Distributed Learning Algorithms for Spectrum Sharing in Spatial Random Access Networks

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***Abstract*— We consider distributed optimization over orthogonal collision channels in spatial multi-channel ALOHA networks. Users are spatially distributed and each user is in the interference range of a few other users. Each user is allowed to transmit over a subset of the shared channels with a certain attempt probability. We study both the non-cooperative and cooperative settings. In the former, the goal of each user is to maximize its own rate irrespective of the utilities of other users. In the latter, the goal is to achieve proportionally fair rates among users. We develop simple distributed learning algorithms to solve these problems. The efficiencies of the proposed algorithms are demonstrated via both theoretical analysis and simulation results.**

I. Introduction

The spectrum scarcity along with the increasing demand for wireless communication have triggered the development of efficient spectrum access schemes for wireless networks. In this paper we focus on Medium Access Control (MAC) schemes in multi-channel wireless networks, in which users transmit over orthogonal channels using Orthogonal Frequency Division Multiple Access (OFDMA).

Consider a spatial wireless network with N users sharing K collision channels. Each user is in the interference range of a few (but not necessarily all) other users, referred to as neighbors (e.g., when the distance between users is small they cause mutual interference). In the beginning of each time slot, each user is allowed to transmit over M channels ($1 \leq M \leq K$) with a certain attempt probability (i.e., using the slotted-ALOHA protocol). If two or more neighbors transmit simultaneously over the same channel, a collision occurs. In multi-channel systems, exploiting the channel diversity plays an important role in designing effective channel allocation protocols. The channel conditions are a function of both the inherent quality of each channel due to fading, shadowing, etc., as well as the interference caused by the users that use the channel. Thus, it is intuitive that users can improve performance by adaptively choosing channels with a higher probability of being idle as well as higher capacity when idle. We are interested in finding a channel allocation and attempt probabilities in a distributed manner so as to optimize certain objectives in the network.

A. Main Results

Spectrum access protocols can be broadly classified into two classes: (i) protocols in which users do not share information with each other, due to security or overhead considerations, and (ii) protocols in which information is shared to achieve a common goal, such as in networks which are controlled by a single provider. Achieving an effective channel allocation for the spectrum access problem in a distributed manner requires users to adaptively adjust their actions (i.e., select channels and attempt probabilities) based on local information about the current state of the system. Thus, the first question of interest is whether the system keeps oscillating due to frequent channel switching, or whether the system converges to a stable operating point. When users do not share information, a stable channel allocation may not be a system-wide optimal solution (though it reduces the undesirable effects of frequent channel switching and also demonstrated good performance in some network models and typical scenarios, as in [1]–[3]). Thus, the second question of interest is whether small amounts of information sharing can lead to a globally-optimal operating point.

We first examine the case where users do not share information with each other. The achievable rate of each user increases with its own attempt probability, when other attempt probabilities are fixed. Thus, a natural approach to achieve a good operating point is to allow every user to maximize its own rate under a constraint on the allowed attempt probability[1] (where different attempt probability constraints are used to prioritize different users in the network), referred to as distributed rate maximization. Previous work [3] has studied the distributed rate maximization problem in a fully connected network (i.e., all users are in the same interference range) and $M = 1$ (i.e., every user is allowed to transmit over a single channel). It was shown in [3] that any improvement path (not necessarily best-response) across users, in which at each iteration the rate of a user increases when it updates its channel-selection strategy given the current system state, reaches an equilibrium in the sense that no user can increase its rate by unilaterally changing its strategy. In this paper, however, we consider a more general case where each user interferes only with its neighbors, and $M \geq 1$. Interestingly, we show that cycles may occur under some improvement paths in this general model. To solve this problem, we use the theory of *best-response (BR) potential games*, introduced by Voorneveld in 2000 [7]. In BR potential

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[1]Similar approaches were applied in [3]–[5] for fully-connected ALOHA networks, all resulting with an individual rate and attempt probability for every user. Another example is the rate-adaptive problem in OFDM systems, where every user maximizes its own rate under an individual power constraint [1], [6].

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games, cycles may occur under some improvement paths, though no cycles occur under a BR dynamics. We prove that the system dynamics can be formulated as a BR potential game and propose a distributed BR learning algorithm that solves the distributed rate maximization problem and converges to an equilibrium in finite time.

