The Economics of Quality Sponsored Data in Wireless Networks

Mohammad Hassan Lotfi, Karthikeyan Sundaresan, Mohammad Ali Khojastepour, and Sampath Rangarajan

***Abstract*—The growing demand for data has driven the Service Providers (SPs) to provide differential treatment of traffic to generate additional revenue streams from Content Providers (CPs). While SPs currently only provide best-effort services to their CPs, it is plausible to envision a model in near future, where CPs are willing to sponsor quality of service for their content in exchange of sharing a portion of their profit with SPs. In this paper, we introduce the problem of *Quality-Sponsored Data* (QSD) in cellular networks and study its implications on market entities in various scenarios. The direct coupling between the scarce wireless resources and the market decisions resulting from QSD is taken into account. In our model, SPs make a portion of their resources available for sponsorship by CPs, and price it appropriately to maximize their payoff, which depends on the monetary revenue and the satisfaction of end-users both for the non-sponsored and sponsored content, while CPs generate revenue through advertisement. We analyze the market dynamics and equilibria, and provide strategies for (i) SPs: to determine if and how to price resources, and (ii) CPs: to determine if and what quality to sponsor. We also discuss about the effects of different parameters of the model on market dynamics.**

I. Introduction

The growing demand for data and the saturating revenue of broadband service providers (SPs) have driven the service providers to provide differential treatment of traffic to generate additional revenue streams from content providers (CPs). This has raised serious concerns among net neutrality advocates, especially with the recent landmark ruling favoring Verizon in its case against the FCC [1]. A similar trend can also be observed in wireless cellular networks, where AT&T has launched a *sponsored data plan* that allows CPs to pay for the data bytes that their users consume, thereby not eating into the users' data quota. In addition to the SPs generating revenue from the CPs, such a model is also beneficial for the CPs to access the SP's user base and generate revenue through advertisements or through increased memberships for their content (through subscription, e.g. Netflix, Hulu+, etc.).

While the SPs only provide best-effort service to its CPs in the current model, it is easy to envision a model in the near future, where CPs require *quality of service* for the data they sponsor. For example, if YouTube wants to increase the number of its active users through sponsoring its videos, it would derive value (utility) from the sponsorship only if the videos are delivered at a good quality. We refer to this model as the *quality-sponsored data* (QSD) model, wherein (spectral) resources at the SP are sponsored to ensure quality for the data bytes being delivered to the end users. This a significant departure from the current model, where by sponsoring only the data bytes in the latter (without any associated quality), there is no direct coupling between the scarce wireless resources at the SP and the market dynamics between the CP, SP and end-users. In contrast, the QSD model brings this coupling to the fore-front.

Hence, the over-arching goal of this work is to analyze and understand the implications of the QSD model on the market dynamics, which we believe is both timely and important. Using game-theoretic [2] tools, we study the market equilibria and dynamics under various scenarios and assumptions involving the three key players of the market, namely the CPs, SPs and end users. In the process, we also devise strategies for the CPs (respectively, SPs) to determine if they should participate in QSD, what quality to sponsor, and how the SPs should price their resources.

In our model, SPs make a portion of their resources available for sponsorship by CPs, and price it appropriately to maximize their payoff, which in turn depends on the monetary revenue and the satisfaction of end-users both for the non-sponsored and sponsored content. Note that the QSD model couples market decisions to the fixed, scarce wireless resources. Thus, resources allocated to sponsored contents will affect the quality of non-sponsored content. Hence, one should consider the impact of the two types of data on each other.

We consider an advertisement revenue model for CPs (Section II), and characterize the myopic pricing strategies for CPs and SPs given the quality of the content that needs to be guaranteed and the available wireless resource (Section III). Assuming the demand for content to be dynamic, wherein the change in the demand is dependent on the quality that end-users experience, we investigate the asymptotic behaviour of the market when decision makers (SP and CP) are short-sighted, i.e. not involving the dynamics of demand in their decision making. We show that depending on certain key parameters, such as the importance of non-sponsored data for SPs and the parameters of the dynamic demand, the market can be asymptotically stable or unstable. Furthermore, four different stable outcomes are possible: 1. no-sponsoring, 2. CPs sponsor all the available resources, 3. CPs sponsor minimum resources to deliver a minimum desired rate to their users, and 4. In-boundary solution in which CPs sponsor more than the minimum but not all the available resources. We study the conditions under which each of these asymptotic outcomes are plausible. We also investigate the role of CPs and SPs with long-sighted business models in stabilizing the market given the parameters of the market. The effects of different market parameters on the asymptotic outcome of the market is investigated through numerical simulations (Section V). Finally, discussion about the results is presented in Section VI.

Our contributions in this work are multi-fold:

- We introduce the problem of quality-sponsored data (QSD) in cellular networks and study its implications

Authors are with NEC Laboratories America. Mohammad Hassan Lotfi is also affiliated with the Department of Electrical and System Engineering, at University of Pennsylvania, Philadelphia, PA. Their email addresses are lotfm@ seas.upenn.edu, and {karthiks,amir,sampath}@ nec-labs.com.

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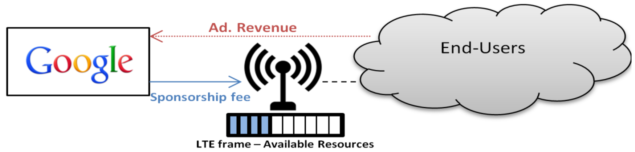

Fig. 1: Market when CP has an advertisement revenue model

on CPs, SPs and end users in various scenarios.

- We analyze the market dynamics and equilibria when CPs have an advertisement revenue model for different sets of market parameters.
- We provide strategies for (i) SPs: to determine if and how to price resources, and (ii) CPs: to determine if and how many resources, i.e. quality, to sponsor.

Related Works:

New pricing schemes in the Internet market either target end-users or CPs. For the end-user side, different pricing schemes have been proposed to replace the traditional flat rate pricing [3], [4], [5]. However, SPs are reluctant adopting such pricing schemes since these schemes are typically not user-friendly. Thus, SPs mainly focus on changing the pricing structure of the CP side, for which they should deal with net-neutrality rules.

This work falls in the category of economic models for a non-neutral Internet [6], [7], [8], [9]. A survey of the existing literature on the economics analysis of the net neutrality debate is presented in [10]. Most of the works in this area study the social welfare of the market under neutrality and non-neutrality regimes with a focus on wired networks.

