results manifest the superiority of the Zipf caching scheme and quantify the hitting probability gains in terms of the number of channels and cache size.**

***Keywords*—Caching system, Stochastic geometry, Multi-channel, F-RAN.**

I. Introduction

The fifth generation (5G) of cellular networks dictates tangible performance leap in terms of network capacity (100 fold) and transmission rate (1000 fold) [1]. Such performance requirement is expected to be fulfilled by an unprecedented network densification phase, in which the small BS (SBS) density can reach hundreds of SBSs per km^2 [2], [3]. Deploying more SBSs in the same geographical location improves the spatial frequency reuse and reduces the number of users associated to the same BS, thus fostering light loaded BSs with high data rate links. However, supplying this large number of SBSs with real-time data services imposes a huge burden on the backhaul links. Consequently, backhauling represents a non-trivial bottleneck to attain the foreseen network densification gains.

Exploiting the common interests among users located within the same geographical location, proactive caching is recently proposed to solve the backhauling problem. With the emergence of social networking, it is expected that a non-negligible percentage of the mobile traffic will be produced from downloading/viewing similar contents [4], denoted as popular files. Proactive caching exploits such phenomena and brings popular files to the network edge (i.e., SBS caches). Consequently, popular files requests are directly served from the caches of the SBS and are not repeatedly requested from the core network via backhaul links, which reliefs the backhaul and core network congestion. Particularly, a user that requests a popular file is directly served from the SBS that stores the requested file. Such networking scheme is currently known as fog radio access networks (F-RANs) [5]–[7].

The performance of F-RAN is mainly determined by the hit probability, which is defined as the probability that a popular file request is successfully catered from the network edge. Hence, maximizing the hitting probability has been a focal research point in the context of F-RAN. For instance, the authors in [8] propose an optimal caching distribution that maximizes the hitting probability in order to alleviate core network delay and achieve minimal file downloading time. However, [8] does not account for the uncertainties in SBSs and users locations. The authors in [9] use stochastic geometry to characterize the hitting probability considering SBSs and users locations uncertainties. However, the proposed paradigm in [9] limits the popular file request to the nearest SBS. Hence, caching diversity among different SBSs is not exploited. The work in [10]–[12] propose optimal file placement in SBSs and allow users to download popular files from the nearest SBS that store them, which is not limited to the geographically nearest SBS to the user. However, only a single channel system is considered, which leads to a pessimistic performance assessment of the hitting probability caused by strong interference of the SBSs closer to the user than its serving SBS. In [13], the authors propose a distributed algorithm to balance the cache-related traffic among the SBSs in a fair way in order to efficiently utilize the available resources assuming that the user may be covered by multiple SBSs. A joint optimal file placement and frequency reuse is proposed in for multi-cast popular file download. However, [14] follows a rigid frequency reuse scheme, which is well-known to underutilize the spectrum resources [15].

In contrast to prior work, this paper exploits the multi-channel system which is well-known in wireless cellular networks [15]–[17]. In the multi-channel system, the total available bandwidth is divided into a set of orthogonal channels that is shared among all SBSs. Each SBS assigns a subset of channels to its associated users according to the applied spectrum access policy. In particular, this paper develops a mathematical framework to characterize the hitting probability in multi-channel F-RAN with universal frequency reuse and dynamic channel assignment. The proposed framework assumes Zipf distributed file popularity and allows users to be served by the nearest SBS that stores the requested popular file, which is denoted as the file catering SBS. Furthermore, the catering SBS exploits opportunistic spectrum access to transmit the popular file to the user. Specifically, if there are vacant channels that are not used by SBSs closer to the user, the catering SBS randomly chooses one of these vacant channels to transmit the file. Otherwise, the catering SBS schedules the file on a randomly selected channel from the complete set of channels. Such transmission scheme aims to avoid dominant interfering SBSs, which significantly reliefs the aggregate interference and improves the hitting probability. To this end, we assess and compare the performance of two caching schemes, namely, the uniform and Zipf caching. The developed mathematical model is based on stochastic geometry, which is a widely used tool for modeling and analyzing cellular networks [18]. Closed-form expressions are obtained for the coverage and hitting probability for a multi-channel caching scenario. These expressions generalize the single channel scenario that is widely adopted in the literature. All theoretical results in this paper are verified via independent Monte Carlo simulations. The numerical results manifest the pessimistic results of the single channel F-RAN and quantify the hitting probability gains in terms of the number of channels, cache size, and caching distribution.

The model under study is presented in Section II, where the performance metric is introduced. In Section III, we find the free channel probability. Service distance distributions are presented in Section IV, which are used to derive the coverage probability expressions in Section V. The proposed mathematical paradigm is verified by the numerical analysis in Section VI. Finally, we conclude the paper in Section VII.