Second, we focus on a cooperative setting, in which the goal is to achieve the optimal channel allocation and attempt probabilities that attain proportionally fair rates in the network. A number of studies exist in the literature addressing the proportional fairness problem over collision channels under various spatial ALOHA models, all assuming the single-channel case, i.e., $K = 1$ (see Section I-B for a discussion on related work). When $K = 1$, users have no freedom to choose among different channels, and the action of each user degenerates to setting the optimal attempt probability for transmission over the single channel. In this paper, however, we address this question for multi-channel networks (i.e., $K \geq 1$) where every user is allowed to choose a single channel for transmission (i.e., $M = 1$) among the K channels and to set the optimal attempt probability for transmission over the channel[2]. Direct computation of the optimal channel allocation and attempt probabilities that attain proportionally fair rates for the multi-channel ALOHA network considered in this paper is a combinatorial optimization problem over a graph which is mathematically intractable. Furthermore, it requires a centralized solution that uses global information which is impractical in large-scale networks. Therefore, we develop a novel cooperative distributed algorithm based on noisy BR dynamics to solve these problems. Specifically, using message exchanges between neighbors only, users take actions with respect to a cooperative utility that balances between their own utilities and the interference level they cause to their neighbors. In noisy BR dynamics, users play the BR that maximizes their cooperative utilities with high probability, while suboptimal responses are taken with small probabilities to escape local maxima. We prove that the proposed cooperative algorithm converges to the global proportional fairness solution with high probability as time increases. Furthermore, we show that every Nash equilibrium attained by the algorithm can be reached in a finite time by playing BR and it is a good operating point in the sense that proportionally fair rates are attained locally among all users sharing the same channel.

B. Related Work

Since distributed algorithms are generally preferred over centralized solutions in large-scale systems, game theoretic models have been widely used to analyze system dynamics in wireless networks. Related work on networking games can be found in [3]–[5], [8]–[23]. Non-cooperative Random access games were studied in [3]–[5], [8], [16], [20]. Cooperative game theoretic optimization has been studied under frequency flat interference channels in the SISO [12], [14], MISO [17], [18] and MIMO cases [15]. The frequency selective interference channels case has been studied in [10], [19]. The collision channels case has been studied under a fully-connected network and without information sharing between users in [23], where the global optimum was attained under the asymptotic regime (i.e., as the number of users N approaches infinity) and the i.i.d assumption on the channel quality. In this paper, however, we study distributed optimization of the user rates under the cooperative setting for spatial networks where information sharing between neighbors is allowed. We show that proportionally fair rates are attained for any number $N \geq 1$ of users without any assumption on the network topology or channel distribution.

ALOHA-based protocols have been widely used in wireless communication primarily because of their ease of implementation and their random nature. Related work on ALOHA-based protocols can be found in [3]–[5], [8], [23]–[28] for fully connected networks and in [29]–[33] for spatial networks. Stability of multi-channel ALOHA systems was studied in [24], [25]. In [31], [32], spatial single-channel ALOHA networks have been studied under interference channels using stochastic geometry. Opportunistic ALOHA schemes that use cross layer MAC/PHY techniques, in which the design of Medium Access Control (MAC) is integrated with physical layer channel information to improve the spectral efficiency, have been studied under both the single-channel [4], [27], [32] and multi-channel [3], [23], [27], [28] cases. A cross-layer MAC/PHY methodology is used in this paper to design efficient distributed algorithms for the problems under study. Achieving proportionally fair rates in spatial ALOHA networks has been studied in [29], [30], [33] under the case of a single collision channel. In this paper we examine a variation of the model considered in [29] under multiple collision channels. It should be noted that achieving proportional fairness under the single-channel case requires information sharing between neighbors. Thus, the implementation complexity of the distributed algorithm developed in this paper for the multi-channel case in terms of the required information sharing does not go significantly beyond existing solutions developed for the single-channel case.

II. Network Model

We consider a wireless network containing sets $\mathcal{N} = \{1, 2, ..., N\}$ of users and $\mathcal{K} = \{1, 2, ..., K\}$ of shared channels (where typically $N > K$). We focus on a spatial wireless network, where each user is in the interference range of a few (but not necessarily all) other users. We assume symmetric interference ranges for all users in the sense that user n is in user r's interference range only if user r is in user n's interference range for all $n, r \in \mathcal{N}$. We refer to users in the same interference range as *neighbors*, and define $\mathcal{I}_n \subseteq (\mathcal{N} \setminus n)$ as the set of user n's neighbors. We assume that users are backlogged, i.e., all N users always have packets to transmit. In the beginning of each time slot, each user (say n) is allowed to transmit over M channels ($1 \leq M \leq K$) with a certain attempt probability (i.e., using the slotted-ALOHA protocol). Let $\mathcal{K}_M$ be the set of all M-element subsets of $\mathcal{K}$ (i.e., $\mathcal{K}_M$ is the set of all channel-selection strategies that a user can choose). Let $\sigma_n = (k_n, p_n)$ be the strategy of user n, where $k_n = \{k_{n,i}\}_{i=1}^{M} \in \mathcal{K}_M$ denotes the set of chosen channels and $0 \leq p_n \leq 1$ denotes the attempt probability of user n. Thus, when user n decides to transmit (which occurs with

[2] Accessing a single-channels is often assumed due to hardware constraints or when it is desired to limit the congestion level over the channels in high-loaded systems. It has been widely assumed in cognitive radio applications, WiFi, sensor networks, etc. It should be noted, however, that developing a tractable optimal solution for the proportional fairness problem under the case where users are allowed to access two or more channels at a time remains an open question.

probability p_n) it uses all the channels in k_n for transmission. We define σ as the strategy profile for all users, and σ_{-n} as the strategy profile for all users except user n.