Works related to the emerging subject of sponsored content in wireless networks are scarce. In [11], [12], [13], and [14], authors investigate the economics aspects of content sponsoring. In these papers, CPs will only pay for the quantity of traffic their users consume. In contrast, we take into the account the quality of the content and the coupling it has with scarce wireless resources. We consider more strategic CPs that decide on the portion of SP's resources they want to sponsor, based on the price SPs quote and the demand from end-users. Thus, in our case, CPs pay for the *quality* and the *quantity* of the traffic carried to the users.

II. Model

We model the wireless ecosystem as a market consisting of three players: CP, SP and end-users, and model the problem of QSD as a sequential game. We seek for a *subgame perfect equilibrium* using *backward induction* to determine the strategies of various players. While the strategies for CPs are to determine if and how much resources to sponsor (i.e. quality) and from which SP, those for the SPs are to determine if and how to price her resources and to which CP, and that for the end-users is to select a CP. Since users in current wireless plans are mostly subject to contracts and large early termination fees, we do not consider choice of SPs for users in this work and defer it for future study. Decisions by the players are made at the beginning of every epoch, which captures the typical time granularity of sponsorship decisions. The duration between epochs can last from hours to several days. For example, one can envision scenarios in which CPs sponsor the quality only in rush hours and stick to best-effort scenario at other times.

We consider one CP and one SP. The CP has an advertisement revenue model; it sponsors r_t resources in an LTE frame (that has a total of R resources) at t^{th} time-epoch to sponsor the quality (i.e., min rate) of at least $\zeta \frac{\text{bit}}{\text{frame}}$ for her content, and pay a price of $\hat{p}_t$ per resource sponsored. A Schematic picture of the market in this case is presented in Figures 1. Assuming a default MCS (e.g. lowest modulation-coding rate, m bits/symbol) and T symbols in a frame, the sponsored resources can be directly translated to the minimum sponsored bits in an LTE frame, b_t , where $b_t = Cr_t$, and $C = mT$.

The CP and the SP choose their strategy at time-epoch t after observing the previous demand, i.e. the number of end-users desiring content from the previous epoch. Note that the demand for the content of the CP changes over time depending on the satisfaction of users, which in turn depends on the resources that the CP decides to sponsor and hence the quality. We suppose that the demand for content updates as follows,

$$d_{t+1} = d_t \left(1 + \gamma \log\left(\kappa_u \frac{b_t}{d_t}\right)\right)^+ \quad (1)$$

where $z^+ = \max\{z, 0\}$, d_t is the demand between epoch t and $t+1$, and $\frac{b_t}{d_t}$ is the rate a single user receives. The parameter $\kappa_u > 0$ links the rate received by users to their satisfaction and subsequently to the change in the demand. If κ_u is high, demand increases even with small rates. In other words, high κ_u is associated with low sensitivity of end-users to the rate they receive. An instance of this types of users are customers of an online shop like Amazon that unlike users of Netflix can be satisfied with lower rates. Thus κ_u captures the sensitivity of the demand of end-users to the quality they perceive. The parameter $\gamma > 0$ is the intensity of changes in demand. We assume a logarithmic function owing to its popularity in the wireless literature. However, our analysis and insights are expected to be applicable to other concave functions with diminishing returns.

Note that the available wireless resources is limited (e.g. R resource blocks in an LTE frame) and hence limits the number of sponsored resources (and in turn bits to $N = CR$). Being related by a constant, we refer to resources and bits synonymously for sponsoring and assume that the number of bits a CP can sponsor in each LTE time frame is bounded above by $\hat{N}$ ($\hat{N} \leq N$). This is a key distinction of our work from previous works as it couples the utility of wireless resources for both sponsored and non-sponsored content with the decisions of the market players. We suppose a complete information setting for the game. The timing of the game at time epoch t is as follow:

1) The SP decides on offering the sponsorship program, y_t, and on the price per sponsored bit in an LTE frame, p_t ($p_t = \frac{\hat{p}_t}{C}$).
2) The CP decides whether to participate in the sponsorship program, z_t, and on the number of bits in an LTE frame (i.e. quality) she wants to sponsor, b_t.

The utility of the CP if she chooses to enter the sponsorship program consists of the utility she receives by sponsoring the content minus the price she pays for sponsoring the sponsored bits. Note that the utility of a CP for sponsoring the content depends on the advertisement revenue which in turns depends on the demand for the content as well as the rate (i.e. content

Symbol	Description
p_t	the price per unit of resources sponsored at time t
b_t	the number of sponsored bits in an LTE frame at time t
d_t	the demand between epoch t and $t+1$
ζ	the minimum quality desired by end-users
γ	intensity of changes in demand
κ_u, κ_{CP}, κ_{SP}	sensitivity to quality for users, CP, and SP, respectively.
$\hat{N}$	the number of available bits for sponsoring
N	the total number of bits (resources) in an LTE frame
$\nu_s(.)$	end-users' satisfaction function
ν_1	the weight end-users assign to the sponsored data
ν_2	the weight end-users assign to the non-sponsored data
D	the total demand of end-users for non-sponsored data
$\frac{1}{\kappa_u}$	the stable quality, the rate that stabilizes the demand
z &y	the participation factor for the CP and SP, 1 =join, 0 =exit

TABLE I: Important Symbols

quality) received by the users (throughput is $\frac{b_t}{d_t}$). Thus, the utility of the CP at time t is considered to be:

$$u_{CP,t}(b_t) = \alpha d_t \log\left(\frac{\kappa_{CP} b_t}{d_t}\right) - p_t b_t \quad (2)$$

where α and κ_{CP} are positive constants. The parameter κ_{CP} models the sensitivity of the CP's profit to the quality she provides for end-users: The higher κ_{CP}, the higher the profit of the CP for small rates sponsored. An example of these kinds of CPs are shopping websites (e.g. Amazon) that in contrast with streaming websites (e.g. Netflix) have a high profit per user rate. Note that In order to have a non-trivial problem assume that $\kappa_{CP}\zeta > e = 2.72$.