II. System Model

A. Network Model

We consider an F-RAN in which the SBSs are spatially deployed in a given geographic area according to a homogenous two-dimensional (2D) Poisson point process (PPP) $\Psi_b = \{y_i, i = 1, 2, 3, ...\}$ with intensity λ_b, where y_i is the location of the i^{th} SBS. Also, the users are modeled as an independent PPP Ψ_u with intensity λ_u. It is assumed that all SBSs transmit with the same power P and share a set of channels $\mathcal{S}$ to serve their associated users. It is assumed that the signal power decays at a rate $r^{-\eta}$ with the transmission distance r between a generic SBS and its associated user, where $\eta > 2$ is the path loss exponent. The channels between the SBSs and their tagged users are assumed to be independent Rayleigh fading channels with unit mean power gains. Without loss of generality, we will focus our analysis on a typical user u_0 located at the origin of the geographical area under study.

B. Caching Model

Consider a finite set of J popular multimedia files $\mathcal{J} = \{c_1, c_2, ..., c_J\}$. We assume that all files are of the same length. However, our analysis can still be applied with files of different length, by chopping such files into equal length packets. The file popularity is assumed to follow the well-known Zipf's distribution [19], i.e., $a_j = \frac{j^{-\gamma}}{\sum_{i=1}^{J} i^{-\gamma}}$, $j = 1, 2, ..., J$, where a_j reflects the probability that a generic user requests file c_j, and γ is the Zipf parameter that controls the skewness of the popularity distribution. Without loss of generality, it is assumed that the files are indexed according to their popularity, i.e., $a_1 \geq a_2 \geq \cdots \geq a_J$. Each SBS is equipped with a cache memory of size $K < J$ files. Thus, it stores a combination of K different files out of the total J files. A set $\mathcal{X} \triangleq \{1, 2, ..., X\}$ denotes all possible combinations with set cardinality $X = \binom{J}{K}$. Caching copies of a file at the same SBS is avoided because it comes with no benefit but clearly restricts the SBS options to store and thus deliver other files. Let p_x denotes the probability that a generic SBS stores a combination $x \in \mathcal{X}$. Thus, the probability that it stores a particular file c_j is given by

$$b_j = \sum_{x \in \mathcal{X}_j} p_x \quad (1)$$

where $\mathcal{X}_j$ is the set of all possible combinations that have file c_j, which is a subset of the set $\mathcal{J}$ with cardinality $\binom{J-1}{K-1}$. The probability p_x is defined according to the applied caching scheme. This paper focuses on the uniform and Zipf caching schemes. In the uniform caching, the K-files combinations are cached randomly and uniformly into the SBSs. Thus, $p_x = \frac{1}{X}$ and from (1), $b_j = \frac{\binom{J-1}{K-1}}{\binom{J}{K}} = \frac{K}{J}$. On the other hand, the Zipf caching follows the popularity of files, i.e., the Zipf distribution. Thus, the probability that a combination $x \in \mathcal{X}$ to be stored at a generic SBS is $p_x = \frac{1}{K} \sum_{j \in x} a_j$.

C. Association Model

Without loss of generality, we assume that the indices of SBSs are ordered according to their distances from the typical user. The typical user is assumed to be served by the nearest SBS that stores the requested file. We also assume that the other users surrounding the typical user are traditionally associated to their geographically closest SBSs based on the maximum received power. It is assumed that each SBS picks a channel at random from the pool $\mathcal{S}$ to serve each of its associated users. A SBS does not assign the same channel to two of its associated users to avoid overwhelming intra-cell interference. Nonetheless, different SBSs can assign the same channel to one of their tagged users and thus inter-cell interference exists between BSs with scheduling ties. According to Slivnyak-Mecke theorem [20], evaluating the network performance for the typical user at the origin is sufficient and applicable to any generic location in the $2D$

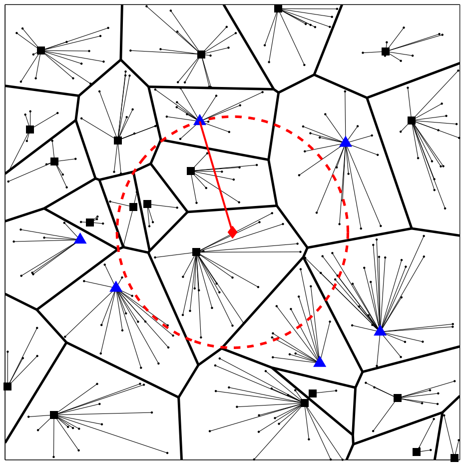

Fig. 1. A realization of the network. The red diamond represents the typical user at the origin and the black dots are the other users. The blue triangles represent the SBSs that store the requested file while the black squares are the others. The red and black lines show the association policies of both the typical user and the other users in the network, respectively. The dashed red circle encloses the SBSs that the file catering SBS should avoids their used channels when assigning channel to serve the typical user.

plane. A realization of the considered system model is shown in Fig. 1.