The topology of the interference model can be represented by an undirected graph $G = (\mathcal{N}, E)$, where the set of users are represented by the vertices and the interference relationships between users are represented by the set of edges E. An edge $(n, r) \in E$ means that users n and r are in the same interference range. The set of user n's neighbors $\mathcal{I}_n$ is represented by vertices directly connected to vertex n excluding vertex n itself. An illustration is given in Fig. 1 in Section V.

We consider transmissions over orthogonal collision channels. Thus, transmission by user n over channel $k_{n,i}$ is successful only if no user $r \in \mathcal{I}_n$ transmits over channel $k_{n,i}$ in the same time-slot. However, if user n and at least one more user in $\mathcal{I}_n$ transmit simultaneously over channel $k_{n,i}$ in the same time slot, a collision occurs. The achievable rate of user n over channel k given that a transmission is successful, referred to as collision-free utility, is denoted by $u_n(k) \geq 0$. We consider long-term rates where $u_n(k)$ remain fixed across time slots during the running-time of the algorithms (e.g., mean-rate, or slow-fading effect).

Define the success probability of user n on channel k given the strategy profile of other users, as follows:

$$v_n(k, \sigma_{-n}) \triangleq \prod_{i \in \mathcal{I}_n} (1 - p_i)^{\mathbf{1}_i(k)} , \quad (1)$$

where $\mathbf{1}_i(k) = 1$ if $k \in k_i$ or $\mathbf{1}_i(k) = 0$ otherwise. Hence, the expected rate of user n over channel $k_{n,i}$ is given by:

$$r_n(k_{n,i}, p_n, \sigma_{-n}) = p_n u_n(k_{n,i}) v_n(k_{n,i}, \sigma_{-n}) . \quad (2)$$

Note that the log-rate of user n over channel $k_{n,i}$ is given by

$$\log r_n(k_{n,i}, p_n, \sigma_{-n}) = \log (u_n(k_{n,i}) p_n) - I_n(k_{n,i}, \sigma_{-n}), \quad (3)$$

where $I_n(k, \sigma_{-n})$ is referred to as the *log-interference* function and is given by:

$$I_n(k, \sigma_{-n}) \triangleq -\log v_n(k, \sigma_{-n}) = \sum_{i \in \mathcal{I}_n} \log \left(\frac{1}{1 - p_i} \right) \mathbf{1}_i(k) . \quad (4)$$

Note that $I_n(k, \sigma_{-n})$ can be viewed as the log-interference that user n experiences over channel k caused by its neighbors that transmit over the same channel. Finally, the expected rate of user n is given by:

$$R_n(\sigma) \triangleq \sum_{i=1}^{M} r_n(k_{n,i}, p_n, \sigma_{-n}) . \quad (5)$$

Throughout the paper, we will develop distributed algorithms to optimize certain objectives in the network. Achieving the desired operating points requires that the algorithms are implemented in a sequential manner, in which only a subset of the users can update their strategies at each iteration as described below. For simplicity, it is assumed that users hold a global clock and may update their strategies only at times $t_1, t_2, ...,$ referred to as *updating times*. At each updating time, every user draws a backoff time from a continuous uniform distribution over the range $[0, B]$ for some $B > 0$. A user whose backoff time expires broadcasts a pilot signal to its neighbors, indicating that its strategy has been updated (or even starts transmitting its data and neighbors can sense activity). Then, all its neighbors keep their strategies fixed until the next updating time. At each updating time, we refer to users that update their strategies as *active users*. The set of active users is denoted by $\mathcal{N}_a$ (which is time-varying across updating times). In Tables I, II (Step 3) we refer to this mechanism as a selection of active users. It should be noted, however, that convergence of the algorithm discussed in Section III-B can be shown even without this coordination mechanism.