The utility of the SP at time t if she chooses to offer the sponsorship program is the revenue she makes by sponsoring the bits plus the users' satisfaction function:

$$u_{SP,t}(p_t) = p_t b_t + \nu_s(p_t) \quad (3)$$

Note that the users' satisfaction function, $\nu_s(p_t)$, is a function of the price of a sponsored bit. This function consists of two parts: (i) the satisfaction of users for access to the sponsored content and its quality, and (ii) the satisfaction of users when using non-sponsored content. This function could be decreasing or increasing depending on the users, the cell condition, etc. We define the satisfaction function as follows,

$$\nu_s(p_t) = \nu_1 d_t \log\left(\frac{\kappa_{SP} b_t}{d_t}\right) + \nu_2 D \log\left(\kappa_{SP}\frac{N - b_t}{D}\right) \quad (4)$$

, where D is the total demand from the end-users for non-sponsored content, N is the total number of bits in an LTE frame (corresponding to the total R resources), and κ_{SP} is a positive constant modelling the sensitivity of the SP's profit to the quality received by her end-users. Note that, despite the dependencies between κ_u, κ_{CP}, and κ_{SP}, these parameters could be potentially different.

A summary of important symbols is presented in Table I.

III. Analysis: Nash Equilibrium

First, we consider that the CP and the SP limit their decision making to the history of the game, i.e. *Short-sighted Business Model*. Later, we will discuss about the scenario in which the decision makers consider the effect of their decision on the future demand and subsequently their future payoff, i.e. *Long-sighted Business Model*.

A. Short-sighted Business Model for the CP and SP

We use the Backward Induction method to find the Sub game Perfect Nash Equilibrium (SPNE) of the game. Thus, first, we find the best response strategy of the CP in the second stage given the strategy of the SP in the first stage and the history of the game. This allows the CP to decide on joining the sponsorship program and also the number of bits to sponsor. Subsequently, using this best response strategy and the history, the SP chooses whether to launch the sponsorship program or not, and the optimum per-bit price, p_t, in the first stage.

CP's Strategy: In the second stage, at each time-epoch t, the CP maximizes her payoff given the constraints, and knowing the decision of the SP at stage one:

$$\begin{aligned} &\max_{b_t>0, z_t} \left(\alpha d_t \log\left(\frac{\kappa_{CP} b_t}{d_t}\right) - p_t b_t\right) z_t \\ s.t. \quad &\frac{b_t}{d_t} \geq \zeta \\ &b_t \leq \hat{N} \\ &z_t \in \{0,1\} \end{aligned} \quad (5)$$

The first constraint is associated with the minimum quality that the CP wants to deliver to her end-users. The second constraint puts an upperbound to the number of bits that a CP can sponsor in an LTE frame. The variable z_t determines whether the CP joins the sponsorship program or not, with $z_t = 1$ implying participation. In addition, note that, $d_t = \left(1 + \gamma \log\left(\kappa_u \frac{b_{t-1}}{d_{t-1}}\right)\right)^+$, and is known as the history of the game is known.

Theorem 1: **Equilibrium Strategy of Stage 2:** The strategy of the CP in the SPNE is as follows:

$$\text{if } 0 < d_t \leq \frac{\hat{N}}{\zeta},$$

$$(b_t^*, z_t^*) = \begin{cases} (\hat{N}, 1) & \text{if} \quad p_t \leq \frac{\alpha d_t}{\hat{N}} \\ (\frac{\alpha d_t}{p_t}, 1) & \text{if} \quad \frac{\alpha d_t}{\hat{N}} \leq p_t \leq \frac{\alpha}{\zeta} \\ (\zeta d_t, 1) & \text{if} \quad \frac{\alpha}{\zeta} \leq p_t \leq \frac{\alpha \log(\kappa_{CP}\zeta)}{\zeta} \\ (-, 0) & \text{if} \quad p_t > \frac{\alpha \log(\kappa_{CP}\zeta)}{\zeta} \end{cases} \quad (6)$$

$$\text{if } d_t > \frac{\hat{N}}{\zeta} \text{ or } d_t = 0, \quad (b_t^*, z_t^*) = (-, 0) \quad (7)$$

Proof: The objective function of (5) is $u_{CP,t}(b_t)z_t$. Thus, in order to solve the optimization, we maximize $u_{CP,t}(b_t)$ given the constraint. If $u_{CP,t}(b_t) < 0$ for all $b_t > 0$ or $d_t = 0$, then $z_t^* = 0$, i.e. CP withdraws from the sponsoring program. If not, then $z_t^* = 1$.

Now, consider $d_t > 0$. Clearly, the utility of the CP (2) is concave. Thus, the first order optimality condition provides us with the candidate optimum answer for (5). The first order condition yields that $\hat{b}_t = \frac{\alpha d_t}{p_t}$.

The constraint on b_t is $\zeta d_t \leq b_t \leq \hat{N}$. Thus, $\hat{b}_t$ is the optimum answer if it satisfies the constraint, i.e. $\frac{\alpha d_t}{\hat{N}} \leq p_t \leq \frac{\alpha}{\zeta}$, and $u_{CP,t}(\hat{b}_t) \geq 0$. The first condition yields another condition that $d_t \leq \frac{\hat{N}}{\zeta}$. The later condition holds if $p_t \leq \frac{\alpha \kappa_{CP}}{e}$. Since $\zeta \kappa_{CP} > e$, $\frac{\alpha}{\zeta} < \frac{\alpha \kappa_{CP}}{e}$. Therefore, the first condition is the sufficient condition for optimality of $\hat{b}_t$. This is the second region from top in (6). If $d_t \leq \frac{\hat{N}}{\zeta}$ and $p_t \leq \frac{\alpha d_t}{\hat{N}}$, then the top boundary condition $b_t^* = \hat{N}$ is the optimum answer since in this region $u_{CP,t}(b_t)$ is positive. On the other hand, if $d_t \leq \frac{\hat{N}}{\zeta}$

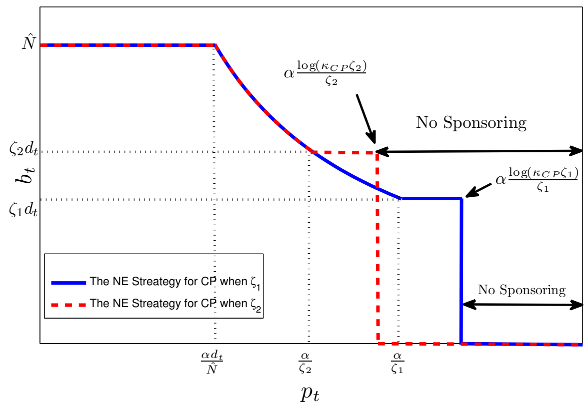

Fig. 2: The optimum strategy of the CP presented in theorem 1, when $0 < d_t \leq \frac{\hat{N}}{\zeta}$ and $\zeta_2 > \zeta_1$.

and $p_t \geq \frac{\alpha}{\zeta}$, then the lower boundary condition $\bar{b}_t = \zeta d_t$ is the candidate optimum answer. In this case, we need to make sure that $u_{CP,t}(\bar{b}_t) \geq 0$. This condition holds if $p_t \leq \frac{\alpha \log(\kappa_{CP}\zeta)}{\zeta}$ which yields the third optimality region in (6). If $p_t > \frac{\alpha \log(\kappa_{CP}\zeta)}{\zeta}$, $u_{CP,t}(b_t) < 0$. Thus, $z_t^* = 0$.