D. Performance Metric

The average hit probability is the main performance metric of the system. It is defined as the probability that the typical user's request of a popular file is successfully fulfilled by the closest SBS that caches that file. Assuming independent caching at the SBSs, the hit probability depends on both the coverage and the free channel existence probabilities and it can be expressed by

$$\mathcal{H} = \sum_{j \in \mathcal{J}} a_j \sum_{n=1}^{\infty} b_j (1-b_j)^{(n-1)} \Big(\mathcal{C} \; P_{free,n} + \; \mathcal{C}' \; (1 - P_{free,n}) \Big) \tag{2}$$

where a_j is the probability that the typical user requests the file c_j; b_j $(1-b_j)^{n-1}$ indicates that the desired file is available at the n^{th} SBS and is not stored in the enclosed $(n-1)$ SBSs; and $\mathcal{C}$ denotes the coverage probability when a free channel is available to serve the typical user, while $\mathcal{C}'$ is the coverage probability when there is no available free channel. $P_{free,n}$ represents the probability that there is a channel that is not used by closer SBSs to the user than the catering SBS.

III. Free Channel Existence Probability

The probability of free channel existence at the file catering n^{th} SBS is equivalent to the probability that the number of occupied channels by the $(n-1)$ SBSs that are closer to the typical user than its file catering SBS is less than the total number of available channels $|\mathcal{S}|$. The free channel existence probability $P_{free,n}$ is given by

$$P_{free,n} = \sum_{\kappa=0}^{|\mathcal{S}|-1} \mathcal{P}_{n-1}(\kappa) = 1 - \mathcal{P}_{n-1}(|\mathcal{S}|) \tag{3}$$

where $\mathcal{P}_{n-1}(\kappa)$ denotes the probability mass function (PMF) of the number of channels used by the $(n-1)$ SBSs. Similar to [21], it can be expressed by the following recursive equation

$$\mathcal{P}_{n-1}(\kappa) = \sum_{v=0}^{\kappa} \mathcal{P}_{n-2}(v) \sum_{w=\kappa-v}^{\kappa} \mathbb{P}[\mathcal{N}_u = w] \binom{w}{w-(\kappa-v)} \cdot \left(\frac{v}{|\mathcal{S}|}\right)^{w-(\kappa-v)} \left(1 - \frac{v}{|\mathcal{S}|}\right)^{\kappa - v} , 0 \le \kappa \le |\mathcal{S}| \tag{4}$$

where $\mathbb{P}[\mathcal{N}_u = k]$ is the PMF of the number of users associated to a generic SBS. Based on the maximum received power association policy, $\mathbb{P}[\mathcal{N}_u = k] = \frac{c^c \Gamma(c+k) (\frac{\lambda_u}{\lambda_b})^k}{\Gamma(k+1)\Gamma(c)(c+\frac{\lambda_u}{\lambda_b})^{k+c}}$, where $c = 3.575$ is a constant related to the PPP Voronoi cell area distribution.

IV. Service Distance Distribution

The first step to analyze the coverage probability of the multi-channel F-RANs is to characterize the service distance which highly affects the signal-to-interference-plus-noise-ratio (SINR). As we assume in the system model, the file catering SBS is the n^{th} nearby SBS to the typical user at the origin with a serving distance r_n. Exploiting the null property of the PPP, the probability density function (PDF) of the serving distance r_n is given by [22, Lemma 3]

$$f_{r_n}(r) = \frac{2(\pi\lambda_b r^2)^n}{r\Gamma(n)} e^{-\pi\lambda_b r^2} \; , 0 \le r \le \infty \tag{5}$$

As aforementioned, the typical user is served over a randomly chosen used channel if there is no available free channel at its file catering SBS. Thus, it suffers from κ interfering SBSs that use the same channel where κ , $1 \le \kappa \le n-1$. It is worth noting that in the single-channel scenario $\kappa = n-1$. To quantify this interference, the conditional distribution $f_{r_i}(x|r_n)$ is needed, where r_i is the distance between the typical user and the interfering SBS. $f_{r_i}(x|r_n)$ is given by [23]

$$f_{r_i}(x|r_n) = \frac{2x}{r_n^2} \quad , 0 \le x \le r_n \tag{6}$$

V. The Coverage Probability

The coverage probability is defined as the probability that the typical user can successfully achieve a specified SINR threshold β. Under the adopted system model, it depends on whether a free channel exists at the catering file SBS or not. Firstly, we need to find the set of SBSs that use the same channel as the serving channel of the typical user. By exploiting the random channel selection policy at the SBSs and the independent thinning property of the PPP, the set of SBSs that interfere with the typical user forms a thinning PPP $\Psi_{b,\mathcal{P}}$ with intensity $\mathcal{P}\lambda_b$. The thinning parameter $\mathcal{P}$ is the probability that a generic SBS randomly selects a particular channel from $\mathcal{S}$, which is given by [24]:

$$\mathcal{P} = 1 - \sum_{k=1}^{|\mathcal{S}|} \mathbb{P}[\mathcal{N}_u = k] \frac{|\mathcal{S}| - k}{|\mathcal{S}|} \tag{7}$$