III. Distributed Rate Maximization: A Non-Cooperative Setting

In this section we consider the case where every user (say n) maximizes its own rate given the current system state under a constraint P_n on its allowed attempt probability, i.e., $p_n \leq P_n$ where $P_n < 1$ (see Section I-A for motivation of this problem). Since maximizing the rate given the current system state results in a transmission with the maximal allowed attempt probability P_n, the strategy for user n degenerates to choosing the subset of channels k_n that maximizes its own rate under a fixed attempt probability P_n. As a result, the strategy played by user n given a fixed strategy profile of other users σ_{-n} is given by $\sigma_n = (k_n^*, P_n)$, where $k_n^* = \{k_{n,i}^*\}_{i=1}^M$ solves the following distributed rate maximization problem[3]:

$$k_n^* = \arg \max_{k_n \in \mathcal{K}_M} \quad R_n(\sigma) \quad \text{s.t.} \quad p_n = P_n . \quad (6)$$

Since $R_n(\sigma) = p_n \sum_{i=1}^M u_n(k_{n,i}) v_n(k_{n,i}, \sigma_{-n})$ and $p_n = P_n$ in (6) is a constant independent of k_n, it suffices to solve:

$$k_n^* = \arg \max_{k_n \in \mathcal{K}_M} \quad \sum_{i=1}^{M} u_n(k_{n,i}) v_n(k_{n,i}, \sigma_{-n}) . \quad (7)$$

For every user n let $\{k_{n,1}^*, k_{n,2}^*, ..., k_{n,K}^*\}$ be a permutation of $\{1, ..., K\}$ such that:

$$\begin{aligned} u_n(k_{n,1}^*) v_n(k_{n,1}^*, \sigma_{-n}) &\geq u_n(k_{n,2}^*) v_n(k_{n,2}^*, \sigma_{-n}) \\ &\geq \cdots \geq u_n(k_{n,K}^*) v_n(k_{n,K}^*, \sigma_{-n}) . \end{aligned} \quad (8)$$

Following (7), the channel-selection strategy that solves (6) at each given updating time is given by:

$$k_n^* = \{k_{n,1}^*, k_{n,2}^*, ..., k_{n,M}^*\} . \quad (9)$$

Note that in practical systems, user n holds an estimate of $u_n(k)$ (from pilot signals for instance). On the other hand, complete information about other user strategies is not required. Monitoring the channels to obtain $v_n(k, \sigma_{-n})$ for all k is sufficient to make a decision[4]. Hence, for purposes of analysis in this section we assume that every user n estimates $v_n(k, \sigma_{-n})$ perfectly (i.e., monitors the channels

[3] For the ease of presentation, we assume continuous random rates $u_n(k)$ to guarantee a uniqueness of the maximizer. Otherwise, channels with the same rate can be ordered arbitrarily.

[4] Note that the number of idle time slots and busy time slots can be used to estimate the success probability. Monitoring the channels can be done by the receiver (which can sense the spectrum and send this information to the transmitter). Another way is to monitor the null period by the transmitter as in cognitive radio systems. Any attempt to access channel k by one user or more results in identifying channel k as busy.

for a sufficient time). Simulation results demonstrate strong performance of the proposed algorithm in practical systems under estimation errors. Next, we examine a distributed algorithm that uses $u_n(k)$, $v_n(k, \sigma_{-n})$ to solve the distributed rate maximization problem.

A. Best-Response Potential Game Formulation

The system dynamics can be viewed as a non-cooperative game, in which every user sequentially updates its strategy to increase its rate given the current system state irrespective of other users' rates, referred to as the *Distributed Rate Maximization (DRM) game*. The strategy k_n^* that solves (6) represents a *best-response (BR)* strategy since a user chooses k_n^* that maximizes its rate given the current system state. On the other hand, switching from strategy k_n to k_n' to increase the rate (but not maximizing it) such that $R_n(k_n', P_n, \sigma_{-n}) > R_n(k_n, P_n, \sigma_{-n})$ is called a *better-response*. A system is in an equilibrium when users cannot increase their rates by unilaterally changing their strategy.

Definition 1: A Nash Equilibrium Point (NEP) for the DRM game is a strategy profile $\sigma^* = (\sigma_n^*, \sigma_{-n}^*)$, where $k_{n'}^* \in \mathcal{K}_M$, $p_{n'}^* = P_{n'}$ for all $n' \in \mathcal{N}$, such that

$$R_n\left(\sigma_n^*, \sigma_{-n}^*\right) \geq R_n(\tilde{\sigma}_n, \sigma_{-n}^*) \quad \forall n\ , \forall \tilde{\sigma}_n = (\tilde{k}_n, P_n)\ ,\ \tilde{k}_n \in \mathcal{K}_M\ . \tag{10}$$

A game has the *finite improvement property (FIP)* if every *improvement path*, in which a sequence of better-responses are executed by users sequentially, is finite. Clearly, a game with FIP converges to a NEP in a finite time under any better-response dynamics. In what follows we use the theory of potential games to analyze the convergence of the BR dynamics to a NEP under the DRM game. In potential games, the incentive of users to switch strategies can be expressed by a global potential function. A NEP for the game is reached at any local maximum of the potential function. Next, we define a class of related potential games to the DRM game at hand.