Now, consider the case in which $d_t > \frac{\hat{N}}{\zeta}$. In this case, $\zeta d_t > \hat{N}$. Therefore, there is no feasible solution for b_t. In other words, the demand is so high that even if the CP sponsor all the $\hat{N}$ bits, it is still unable to meet the minimum quality ζ. Thus, the CP exits from the sponsorship program, i.e. $z_t^* = 0$, which completes the proof. ■

Figure 2 illustrates the optimum strategy of the CP and the regions described in Theorem 2 for two different values of ζ. Note that for a given desired minimum quality (ζ), if the price per sponsored bit (p_t) is lower than a threshold ($\frac{\alpha d_t}{\hat{N}}$), the CP sponsors all the available bits. If p_t is higher than the threshold $\frac{\alpha}{\zeta}$, the CP reserves only the amount to satisfy the minimum desired quality, and for p_t higher then $\alpha\frac{\log(\kappa_{CP}\zeta)}{\zeta}$, the CP exits the sponsorship program. In addition, the higher the minimum desired quality, the lower the thresholds on p_t after which the CP sponsors only the minimum quality or exits the sponsorship program.

SP's Strategy: Now, having the optimum strategy of the CP in stage 2, we can find the optimum strategy for the SP in stage 1:

Theorem 2: **Equilibrium Strategy of Stage 1:** The optimum strategies of the SP are:

$$\text{if} \quad 0 < d_t \leq \frac{\hat{N}}{\zeta},$$
$$(p_t^*, y_t^*) = \begin{cases} (\text{argmax}\{u_{SP,t}(p_t) : p_t \in P^*\}, 1) & \text{if } u_{SP,t}(p_t^*) \geq 0 \\ (-, 0) & \text{if } u_{SP,t}(p_t^*) < 0 \end{cases}$$
$$\text{if} \quad d_t > \frac{\hat{N}}{\zeta} \text{ or } d_t = 0, \quad (p_t^*, y_t^*) = (-, 0) \tag{8}$$

where $P^* = \{\frac{\alpha d_t}{\hat{N}}, \frac{\alpha \log(\kappa_{CP}\zeta)}{\zeta}, \alpha\frac{\nu_1 d_t + \nu_2 D}{\nu_1 N}\}$ is the set of candidate optimum pricing strategies. Note that the variable y_t determines whether the SP offers the sponsorship program or not, with $y_t = 1$ implying the offering.

Proof: If $d_t < 0$, clearly $y_t^* = 0$. Now, consider $d_t > 0$. The SP maximization problem is,

$$\max_{p_t} u_{SP,t}(p_t) = \max_{p_t} \Bigg(p_t b_t^* + \nu_1 d_t \log\left(\frac{\kappa_{SP} b_t^*}{d_t}\right) + + \nu_2 D \log\left(\kappa_{SP}\frac{N - b_t^*}{D}\right)\Bigg), \tag{9}$$

where b_t^* is the equilibrium outcome of the second stage. First, consider the case in which $d_t \leq \frac{\hat{N}}{\zeta}$. If $p_t \leq \frac{\alpha d_t}{\hat{N}}$, then $b_t^* = \hat{N}$. Thus, $u_{SP,t}(p_t)$ is a strictly increasing function of p_t. Therefore, all prices less than $\frac{\alpha d_t}{\hat{N}}$ yields a strictly lower payoff than $p_{1,t}^* = \frac{\alpha d_t}{\hat{N}}$, which is the first candidate pricing strategy. If $\frac{\alpha}{\zeta} \leq p_t \leq \frac{\alpha \log(\kappa_{CP}\zeta)}{\zeta}$, then $b_t^* = \zeta d_t$. Again, in this region, $u_{SP,t}(p_t)$ is a strictly increasing function of p_t. Thus, $p_{2,t}^* = \frac{\alpha \log(\kappa_{CP}\zeta)}{\zeta}$ strictly dominates all other prices in this interval, which yields the second candidate pricing strategy. If $\frac{\alpha d_t}{\hat{N}} \leq p_t \leq \frac{\alpha}{\zeta}$, then $b_t^* = \frac{\alpha d_t}{p_t}$. In this region, the first order condition on $u_{SP,t}(p_t)$ provides us with the local extremum,

$$p_{3,t}^* = \alpha\frac{\nu_1 d_t + \nu_2 D}{\nu_1 N} \tag{10}$$

Since the second order derivative can be negative or positive, the first order condition provides us with only a candidate optimum answer, which is the third candidate pricing strategy. Thus, the optimum pricing strategy if the SP decides to offer the sponsorship program is $p_t^* = \text{argmax}\{u_{SP,t}(p_t) : p_t \in P^*\}$. If $u_{SP,t}(p_t^*) \geq 0$, then $y_t^* = 1$, otherwise $y_t^* = 0$.

If $d_t > \frac{\hat{N}}{\zeta}$, the CP does not participate in the sponsoring program if there will be any. Thus, the SP does not offer the program, i.e. $y_t^* = 0$. ■

Based on Theorem 2, if $0 < d_t \leq \frac{\hat{N}}{\zeta}$, the SP chooses one of these options: the highest price that makes the CP to reserve all the available bits, the highest price that makes the CP to reserve only to satisfy the minimum desired quality, or an in-boundary optimum price. This choice is conditional on getting a non-negative payoff from the price chosen. Otherwise, the SP exits the sponsorship program.

Corollary 1: Choosing the price $\frac{\alpha \log(\kappa_{CP}\zeta)}{\zeta}$ by the SP, i.e. the highest price by which the CP sponsors only to guarantee the minimum quality, makes the utility of the CP to be zero, and the CP to be indifferent between joining or not joining the sponsorship program. Note that we have assumed that whenever the CP or the SP are indifferent, they choose to join the sponsorship program.

Outcome: Now that we have characterized the SPNE at each time-epoch for a short-sighted CP and SP, the next step is to analyze the asymptotic behaviour of the market given the demand update function (1). The goal is to characterize the asymptotically stable 5-tuple equilibrium outcome of the game, i.e. (d, y, p, z, b) (please refer to table I), if it were to exist. In Theorem 3, all possible asymptotically stable outcomes are listed, However, the existence of such a stable outcome is not guaranteed, and the market can be unstable in some cases.