Then, the coverage probabilities $\mathcal{C}$ and $\mathcal{C}'$ are given by

$$
\begin{aligned}
\mathcal{C} &= \mathbb{P}[SINR \geq \beta] \\
&= \mathbb{P}\Big[\frac{P h_n r_n^{-\eta}}{\sigma_n^2 + \sum_{y_i \in \Psi_{b,\mathcal{P}}, r_i > r_n} P h_i r_i^{-\eta}} \geq \beta\Big] \\
&= \mathbb{P}\Big[\frac{P h_n r_n^{-\eta}}{\sigma_n^2 + \mathcal{I}_{out}} \geq \beta\Big] \\
\mathcal{C}' &= \mathbb{P}\Big[\frac{P h_n r_n^{-\eta}}{\sigma_n^2 + \sum_{y_i \in \Psi_{b,\mathcal{P}}, r_i < r_n} P h_i r_i^{-\eta} + \mathcal{I}_{out}} \geq \beta\Big] \\
&= \mathbb{P}\Big[\frac{P h_n r_n^{-\eta}}{\sigma_n^2 + \mathcal{I}_{in} + \mathcal{I}_{out}} \geq \beta\Big]
\end{aligned}
\tag{8}
$$

where h_n(resp. h_i) is the channel gain between the typical user and its file catering SBS (resp. the i^{th} interfering SBS). r_n(resp. r_i) is the Euclidean distance between the typical user and its file catering SBS (resp. the i^{th} interfering SBS). σ_n^2 is the serving channel noise. $\mathcal{I}_{in}$ and $\mathcal{I}_{out}$ represent the aggregate interference of the SBSs closer to and farther to the typical user from its file catering SBS, respectively.

Conditioning on the distance r_n between the typical user at the origin and its file catering SBS, the conditional coverage probabilities are given by

$$
\begin{aligned}
\mathcal{C}(r_n) &= e^{\frac{-\beta r_n^{\eta} \sigma_n^2}{P}} \mathcal{L}_{\mathcal{I}_{out}}\Big(\frac{\beta r_n^{\eta}}{P}\Big) \\
\mathcal{C}'(r_n) &= e^{\frac{-\beta r_n^{\eta} \sigma_n^2}{P}} \mathcal{L}_{\mathcal{I}_{out}}\Big(\frac{\beta r_n^{\eta}}{P}\Big) \mathcal{L}_{\mathcal{I}_{in}}\Big(\frac{\beta r_n^{\eta}}{P}\Big)
\end{aligned}
\tag{9}
$$

where $\mathcal{L}_{\mathcal{I}}(t) = \mathbb{E}[e^{-t\mathcal{I}}]$ denotes the Laplace transform (LT) of $\mathcal{I}$. $\mathcal{L}_{\mathcal{I}_{in}}$ and $\mathcal{L}_{\mathcal{I}_{out}}$ are the Laplace transforms (LTs) of $\mathcal{I}_{in}$ and $\mathcal{I}_{out}$, respectively.

the LT of the interference from the outside r_n SBSs can be obtained similar to [23] with considering only the SBSs that use the same channel of the typical user. Thus, $\mathcal{L}_{\mathcal{I}_{out}}$ is given by

$$
\mathcal{L}_{\mathcal{I}_{out}}\left(\frac{\beta r_n^{\eta}}{P}\right) = \exp\left(-\pi \mathcal{P} \lambda_b r_n^2 \beta^{\frac{2}{\eta}} \int_{z=\beta^{\frac{-2}{\eta}}}^{\infty} \frac{1}{1+z^{\frac{\eta}{2}}} dz\right) \tag{10}
$$

On the other hand, the LT of the interference from the SBSs inside r_n $\mathcal{L}_{\mathcal{I}_{in}}$ if there is no available channel is given by the following Lemma

Lemma 1: The LT of the interference from the inner SBSs is given by

$$
\mathcal{L}_{\mathcal{I}_{in}}\left(\frac{\beta r_n^{\eta}}{P}\right) = \left(\frac{2\mathcal{P}}{r_n^2} \int_{r=0}^{r_n} \frac{1}{1+\beta r_n^{\eta} r^{-\eta}} r dr + 1 - \mathcal{P}\right)^{n-1} - (1-\mathcal{P})^{n-1} \tag{11}
$$

Proof: See Appendix A

By averaging over the PDF of the serving distance r_n between the typical user at the origin and its file catering SBS, $f_{r_n}(r)$, the unconditional coverage probabilities are given by (12).