Definition 2 ([7]): The DRM game is referred to as a *best-response potential game* if there is a best-response potential function $\phi : \sigma \rightarrow \mathbb{R}$ such that for every user n and for every $\sigma_{-n} = \{k_i, p_i\}_{i \neq n}$, where $k_i \in \mathcal{K}_M$, $p_i = P_i$, the following holds:

$$\arg \max_{k_n \in \mathcal{K}_M} R_n(k_n, P_n, \sigma_{-n}) = \arg \max_{k_n \in \mathcal{K}_M} \phi(k_n, P_n, \sigma_{-n})\ . \tag{11}$$

Differing from other classes of potential games (e.g., exact, ordinal) which have the FIP, cycles may occur in BR potential games under some improvement paths. Nevertheless, no cycle occurs when playing BR dynamics since the potential function increases at any BR. In the DRM game, some improvement paths may result in cycles when $M > 1$ (see [34] for examples). Nevertheless, the following theorem shows that the DRM game is a best-response potential game.

Theorem 1: The DRM game is a best-response potential game, with the following best-response potential function:

$$\phi(\sigma) = \sum_{n=1}^{N} \log\left(\frac{1}{1 - P_n}\right) \times \sum_{i=1}^{M} \left(\log u_n(k_{n,i}) - \frac{I_n(k_{n,i}, \sigma_{-n})}{2}\right)\ . \tag{12}$$

The proof is given in the extended version of this paper [34].

TABLE I
BR-DRM ALGORITHM

1)	Initialize: each user (say n) estimates $u_n(k)$ for all k, and selects the M channels with the highest $u_n(k)$
2)	repeat (at each updating time):
3)	select a set of active users $\mathcal{N}_a$
4)	for any active user $n \in \mathcal{N}_a$ do:
5)	estimate $v_n(k, \sigma_{-n})$ for all k
6)	$k_n^* \leftarrow$ solution of (9)
7)	$(k_n, p_n) \leftarrow (k_n^*, P_n)$
8)	end for
10)	until convergence

B. Best-Response Algorithm for Distributed Rate Maximization

Following Theorem 1, we propose a non-cooperative BR algorithm to solve the constrained distributed rate maximization problem in the spatial multi-channel ALOHA networks, dubbed BR for Distributed Rate Maximization (BR-DRM) algorithm. We initialize the algorithm by a simple solution where every user picks the M channels with the highest collision-free utility $u_n(k)$. In the learning process step, each user monitors the load on the channels to obtain $v_n(k, \sigma_{-n})$ for all k (see the beginning of Section III for more details on the monitoring process). Then, at each updating time the selected active users (selected according to the mechanism described in Section II) update their strategies by selecting the channels according to (9). When users cannot increase their rates by unilaterally changing their strategy, an equilibrium is obtained. The BR-DRM Algorithm is given in Table I. Users may repeat updating strategies for a predetermined number of iterations. During the running time of the algorithm the loads on the channels are changed dynamically and affect user decisions across time. Convergence is guaranteed following Theorem 1, since the best response potential function is upper bounded (by $\phi(\sigma) \leq M \sum_{n=1}^{N} \log\left(\frac{1}{1-P_n}\right) \max_k \log\left(u_n(k)\right)$) and any local maxima is a NEP for the game (since no user can increase its rate by unilaterally changing its strategy). It should be noted that convergence in a finite time of BR dynamics in the DRM game is preserved as long as all active users are not neighbors (since the log-interference that user n experiences $I_n(k, \sigma_{-n})$ is affected only by users in $\mathcal{I}_n$) as designed by the mechanism that selects the active users described in Section II (for more details see [34]).

Corollary 1: Assume that users update their strategy according to the mechanism described in Section II. Then, the BR-DRM algorithm, given in Table I, converges to a NEP in finite time.

IV. ACHIEVING GLOBAL PROPORTIONAL FAIRNESS: A COOPERATIVE SETTING

In the previous section we have shown that the distributed rate maximization can be solved without sharing any information between users. In this section, however, we show that much better performance from a system-wide fairness perspective can be expected using information sharing between neighbors only. Specifically, instead of solving a distributed rate maximization as done in the preceding section, here we are interested in developing a distributed algorithm that attains proportionally fair rates in the network. Cooperation in this section refers to a social behavior (by designing a social utility function for each user) that can lead to a globally-optimal operating point. Nevertheless, the model is still cast as a non-cooperative game in the sense that users act with respect to their own social utility.