Theorem 3: The possible asymptotically stable outcomes of the game are:

1) $(-, 0, -, 0, -)$: No sponsoring is offered, none taken.

2) $\left(\kappa_u \hat{N}, 1, \alpha\kappa_u, 1, \hat{N}\right)$, if $\kappa_u \leq \frac{1}{\zeta}$: Maximum bit sponsorship.
3) $\left(d, 1, \frac{\alpha \log(\kappa_{CP}\zeta)}{\zeta}, 1, \zeta d\right)$, if $\kappa_u = \frac{1}{\zeta}$ and $0 < d \leq \frac{\hat{N}}{\zeta}$: the minimum quality.
4) $\left(N\kappa_u - \frac{\nu_2}{\nu_1}D, 1, \alpha\kappa_u, 1, N - \frac{\nu_2}{\kappa_u \nu_1}D\right)$, if $\kappa_u \leq \frac{1}{\zeta}$ and $0 < b \leq \hat{N}$: In-boundaries stable point.

Proof: The first candidate stable outcome is trivial: as soon as one the CP or SP exits the sponsorship program, or $d_t = 0$, the program will not be resumed. If demand becomes stable, an stable market is expected. From (1), it can be concluded that the demand is stable when $d_t = \kappa_u b_t$. One possible scenario is when sponsoring happens, i.e. $y_t = z_t = 1$, $b_t = \hat{N}$, and $d_t = \kappa_u \hat{N}$. In this case, $p_t = \frac{\alpha d_t}{\hat{N}} = \alpha\kappa_u$. Note that this case is valid only if $d_t = \kappa_u \hat{N} \leq \frac{\hat{N}}{\zeta}$. Thus, $\kappa_u \leq \frac{1}{\zeta}$.

Another possible stable outcome happens when $b_t = \zeta d_t$, and $\kappa_u \zeta = 1$. In this case, $d_t = \kappa_u b_t = \kappa_u \zeta d_t = d_t$. Therefore, the demand is stable. This case happens when the SP sets the price $p_t = \frac{\alpha \log(\kappa_{CP}\zeta)}{\zeta}$. The demand, in this case, could be any positive value less than $\frac{\hat{N}}{\zeta}$.

The last possible stable outcome happens when $\frac{\alpha d_t}{\hat{N}} \leq p_t \leq \frac{\alpha}{\zeta}$. In this case, $b_t = \frac{\alpha d_t}{p_t}$. In order to have a stable outcome, $p_t = \alpha\kappa_u$. Note that, in this case, $p_t = p = \alpha\frac{\nu_1 d_t + \nu_2 D}{\nu_1 N} = \alpha\kappa_u$. Thus, $d_t = d = N\kappa_u - \frac{\nu_2}{\nu_1}D$ and $b_t = b = N - \frac{\nu_2}{\nu_1 \kappa_u}D$. In order to be valid, $0 < b \leq \hat{N}$ and $\frac{\alpha d_t}{\hat{N}} \leq p_t = \alpha\kappa_u \leq \frac{\alpha}{\zeta}$. The latter yields that $\kappa_u \leq \frac{1}{\zeta}$ and $b_t = \frac{\alpha d_t}{p_t} \leq \hat{N}$. The result follows. ■

Corollary 2: **The Stable Quality:** The market is stable if and only if the quality $\frac{b}{d} = \frac{1}{\kappa_u}$ is sponsored for end-users. We call $\frac{1}{\kappa_u}$ the stable quality. Note that if $\kappa_u = \frac{1}{\zeta}$, the stable quality is equal to ζ, i.e. the minimum quality to be satisfied.

Corollary 3: There is no stable outcome involving sponsoring for the game if the stable quality is smaller than the minimum quality set by the CP, i.e. $\frac{1}{\kappa_u} < \zeta$. In other words, when end-users grow drastically even with small rates and the CP over-provision the minimum quality for customers satisfaction, then either the market is unstable or the CP and SP exit the sponsorship program.

Corollary 4: The third possible stable point, i.e. $\left(d, 1, \frac{\alpha \log(\kappa_{CP}\zeta)}{\zeta}, 1, \zeta d\right)$, characterizes a wide range of stable demands when $\zeta = \frac{1}{\kappa_u}$, i.e. when the stable quality is equal to the minimum desired quality set by the CP. Whenever the SP sets $p = \frac{\alpha \log(\kappa_{CP}\zeta)}{\zeta}$, the CP sets $\frac{b_t}{d_t} = \zeta$, and the market will be stable. By choosing that price, the SP ensures that she will extract all the profit of the CP and makes her indifferent between joining the sponsorship program and opting out, i.e. $u_{CP}(b) = 0$. In Theorem 4, we derive the desired stable demand that maximize the payoff of the SP in this set of stable points.

In the next theorem, we find the stable demand that maximizes the payoff of the SP when she chooses the third stable point, i.e. the minimum quality.

Theorem 4: The payoff of SP when the 5-tuple $\left(d, 1, \frac{\alpha \log(\kappa_{CP}\zeta)}{\zeta}, 1, \zeta d\right)$ is the stable outcome of the market is maximized when $d = \min\{d^*, \frac{\hat{N}}{\zeta}\}$ if $d^* \geq 0$ and $d = 0$ otherwise, where $d^* = \frac{N}{\zeta} - \frac{1}{(\alpha+\nu_1)\log(\kappa_{SP}\zeta)}$.

Proof: The utility of the SP when choosing the tuple $\left(d, 1, \frac{\alpha \log(\kappa_{CP}\zeta)}{\zeta}, 1, \zeta d\right)$ is:

$$u_{SP} = \alpha d \log(\kappa_{SP}\zeta) + \nu_1 d \log(\kappa_{SP}\zeta) + \nu_2 D \log\left(\kappa_{SP}\frac{N - \zeta d}{D}\right)$$

First, note that the expression of the utility is concave. Thus, the first order condition gives the optimum answer. The solution of the first order condition is:

$$d^* = \frac{N}{\zeta} - \frac{1}{(\alpha + \nu_1)\log(\kappa_{SP}\zeta)}$$

If $d^* > 0$, d^* is the optimum answer if the number of bits reserved corresponding to this demand (b^*) satisfies the constraint $b^* \leq \hat{N}$. Thus, $\zeta d^* \leq \hat{N}$, and $d^* > 0$ is the optimum answer if $d^* \leq \frac{\hat{N}}{\zeta}$. If $d^* < 0$, zero demand for the content is optimum. The result follows. ■

Note that from Theorem 4 the optimum demand is an increasing function of resources (N and $\hat{N}$) and a decreasing function of the minimum rate, ζ.