In the special case of interference-limited network (i.e., $\sigma_n^2 = 0$) and path loss exponent $\eta = 4$ (which is common for wireless networks), and with integral manipulations, the unconditional coverage probabilities in (12) turn to the simple closed-form expressions explained in the following corollary

Corollary 1: The closed-form for the general expressions of the coverage probabilities are given by

$$
\begin{aligned}
\mathcal{C} &= \left(1 + \mathcal{P}\sqrt{\beta}\ \arctan(\sqrt{\beta})\right)^{-n} \\
\mathcal{C}' &= \left[\left(1 - \mathcal{P}\sqrt{\beta}\ \arctan(\frac{1}{\sqrt{\beta}})\right)^{n-1} - (1-\mathcal{P})^{n-1}\right] \\
&\quad \cdot \left(1 + \mathcal{P}\sqrt{\beta}\ \arctan(\sqrt{\beta})\right)^{-n}
\end{aligned}
\tag{13}
$$

Finally, by substituting from (12) and (13) into (2), we end up with the general and closed-form expressions of the hit probability for the proposed multi-channel system.

It is worth noting that the hit probability of the single-channel F-RAN, which is widely used in the literature can be derived from the multi-channel system. In the single-channel scenario, all the SBSs use the same channel to serve their associated users. Therefore, the typical user suffers from interference from all SBSs except its file catering SBS. Following the previous analysis, $\mathcal{P}$ is omitted and k is deterministic with value $n-1$. Thus, the single-channel system hit probability $\mathcal{H}_{\text{single}}$ is given by the following Lemma.

Lemma 2: The hit probability of the single-channel system is given by

$$
\begin{aligned}
\mathcal{H}_{\text{single}} &= \sum_{j \in \mathcal{J}} a_j \sum_{n=1}^{\infty} b_j (1-b_j)^{(n-1)}\ \mathcal{C}_{\text{single}} \\
\mathcal{C}_{\text{single}} &= \frac{2(\pi\lambda_b)^n}{\Gamma(n)} \int_0^{\infty} v^{2n-1} \left(\frac{2}{v^2} \int_{w=0}^{v} \frac{1}{1+\beta v^{\eta}\ w^{-\eta}} w dw\right)^{(n-1)} \\
&\quad \cdot \exp\left(-\pi\lambda_b v^2 \beta^{\frac{2}{\eta}} \int_{z=\beta^{\frac{-2}{\eta}}}^{\infty} \frac{1}{1+z^{\frac{\eta}{2}}} dz\right) e^{\frac{-\beta v^{\eta} \sigma_n^2}{P}} e^{-\pi\lambda_b v^2} dv
\end{aligned}
\tag{14}
$$

In the special case of interference-limited network and $\eta = 4$, the coverage probability in (14) turns to a closed-form expression that is given by

$$
\mathcal{C}_{\text{single}} = \left(1 + \sqrt{\beta}\ \arctan(\sqrt{\beta})\right)^{-n} \left(1 - \sqrt{\beta}\ \arctan(\frac{1}{\sqrt{\beta}})\right)^{n-1} \tag{15}
$$

VI. Numerical results

In this section, the proposed multi-channel system model is compared with the single-channel system. The parameters $\lambda_b = 4$ SBSs/Km^2 and $\lambda_u = 40$ users/Km^2 were chosen to conduct the analysis. All the SBSs have the same transmitted power $P = 1$ watt. The path loss exponent $\eta = 4$ is considered. Both the uniform caching and the Zipf caching are considered.

Fig. 2 illustrates the hit probability versus the SINR threshold for both the multi-channel and the single-channel systems using uniform and Zipf caching schemes considering the requested file is $c_j = 1$. A clear match can be noticed between our derived expressions and the simulation results, for both single and multi-channel systems. Furthermore, Fig. 2 quantifies the gain of the multi-channel system over the single-channel one and reveals the significant improvement of system performance using the Zipf caching compared with the uniform caching scheme. However, focusing on the first popular file of the highest popularity a_1 results in an unfair study.

$$\mathcal{C} = \frac{2(\pi\lambda_b)^n}{\Gamma(n)} \int_0^\infty v^{2n-1} e^{\frac{-\beta v^\eta \sigma_n^2}{P}} \exp\left(-\pi\mathcal{P}\lambda_b v^2 \beta^{\frac{2}{\eta}} \int_{z=\beta^{\frac{-2}{\eta}}}^{\infty} \frac{1}{1+z^{\frac{\eta}{2}}} dz \right) e^{-\pi\lambda_b v^2} dv$$

$$\mathcal{C}' = \frac{2(\pi\lambda_b)^n}{\Gamma(n)} \int_0^\infty v^{2n-1} e^{\frac{-\beta v^\eta \sigma_n^2}{P}} \exp\left(-\pi\mathcal{P}\lambda_b v^2 \beta^{\frac{2}{\eta}} \int_{z=\beta^{\frac{-2}{\eta}}}^{\infty} \frac{1}{1+z^{\frac{\eta}{2}}} dz \right) e^{-\pi\lambda_b v^2} \cdot \left\{ \left(1 - \mathcal{P} + \frac{2\mathcal{P}}{v^2} \int_{w=0}^{v} \frac{1}{1+\beta v^\eta\, w^{-\eta}} w dw \right)^{n-1} - (1-\mathcal{P})^{n-1} \right\} dv \quad (12)$$