We consider the case where $M = 1$. Thus, $k_n \in \mathcal{K}$ is a natural number denoting a single channel chosen by user n. Formally, the problem is to find a strategy profile that maximizes the sum-log rate in the network:

$$\sigma^* = \arg \max_{\{k_n \in \mathcal{K}, 0 \leq p_n \leq 1\}_{n=1}^N} \sum_{n=1}^N \log R_n(\sigma) \ . \qquad (13)$$

The above optimization problem (13) was formulated in [29] under a variation of the model considered in this paper for single-channel systems (i.e., $K = 1$) and equal rates for all links. A simple optimal algorithm was developed that uses message exchanges between neighbors only. In this section we address this problem under the multi-channel case.

A. Exact Potential Game Formulation

In Section III we have shown that any NEP of the DRM game is a local maximum of its potential function (12). In this section, however, we are interested in finding a *global maximum* of (13) since it attains a global proportional fairness in the network.

Let $\mathcal{I}_n(k)$ be the set of user n's neighbors that transmit over channel k, and let

$$\begin{aligned} &F_n(k_n, p_n, \sigma_{-n}) \\ &\triangleq \log(u_n(k_n)p_n) - I_n(k_n, \sigma_{-n}) - \log\left(\frac{1}{1-p_n}\right)|\mathcal{I}_n(k_n)| \ , \end{aligned} \qquad (14)$$

be the *cooperative utility* (or fair utility) for user n. Note that the cooperative utility balances between individual and social utilities. The term $\log(u_n(k_n)p_n) - I_n(k_n, \sigma_{-n})$ is the individual utility for user n, where $\log\left(\frac{1}{1-p_n}\right)|\mathcal{I}_n(k)|$ represents the aggregated log-interference that user n causes to its neighbors. Throughout this section it is assumed that user n can compute its cooperative utility when making decisions (see a discussion on a practical implementation in section IV-C). We refer to this game as the *fairness game*.

Next, we show that the fairness game is an exact potential game where $\sum_n \log R_n(\sigma)$ is a potential function of the game.

Definition 3 ([35]): The fairness game is called an *exact* potential game if there is an exact potential function $\phi : \sigma \to \mathbb{R}$ such that for every user n and for every $\sigma_{-n} = \{k_i, p_i\}_{i \neq n}$, where $k_i \in \mathcal{K}$, $0 \leq p_i \leq 1$, the following holds:

$$\begin{aligned} &F_n(\sigma_n^{(2)}, \sigma_{-n}) - F_n(\sigma_n^{(1)}, \sigma_{-n}) \\ &\qquad = \phi(\sigma_n^{(2)}, \sigma_{-n}) - \phi(\sigma_n^{(1)}, \sigma_{-n}) \ , \\ &\forall \sigma_n^{(1)} = (k_n^{(1)}, p_n^{(1)}), \sigma_n^{(2)} = (k_n^{(2)}, p_n^{(2)}) \ , \\ &\qquad k_n^{(1)}, k_n^{(2)} \in \mathcal{K} \ , 0 \leq p_n^{(1)}, p_n^{(2)} \leq 1 \ . \end{aligned} \qquad (15)$$

Theorem 2: The fairness game is an exact potential game, with the following exact potential function:

$$\phi(\sigma) = \sum_{n=1}^N \log R_n(\sigma) \ . \qquad (16)$$

The proof is given in the extended version of this paper [34].

B. Nash Equilibrium of the fairness game

Since the fairness game is an exact potential game with an upper bounded potential function (by $\phi(\sigma) < \sum_{n=1}^N \max_k \log(u_n(k))$), any BR dynamics converges to a NEP in the sense that users cannot increase their cooperative utility by unilaterally changing their strategies. However, any local maximum of the potential function (16) is a NEP of the game. Thus, here we first characterize the NEPs' structure of the fairness game. In Section IV-C we will use this result to develop an algorithm that achieves the best NEP in the sense that the global maximum of (16) is attained.

Definition 4: A Nash Equilibrium Point (NEP) for the fairness game is a strategy profile $\sigma^* = (\sigma_n^*, \sigma_{-n}^*)$, where $k_i^* \in \mathcal{K}$, $0 \leq p_i^* \leq 1$ for all $i \in \mathcal{N}$, such that

$$\begin{aligned} &F_n(\sigma_n^*, \sigma_{-n}^*) \geq F_n(\tilde{\sigma}_n, \sigma_{-n}^*) \\ &\qquad \forall n \ , \forall \tilde{\sigma}_n = (\tilde{k}_n, \tilde{p}_n) \ , \ \tilde{k}_n \in \mathcal{K} \ , \ 0 \leq \tilde{p}_n \leq 1 \ . \end{aligned} \qquad (17)$$

Theorem 3: A strategy profile $\sigma^* = \{k_n^*, p_n^*\}_{n=1}^N$ is a NEP for the fairness game if $k_n^* \in \mathcal{K}$, $p_n^* = \frac{1}{|\mathcal{I}_n(k_n^*)| + 1}$ for all $n \in \mathcal{N}$.

The proof is given in the extended version of this paper [34].