In Section V, we explain more about the effects of key parameters of the market on stabilization by experimenting several sets of parameters. Results reveal that if $\frac{1}{\kappa} > \zeta$, i.e. the stable quality is larger than the minimum quality desired by the CP, then it is expected to have an unstable market or the non-sponsoring stable point, i.e. $(-, 0, -, 0, -)$, for a wide range of parameters. This is because of the fact that the conditions for having a stable sponsoring outcome if $\kappa_u \leq \frac{1}{\zeta}$ and $0 < b \leq \hat{N}$, are more tight than those when $\kappa_u \zeta = 1$. Considering decision makers with long-sighted vision about the market may ensure a stable sponsoring outcome for the market, when parameters of the market are such that a stable sponsorship outcome is not plausible.

IV. Long-sighted Business Model

In this section, we consider scenarios in which stakeholders of the market have long-sighted business model, i.e. when they not only consider the history of the game in their decision making, but also take into the account the future of the market. First, we consider a CP with a short-sighted business model and an SP with a long-sighted business model, and then a long-sighted CP and a short-sighted SP. Finally, we discuss about the market in which both the CP and the SP have long-sighted business models.

A. Long-Sighted SP, Short-Sighted CP

A long-sighted SP sets the per-bit sponsorship fee in order to achieve a stable market, i.e. a stable demand for the content, and also to maximize the payoff in the long-run:

$$U_{SP,Long\ Run}(\vec{p}) = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} u_{SP,t}(p_t) \tag{11}$$

Note that in this scenario, the SP is the leader of the game and therefore can set the equilibrium of the game individually by knowing that the CP is a myopic optimizer unit and follows the results in Theorem 1. Thus, the SP chooses the plausible stable sponsoring point (presented in Theorem 3) that yields the highest payoff, and sets appropriate sponsoring fees in order to lock the stable outcome of the market in this point. In the next theorem, we prove that in the case that $\kappa_u = \frac{1}{\zeta}$, the third stable point yields the highest payoff. This is because of the fact that in this case, the SP extracts all the profits of CPs from sponsoring and makes it indifferent between sponsoring and not sponsoring, i.e. $u_{CP} = 0$.

Theorem 5: The minimum quality stable point, i.e. $\left(d, 1, \frac{\alpha \log(\kappa_{CP}\zeta)}{\zeta}, 1, \zeta d\right)$, with the demand characterized in Theorem 4 yields the highest payoff for the SP. Thus, a long-sighted SP sets this stable point as the asymptotic outcome of the market when $\kappa_u = \frac{1}{\zeta}$.

Proof: First, note that in Theorem 3, in all the stable sponsoring points, $\frac{b}{d} = \frac{1}{\kappa_u}$. Thus, when $\kappa_u = \frac{1}{\zeta}$, all the stable sponsoring points yield the quality of ζ for end-users. In this case, the payoff of the SP is:

$$u_{SP} = p\zeta d + \nu_1 d \log(\kappa_{SP}\zeta) + \nu_2 D \log\left(\kappa_{SP}\frac{N - \zeta d}{D}\right)$$

Note that from (6), the CP operates at the minimum desired quality, i.e. ζ, when $\frac{\alpha}{\zeta} \leq p \leq \frac{\alpha \log(\kappa_{CP}\zeta)}{\zeta}$. Since the payoff of the SP in this case is an increasing function of the price p, and the demand d is dependent solely on the quality ζ and not the price set by the SP, the SP chooses $p = \frac{\alpha \log(\kappa_{CP}\zeta)}{\zeta}$ as the optimum asymptotic price, which is the third stable outcome of the market. The optimum demand is chosen by Theorem 4 as discussed before. The result follows. ■

If $\kappa_u < \frac{1}{\zeta}$, depending on the parameters of the market, the stable point 2, i.e. maximum bit sponsoring, or 4, i.e. in-boundary stable point, is chosen by the SP. In this case, if ν_2, i.e. the importance of non-sponsored data for end-users and SP, is high enough, the stable point 4 is chosen and set by the SP. In addition, increasing the total number of resources available with the SP, i.e. N, makes the stable point 2, i.e. maximum bit sponsoring, more favorable for the SP.

B. Short-Sighted SP, Long-Sighted CP

Now, consider a CP that chooses b_t in order to achieve a stable demand for its content, and also to maximize the payoff in the long-run:

$$U_{CP,Long\ Run}(\vec{b}) = \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} u_{CP,t}(b_t) \quad (12)$$

In the next theorem, we prove that for a long-sighted CP, the maximum bit sponsorship yields the highest payoff.

Theorem 6: The 5-tuple plausible stable sponsoring points in a decreasing order of the utility they yield to the CP are: 1. maximum bit sponsorship, 2. in-boundary stable point, 3. minimum quality.

Proof: First note that the minimum quality stable point, i.e. $\left(d, 1, \frac{\alpha \log(\kappa_{CP}\zeta)}{\zeta}, 1, \zeta d\right)$, yields a payoff of zero for the CP. From Corollary 2, in both the maximum bit sponsorship, i.e. $\left(\kappa_u \hat{N}, 1, \alpha\kappa_u, 1, \hat{N}\right)$, and in-boundary stable point, i.e. $\left(N\kappa_u - \frac{\nu_2}{\nu_1}D, 1, \alpha\kappa_u, 1, N - \frac{\nu_2}{\kappa_u \nu_1}D\right)$, the stable quality ($\frac{b}{d}$)

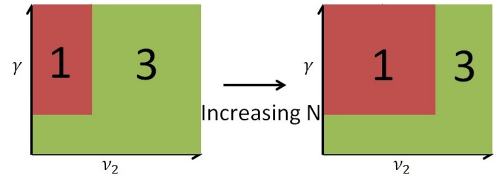

Fig. 3: Market Asymptotic Outcomes with Short-Sighted Decision Makers when $\kappa_u = \frac{1}{\zeta}$