Therefore, Fig. 3 shows the hit probability versus the requested file index. It can be observed that the hitting probability is independent from the file index for uniform caching scheme. This is because the uniform caching scheme distributes the files into SBSs randomly and uniformly irrespective to their popularity. On the other hand, the hit probability of the Zipf caching scheme highly depends on the requested file popularity. The hitting probability dramatically degrades when the typical user requests any other files rather than the first one in the case of high Zipf parameter. This figure also indicates that the multi-channel system always outperforms the single-channel scenario. In addition, the uniform caching slightly outperforms the Zipf caching when the files with low popularity are requested.

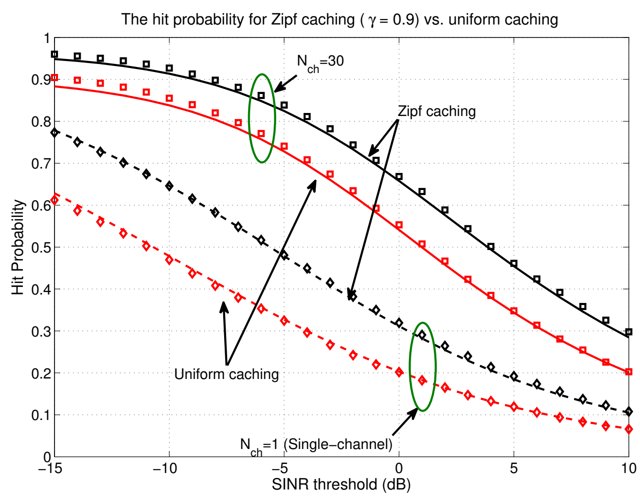

Fig. 2. The hit probability vs. the SINR with $J = 15$ & $K = 5$ & $\frac{\lambda_u}{\lambda_b} = 10$ and $c_j = 1$

Fig. 4 illustrates the hit probability versus the cache size. It can be noticed that the performance of the system improves as the cache size increases. This highlights the tradeoff between the F-RAN performance and the network storage resources. Based on the SINR threshold, the hit probability shows different trends with the cache size. Such SINR threshold dependent behavior can be explained by the fact that at most m SBSs can have signal-to-interference ratio (SIR) greater than $\frac{1}{m}$ for any positive integer m [25]. Thus, there are at most 10 SBSs have SIR -10 dB and only one SBS with SIR 0 dB. This fact explains the non-linearity of the hit probability at -10 dB scenario for all the displayed number of channels. On the other hand, almost all the trends are linear at the 0 dB scenario. The dependency on the number of channels comes from the fact that increasing the number of channels enhances the chance that typical user is served by multiple SBSs and this combination ends up with the non-linear trend.

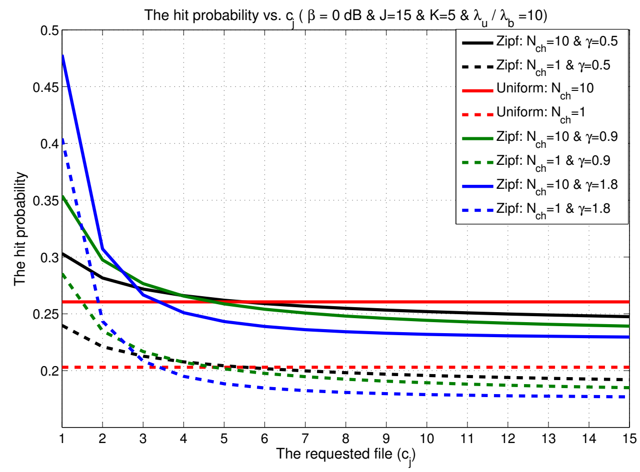

Fig. 3. The hit probability vs. the requested file c_j with $J = 15$ & $K = 5$ & $\frac{\lambda_u}{\lambda_b} = 10$ at different values of Zipf parameter.

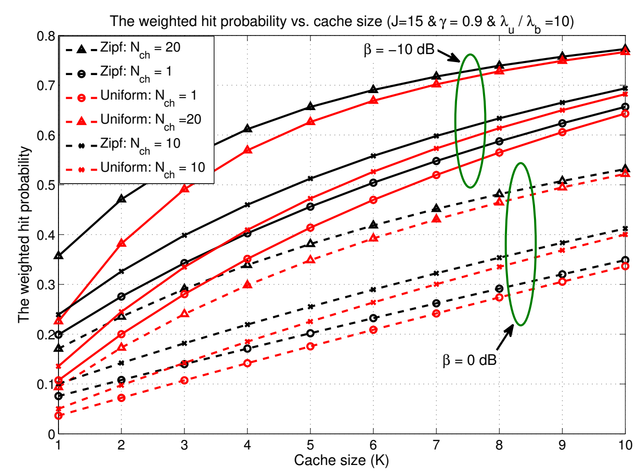

Fig. 4. The hit probability vs. the cache size K with $\beta = 0$ dB & $J = 15$ & $\gamma = 0.9$ & $\frac{\lambda_u}{\lambda_b} = 10$ at different number of channels.