Corollary 2: A local maximum of (16) is attained only if every user n is associated with an attempt probability $p_n = \frac{1}{|\mathcal{I}_n(k_n)| + 1}$. In particular, the strategy profile that attains proportionally fair rates (i.e., the solution to (13)) must satisfy $p_n = \frac{1}{|\mathcal{I}_n(k_n)| + 1}$ for all n.

Theorem 4: Let $\{k_n^*\}_{n=1}^N$ be a given channel allocation for all users. A strategy profile

$$\sigma^* = \left\{k_n^*, p_n^* = \frac{1}{|\mathcal{I}_n(k_n^*)| + 1}\right\}_{n=1}^N \qquad (18)$$

is the unique solution to the following optimization problem:

$$\begin{aligned} \{p_n^*\}_{n=1}^N = \arg \max_{\{0 \leq p_n \leq 1\}_{n=1}^N} \sum_{n \in \mathcal{N} : k_n^* = k} \log R_n\left(\{k_n^*, p_n\}_{n=1}^N\right) \\ \forall k \in \mathcal{K} \ . \end{aligned} \qquad (19)$$

The proof is given in the extended version of this paper [34].

Combining Theorems 3 and 4 yields:

Corollary 3: A strategy profile $\sigma^* = \{k_n^*, p_n^*\}_{n=1}^N$ is a NEP for the fairness game if $\{p_n^*\}_{n=1}^N$ solves (19).

Corollary 2 follows directly from the NEPs' structure characterized in Theorem 3. We will use the fact that attaining the global maximum of (16) implies $p_n = \frac{1}{|\mathcal{I}_n(k_n)| + 1}$ for all n to design a distributed learning algorithm that converges to the solution of (13). Corollary 3 sheds a light on the operating points of the system. Learning algorithms used to converge to a global optimum may spend some time at local maxima of the objective function (i.e., a NEP). Corollary 3 shows that the local maxima of the potential function may not be so bad. Specifically, every NEP of the fairness game can be viewed as a local proportional fairness in the sense that proportionally fair rates are attained among all users that share channel k for all $k \in \mathcal{K}$.

C. Distributed Cooperative Learning Algorithm

The optimization problem in (13) is a combinatorial optimization problem over a graph which is mathematically intractable. Furthermore, it requires a centralized solution that uses global information which is impractical in large-scale networks. Therefore, we propose a probabilistic approach to solve the problem in a distributed manner. We develop a distributed cooperative learning algorithm, dubbed Noisy BR for Fairness (NBRF) algorithm, with the goal of solving (13) using limited message exchanges between neighbors only. NBRF is a cooperative algorithm in the sense that users make decisions with respect to the cooperative utility that balances between their own utilities and the interference level they cause to their neighbors.

Recall that BR dynamics may lead to local maxima of the potential function. Hence, instead of playing purely BR, in NBRF users play noisy BR (also known as spatial adaptive play) when updating their strategies [36], [37]. In NBRF, active users construct a probability mass function (pmf) over their actions and draw their actions according to this distribution. Typically, the BR is played with high probability, while other strategies are played with a probability that decays exponentially fast with the myopic utility loss in order to escape local maxima. Specifically, the pmf over the available actions is given by:

$$\Pr((k_n, p_n) = (k, p)) = \frac{e^{\beta F_n(k,p,k_{-n},p_{-n})}}{\sum_{k'=1}^{K} \sum_{r=1}^{|\mathcal{I}_n|+1} e^{\beta F_n(k',1/r,k_{-n},p_{-n})}} \tag{20}$$

for some exploration parameter $\beta > 0$. Note that when $\beta = 0$ the pmf assigns equal weights on all strategies, while the probability of playing BR[5] approaches one as $\beta \to \infty$ (a discussion on the setting of β based on simulated annealing analysis [38] is provided in the end of this section). The NBRF Algorithm is given in Table II. In NBRF, active users must send complete information about their updated strategies to their neighbors (Step 8) such that all users can compute their cooperative utility at each given updating time. A similar mechanism as described in Section II can be applied, where the pilot signal is now replaced by a packet containing complete information about the updated strategy. Users may repeat updating strategies for a predetermined number of iterations and then stick their BR (see a discussion in the end of this section)

The following theorem shows that NBRF attains proportional fairness with an arbitrarily high probability as time increases.

[5]For the ease of presentation, we assume continuous random rates $u_n(k)$ to guarantee a uniqueness of the maximizer. Otherwise BRs are drawn uniformly.

TABLE II
NBRF ALGORITHM

1) Initialize:
based on message exchanges between neighbors each user (say n) set $k_n \leftarrow \arg\max_k \{u_n(k)\}$ and $p_n \leftarrow 1/\left(|\mathcal{I}_n(k_n)| + 1\right)$.