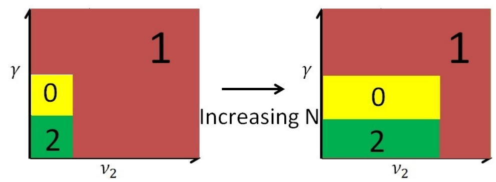

Fig. 4: Market Asymptotic Outcomes with Short-Sighted Decision Makers when $\kappa_u < \frac{1}{\zeta}$

is $\frac{1}{\kappa_u}$. Thus, the payoff of the CP in these plausible stable outcomes is:

$$u_{CP} = \alpha d\left(\log\left(\frac{\kappa_{CP}}{\kappa_u}\right) - 1\right)$$

Note that from the condition for plausibility of these stable points ($\kappa_u \leq \frac{1}{\zeta}$), and our previous assumption that $\kappa_{CP}\zeta > e$[1], $\frac{\kappa_{CP}}{\kappa_u} > e$. Thus, the payoff of the CP is strictly increasing with respect to the demand. Given that the quality, i.e the ratio of the bit sponsored and the demand, is $\frac{1}{\kappa_u}$ and is a constant, the higher the number of sponsored bits, the higher the payoff of the CP. Thus, the maximum bit sponsoring point yields the highest payoff. The result follows. ■

Therefore, a long-sighted CP is willing to sponsor all the available bits for sponsoring. Thus, given that the SP is short-sighted, the CP sets the number of bits for sponsoring appropriately, in order to achieve the demand of $\kappa_u \hat{N}$. Subsequently, she sponsors all the available bits, i.e. $\hat{N}$, and the market will be locked in this stable point.

C. Long-Sighted SP, Long-Sighted CP

Now, consider the case in which both the SP and the CP are long-sighted. In this case, if the SP prefers the maximum bit sponsorship over other plausible stable points, since this stable point is also preferred by the CP, the maximum bit sponsorship point is set by the SP and CP. Otherwise, a bargaining game should be played by the CP and the SP to set the stable outcome. Depending on their power over the market, one of the plausible stable outcomes is set by the SP and the CP. This bargaining game is beyond the scope of this paper and is a topic of future work.

V. Numerical Evaluations

In this section, we consider the SP and the CP to have a short-sighted business model. The fixed parameters considered in this section are $\nu_1 = 1$, $\hat{N} = 25$, $D = 50$, $\kappa_{SP} = \kappa_{CP} = 10$, and $\zeta = 0.3$. We want to observe the effect of important parameters such as the sensitivity of the demand to the quality (κ_u), intensity of changes in the demand (γ), the weight an SP

[1]The condition to have a non-trivial problem.

assigns to the non-sponsored data (ν_2), and the total number of available bits in an LTE frame (N) on the asymptotic outcome of the market.

Market asymptotic outcomes for different parameters for the two cases of $\kappa_u = \frac{1}{\zeta}$ and $\kappa_u = \frac{1}{2\zeta} < \frac{1}{\zeta}$ are presented in Figures 3 and 4, respectively. Recall from Theorem 3 that the asymptotically stable outcome of the game is one the four candidates: 1. No-Sponsoring, 2. Maximum bit sponsoring, 3. Minimum quality: the minimum quality that the CP wants to sponsor, and 4. In-boundary stable point. In the figures, we denote the unstable outcome by 0.

1) $\kappa_u = \frac{1}{\zeta}$ *:* In this case, since the stable quality, $\frac{1}{\kappa_u}$, is equal to the lower bound of quality, i.e. ζ, by (1), the demand for the data is non-decreasing over time. Thus, demand increases up to the point that the CP is forced to work on the minimum quality and market is stabilized after that.

Results in Figure 3 reveal that in this case, the market is stable for a wide range of parameters. The stable points are either the third outcome, i.e. the minimum quality point ($\left(d, 1, \frac{\alpha \log(\kappa_{CP}\zeta)}{\zeta}, 1, \zeta d\right)$), or the no-sponsoring point.

Impact of intensity of changes in demand, γ: Note that when v_2 is smaller than a threshold, fixing v_2 and increasing γ shift the stable point of the market from 3, i.e. the minimum quality, to 1, i.e. no-sponsoring. The parameter γ can be considered as the parameter to fine-tune the demand. When γ is small, the market will be tuned on the minimum quality stable point, whereas in the case of large γ, the demand may exceed $d_{max} = \frac{\hat{N}}{\zeta}$, which is the highest number of end-users that can be satisfied with the minimum quality. Thus, market will be set on a stable point of no-sponsoring.

Impact of the importance of non-sponsored content, ν_2: Results reveal that by increasing v_2 and after a threshold, the stable outcome is not sensitive to γ, and market operates at the minimum quality stable point even when γ is large. The parameter v_2 being large, when v_1 is normalized to one, represents the fact that the SP assigns more weight to the satisfaction of users for using non-sponsored content. Thus, the best strategy for the SP is to set her per-bit sponsorship fee high enough so that the CP sponsors a smaller number of bits. This means that the CP starts the sponsoring program with a quality near the minimum quality, and demand increases more slowly. Therefore, similar to having a smaller γ, the market can be operated on the minimum quality stable point.

Impact of total available resources, N: Figure 3 reveals that increasing the of number available bits (resources) increases the area of no-sponsoring region. This is because of the fact that by increasing N, i.e. the number of bits in an LTE frame or the total number of resources of the SP, the value of the SP for each bit decreases and even when v_2 is large, the SP sets a lower sponsoring fees. This leads to sponsoring more bits by the CP, which yields the same outcomes as the case in which v_2 is small: the minimum quality stable point when γ is small and no-sponsoring when γ is large.

2) $\kappa_u = \frac{1}{2\zeta} < \frac{1}{\zeta}$*:* In this case, the market is either unstable or has one of the asymptotically stable candidate outcomes of 1. no-sponsoring, 2. maximum bit sponsorship, or 4. in-boundary stable point depending on the parameters of the market. The stable and unstable regions is presented in Figure 4, with 0 denoting an unstable market.

Impact of intensity of changes in demand, γ: Similar to the case of $\kappa_u = \frac{1}{\zeta}$, when γ is large, the demand cannot be fine-tuned in a stable sponsoring point. Thus, the market is either unstable or the demand exceeds the highest number of end-users that can be satisfied even when sponsoring the maximum resources available and providing the minimum quality for end-users, and the market will be set on a stable point of no-sponsoring.

Impact of the importance of non-sponsored content, v_2: when v_2 is small and the SP cares mostly about the sponsored data, she maximizes her utility by quoting a price that provides enough incentive for the CP to sponsors all the bits. Thus, in certain range of γ, the stable outcome is $\left(\kappa_u \hat{N}, 1, \alpha\kappa_u, 1, \hat{N}\right)$, i.e. to sponsor all the available bits.