VII. Conclusion

This paper considers an opportunistic spectrum access approach for F-RANs. We adopt multi-channel scenario with opportunistic spectrum access to improve the F-RANs's overall performance. We develop a tractable mathematical model for the hit probability, which is reduced to closed form expression in special cases. Our numerical results both verify the derived analytical paradigms and quantify the gains of the considered multi-channel model. We end up by the achieved improvement of the joint multi-channel cached-enabled F-RAN over the single-channel system.

Appendix A: Proof of Lemma 1

In the scenario of no available free channel, the typical user is served over a randomly chosen channel by the file catering SBS. Thus, the number of inside r_n interfering SBSs κ is a random variable with a maximum value $(n-1)$. The PMF of κ is given by

$$\mathbb{P}[\kappa = a] = \binom{n-1}{a} \mathcal{P}^a \, (1-\mathcal{P})^{n-a-1} \quad ,1 \leq a \leq (n-1) \tag{16}$$

Given that the number of the inside r_n interfering SBSs $\kappa = a$, the LT $\mathcal{I}_{in,\kappa}$ is given by

$$\begin{aligned}
\mathcal{L}_{\mathcal{I}_{in,\kappa}}(t) &= \mathbb{E}_{\mathcal{I}_{in},\kappa}\left[e^{-t\mathcal{I}_{in}} \big| \kappa = a\right] \\
&= \mathbb{E}_{\kappa}\left[\mathbb{E}_{y_i \in \Psi_{b,\kappa}, r_i < r_n}\left[e^{-t\sum_i P h_i r_i^{-\eta}}\right] \big| \kappa = a\right] \\
&\overset{(a)}{=} \mathbb{E}_{\kappa}\left[\mathbb{E}_{r_{n-1}}\left[\mathbb{E}_h[e^{-tPhr^{-\eta}} \mid r = r_n]\right]^{\kappa} \big| \kappa = a\right] \\
&\overset{(b)}{=} \mathbb{E}_{\kappa}\left[\left[\int_{r=0}^{r_n} \frac{1}{1+tPr^{-\eta}} \, f_{r_{n-1}}(r|r_n) dr\right]^{\kappa} \big| \kappa = a\right]
\end{aligned} \tag{17}$$

The equality (a) is by the fact that the PPP consists of uniformly distributed nodes, so intuitively the conditional PDF of the κ nodes on the n^{th} node is equivalent to the κ times the conditional PDF of two consecutive nodes. Equality (b) from the channel gain exponential distribution with averaging over the conditional PDF $f_{r_{n-1}}(r|r_n)$ that is given in (6). Replacing t in the above equation with $(\frac{\beta r_n^\eta}{P})$ and averaging over the random variable κ, the unconditional LT $\mathcal{L}_{\mathcal{I}_{in}}$ can be obtained. Finally, by applying the binomial theorem, Lemma 1 can easily verified.

Acknowledgment

This research was funded by a grant from the office of competitive research funding (OCRF) at the King Abdullah University of Science and Technology (KAUST).

The work was also supported by the Deanship of Scientific Research (DSR) at King Fahd University of Petroleum and Minerals (KFUPM), Dhahran, Saudi Arabia, through project number KAUST-002.

References

[1] J. G. Andrews, S. Buzzi, W. Choi, S. V. Hanly, A. Lozano, A. C. Soong, and J. C. Zhang, "What will 5g be?" *IEEE Journal on Selected Areas in Communications*, vol. 32, no. 6, pp. 1065–1082, 2014.

[2] J. G. Andrews, T. Bai, M. Kulkarni, A. Alkhateeb, A. Gupta, and R. W. Heath Jr, "Modeling and analyzing millimeter wave cellular systems," *arXiv preprint arXiv:1605.04283*, 2016.

[3] S. G. Larew, T. A. Thomas, M. Cudak, and A. Ghosh, "Air interface design and ray tracing study for 5g millimeter wave communications," in *2013 IEEE Globecom Workshops (GC Wkshps)*. IEEE, 2013, pp. 117–122.

[4] E. Bastug, M. Bennis, and M. Debbah, "Living on the edge: The role of proactive caching in 5g wireless networks," *IEEE Communications Magazine*, vol. 52, no. 8, pp. 82–89, 2014.

[5] R. Tandon and O. Simeone, "Harnessing cloud and edge synergies: toward an information theory of fog radio access networks," *IEEE Communications Magazine*, vol. 54, no. 8, pp. 44–50, 2016.