As v_2 increases, the SP cares more and more about the quality of non-sponsored data. Thus, the SP will set her price such that fewer bits are sponsored by the CP, and the CP operates at the minimum quality point. Note that in this case the stable quality is higher than the minimum quality requested by the CP. In other words, the CP under-provision the minimum quality for customers, or mathematically $\frac{1}{\kappa_u} > \zeta$. Thus, the number of end-users decreases when the CP sponsors the minimum quality ζ, and the demand of end-users diminishes to zero. Therefore, when v_2 is large, market will asymptotically set to the no-sponsoring point.

Impact of total available resources, N: In Figure 4, similar to the case of $\kappa_u = \frac{1}{\zeta}$, increasing N increases the area of no-sponsoring region. The reason is similar to what was previously stated in the earlier case.

Remark: Figure 4 illustrates that the stable point 4, i.e. in-boundary point, does not emerge, and only stable points 1 and 2 occur: in a stable outcome, when decision makers are short-sighted, either the CP sponsors all the available resources or no sponsoring occurs. Note that in the stable point 4 the number of bits sponsored by the CP in the equilibrium is $N - \frac{\nu_2}{\kappa_u \nu_1} D$. In addition, a stable sponsoring 5-tuple occurs only when ν_2 is small which makes $N - \frac{\nu_2}{\kappa_u \nu_1} D > \hat{N}$ for a wide range of parameters. Thus, the stable point 4 does not emerge in many scenarios. One can argue that by decreasing N or increasing D, we may have a scenario in which $N - \frac{\nu_2}{\kappa_u \nu_1} D < \hat{N}$. However, note these changes, increase the value of each bit for the SP, and decrease the threshold after which the SP is willing to set a price so high that leads the CP and the market to a no-sponsoring outcome. Thus, again in the regions that support sponsoring an in-boundary amount of resources ($N - \frac{\nu_2}{\kappa_u \nu_1} D > \hat{N}$), the stable outcome 4 does not occur.

VI. Discussion

A. Conducive/Detrimental Factors for Sponsoring:

The importance of non-sponsored data, ν_2: the SP may assign a large weight to the quality experienced by her end-users for the non-sponsored data (large ν_2) to maintain their satisfaction and avoid losing her market share in the future. In this case, the SP sets her price for resources high enough to ensure that enough resources remain to satisfy the demand for the non-sponsored content. Thus, increasing ν_2, may move the stable strategy of the CP from sponsoring all the available resources to in-boundary strategies, and then to the strategy in which the CP sponsors only a minimum quality. Large ν_2 may eventually lead to a no-sponsoring outcome since the amount of resources the SP is willing to offer is not sufficient even for sponsoring the minimum quality. Therefore, when ν_2 is large, only a CP that requires a lower rate (smaller ζ) can benefit from quality-sponsored plan.

Intensity of changes in demand, γ: In a market with short-sighted business entities, the intensity of changes in the demand, i.e. γ, greatly influences the stability of the market. When γ is high, the demand of end-users increases/decreases drastically with small changes in the rate perceived by them. Thus, the market will be asymptotically unstable, or players will exit the sponsorship program. Thus, a CP that has a well-established end-user side, i.e. a more stable demand, is preferable for the QSD when market entities are short-sighted.

The minimum quality (ζ) and the stable quality ($\frac{1}{\kappa_u}$): In Corollary 2, we discussed that the demand update function (1) has a stable quality, and the demand is stable if and only if the CP sponsors this quality. In addition, depending on the minimum quality set by the CP for the quality that end-users receive, the market has different stability conditions in short-sighted and long-sighted scenarios:

• If the CP *over-provisions* the minimum quality for the satisfaction of users ($\zeta > \frac{1}{\kappa_u}$), there is no stable sponsoring outcome since the demand of users grows drastically forcing the SP and CP to exit the sponsoring program.

• If the CP sets the minimum quality equal to the stable quality ($\zeta = \frac{1}{\kappa_u}$), the market has a wide range of stable sponsoring outcomes. However, the price is set by the SP to extract all the profit of the CP and make it indifferent between joining or opting out of the sponsoring program. Simulation results reveal that when the SP and the CP are short-sighted, the market will be set at the minimum quality stable point for a wide range of parameters. In this case, the market is set to the stable point of no-sponsoring only when the intensity of changes in the demand is high (large γ), and the SP is willing to set a low price to increase the number of bits sponsored. In this case, the CP secures a quality more than the stable quality which results in the rapid growth of the demand for the content and forces the SP and the CP to exit the sponsoring program.

• If the CP *under-provisions* the minimum quality ($\zeta < \frac{1}{\kappa_u}$), simulation results reveal that the market is expected to be unstable or has the stable outcome of no-sponsoring for a wide range of parameters. In this case, the market will be set in the stable point of maximum bit sponsoring only if the intensity of changes in the demand is low, and the SP is willing to set a low price for a sponsored bit.

B. The bargaining game:

Note that From Theorem 5, the SP prefers the minimum quality stable point over all other possible stable outcomes of the market, since in this case the profit of the CP from sponsoring can be fully extracted. On the other hand, for the same reason, this stable point is the least favorite one for the CP. Note that this stable outcome only happens when the CP sets the desired minimum quality equal to the stable quality. Thus, to avoid the minimum quality stable point, the CP has the incentive to increase their desired minimum rate in order to force the SP to set the stable outcome at points 2 or 3, in which the payoff of the CP is positive. We discussed that with short-sighted SP and CP, this scenario results in a stable outcome of no-sponsoring for a wide set of parameters. This illustrates the importance of setting up a bargaining structure between SPs and CPs.

Note that when the SP and the CP are long-sighted a profit sharing mechanism can be obtained using bargaining theory in order to avoid an unstable market or a no-sponsoring stable point. This constitutes a topic of future research.

VII. Conclusion

We introduced the problem of quality-sponsored data (QSD) in cellular networks and studied its implications on market entities in various scenarios. The direct coupling between the scarce wireless resources and the market decisions resulting from QSD has been taken into account, and the market dynamics and equilibira have been investigated. In addition, we provided strategies for (i) SPs: to determine if and how to price resources, and (ii) CPs: to determine if and how many resources to sponsor (what quality).

A topic of future work is to consider a bargaining structure between long-sighted CPs and SPs. In addition, note that since end-users in current wireless plans are mostly subject to contracts, we assumed that end-users are locked with SPs. However, considering the recent trend among SPs to offer no-contract plan services, another possible direction for future work is to consider end-users that can switch their SPs.

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