[6] S.-H. Park, O. Simeone, and S. Shamai, "Joint optimization of cloud and edge processing for fog radio access networks," *arXiv preprint arXiv:1601.02460*, 2016.

[7] R. Tandon and O. Simeone, "Cloud-aided wireless networks with edge caching: Fundamental latency trade-offs in fog radio access networks," in *Information Theory (ISIT), 2016 IEEE International Symposium on*. IEEE, 2016, pp. 2029–2033.

[8] K. Shanmugam, N. Golrezaei, A. G. Dimakis, A. F. Molisch, and G. Caire, "Femtocaching: Wireless content delivery through distributed caching helpers," *IEEE Transactions on Information Theory*, vol. 59, no. 12, pp. 8402–8413, 2013.

[9] E. Baştuğ, M. Bennis, and M. Debbah, "Cache-enabled small cell networks: Modeling and tradeoffs," in *2014 11th International Symposium on Wireless Communications Systems (ISWCS)*, Aug 2014, pp. 649–653.

[10] B. Blaszczyszyn and A. Giovanidis, "Optimal geographic caching in cellular networks," in *2015 IEEE International Conference on Communications (ICC)*. IEEE, 2015, pp. 3358–3363.

[11] B. Serbetci and J. Goseling, "On optimal geographical caching in heterogeneous cellular networks," *arXiv preprint arXiv:1601.07322*, 2016.

[12] M. Afshang and H. S. Dhillon, "Optimal geographic caching in finite wireless networks," *arXiv preprint arXiv:1603.01921*, 2016.

[13] J. Krolikowski, A. Giovanidis, and M. Di Renzo, "Fair distributed user-traffic association in cache equipped cellular networks," *arXiv preprint arXiv:1610.03727*, 2016.

[14] Y. Cui, Y. Wu, and D. Jiang, "Analysis and optimization of caching and multicasting in large-scale cache-enabled information-centric networks," in *2015 IEEE Global Communications Conference (GLOBECOM)*. IEEE, 2015, pp. 1–7.

[15] A. H. Sakr and E. Hossain, "Cognitive and energy harvesting-based d2d communication in cellular networks: Stochastic geometry modeling and analysis," *IEEE Transactions on Communications*, vol. 63, no. 5, pp. 1867–1880, 2015.

[16] H. ElSawy and E. Hossain, "On cognitive small cells in two-tier heterogeneous networks," in *Modeling & Optimization in Mobile, Ad Hoc & Wireless Networks (WiOpt), 2013 11th International Symposium on*. IEEE, 2013, pp. 75–82.

[17] ——, "Two-tier hetnets with cognitive femtocells: Downlink performance modeling and analysis in a multichannel environment," *IEEE Transactions on Mobile Computing*, vol. 13, no. 3, pp. 649–663, 2014.

[18] H. ElSawy, E. Hossain, and M. Haenggi, "Stochastic geometry for modeling, analysis, and design of multi-tier and cognitive cellular wireless networks: A survey," *IEEE Communications Surveys & Tutorials*, vol. 15, no. 3, pp. 996–1019, 2013.

[19] M. Cha, H. Kwak, P. Rodriguez, Y.-Y. Ahn, and S. Moon, "I tube, you tube, everybody tubes: analyzing the world's largest user generated content video system," in *Proceedings of the 7th ACM SIGCOMM conference on Internet measurement*. ACM, 2007, pp. 1–14.

[20] S. N. Chiu, D. Stoyan, W. S. Kendall, and J. Mecke, *Stochastic geometry and its applications*. John Wiley & Sons, 2013.

[21] H. ElSawy and E. Hossain, "Channel assignment and opportunistic spectrum access in two-tier cellular networks with cognitive small cells," in *2013 IEEE Global Communications Conference (GLOBECOM)*. IEEE, 2013, pp. 4477–4482.

[22] N. Lee, D. Morales-Jimenez, A. Lozano, and R. W. Heath, "Spectral efficiency of dynamic coordinated beamforming: A stochastic geometry approach," *IEEE Transactions on Wireless Communications*, vol. 14, no. 1, pp. 230–241, 2015.

[23] R. Arshad, H. ElSawy, S. Sorour, T. Y. Al-Naffouri, and M.-S. Alouini, "Handover management in dense cellular networks: A stochastic geometry approach," in *Communications (ICC), 2016 IEEE International Conference on*. IEEE, 2016, pp. 1–7.

[24] H. ElSawy, A. Sultan-Salem, M.-S. Alouini, and M. Z. Win, "Modeling and analysis of cellular networks using stochastic geometry: A tutorial," *IEEE Communications Surveys & Tutorials*, 2016.

[25] H. S. Dhillon, R. K. Ganti, F. Baccelli, and J. G. Andrews, "Coverage and ergodic rate in k-tier downlink heterogeneous cellular networks," in *Communication, Control, and Computing (Allerton), 2011 49th Annual Allerton Conference on*. IEEE, 2011, pp. 1627–1632.