2) repeat (at each updating time):
3) select a set of active users $\mathcal{N}_a$
4) for any active user $n \in \mathcal{N}_a$ do:
5) compute $F_n(k, p, k_{-n}, p_{-n})$ for all $k = 1, ..., K$, $p = 1, 1/2, ..., 1/(|\mathcal{I}_n| + 1)$.
6) construct pmf given in (20) for all $k = 1, ..., K$, $p = 1, 1/2, ..., 1/(|\mathcal{I}_n| + 1)$.
7) draw (k_n, p_n) randomly according to the distribution in Step 6.
8) send a packet containing (k_n, p_n) to inform all neighbors $\mathcal{I}_n$
9) end for
10) until convergence

Theorem 5: Let $\sigma^{NBRF(\beta)}(t), \sigma^*$ be the strategy profile under NBRF (with a parameter β) at time t and the strategy profile that solves (13), respectively. For any $\epsilon > 0$ there exists $\beta > 0$ such that

$$\lim_{t\to\infty} \Pr\left(\sigma^{NBRF(\beta)}(t) = \sigma^*\right) \geq 1 - \epsilon \,. \tag{21}$$

Proof: The proof is based on the results reported in Section IV-B and the fact that a noisy best response dynamics following (20) in exact potential games converges to a stationary distribution of the Markov chain corresponding to the game [36]. By Theorem 2, the fairness game with the cooperative utility F_n is an exact potential game with an exact potential function ϕ given in (16). Since NBRF plays noisy BR with respect to F_n, the stationary distribution of the strategy profile is given by [36]:

$$Pr(\sigma^{NBRF(\beta)} = \sigma) = \frac{e^{\beta\phi(\sigma)}}{\sum_{\tilde{\sigma}} e^{\beta\phi(\tilde{\sigma})}} \,. \tag{22}$$

Next, note that the number of user n's neighbors that transmit over channel k_n, $|\mathcal{I}_n(k_n)|$, is lower bounded by $|\mathcal{I}_n|$ for all n. Therefore, following Corollary 2, the strategy profile σ^* that attains the global maximum of (16) lies inside the action space played by NBRF. Therefore, for every $\epsilon > 0$ we can choose $\beta > 0$ sufficiently large such that the stationary distribution puts a sufficiently high weight on the strategy profile that maximizes (16) (i.e., ϕ in (22)). Thus, (21) is satisfied as time approaches infinity. ■

Following the proof of Theorem 5 we infer that as the probability of playing BR increases (i.e., $\beta \to \infty$) the probability of attaining the global maximum of the potential function (16) increases with time. However, increasing β too fast may push the algorithm into a local maximum for a long time (since the probability of not playing BR is too small). Following simulated annealing analysis [38], a rule of thumb used to achieve fast convergence is to increase β as $\sim \log(t+1), t = 0, 1, \ldots$. As a result, users explore strategy profiles in the beginning of the running time and will stick their BR as time approaches infinity. In cases where the optimal operating point is not unique, the algorithm may converge to one of the optimal operating points. Simulation results demonstrate fast convergence to the optimal channel allocation and attempt probabilities under typical scenarios.

V. Numerical Examples

In this section we provide numerical examples to illustrate the performance of the algorithms. We simulated the following network: N users was randomly dispersed (uniformly) in a circle region with a radius of 10 meters. Each user causes interference to all users in a radius of 5 meters. Every user can choose one channel for transmission among K channels. We assume equal achievable rates $u_n(k) = 100$Mbps for all users on all channels when channels are free (i.e., collision-free utility). We focus on the cooperative setting, where the goal is to find a channel allocation and attempt probabilities in a distributed manner so as to attain proportionally fair rates among users. We compare the NBRF algorithm, given in Table II, with the random channel allocation scheme, where the optimal attempt probabilities were set under any random channel allocation (i.e., $p_n = 1/(|I_n(k_n)|+1)$ for all n). In the NBRF algorithm, we set $\beta = \log t$ (where $t = 1, 2, \ldots$ indicates the iteration number) to construct the pmf in Step 6. We first examine a small connected network with $N = 10$ users sharing $K = 2$ channels, so as the centralized optimal exhaustive search solution can be computed and serve as a benchmark for comparison. An illustration of the small network is depicted in Fig. 1. In Fig. 2 we present the average log rate to demonstrate the performance in terms of proportional fairness and also the average rate to demonstrate the achievable effective rates. It can be seen that NBRF significantly improves performance as compared to a random channel allocation (even though the attempt probabilities are optimal given any random channel allocation) in terms of both fairness and efficiency. It can be seen that NBRF approaches the optimal centralized solution as time increases. This result demonstrates the efficiency of the proposed distributed learning algorithm in achieving the global proportional fairness in the network. Next, we considered a larger network with $N = 50$ users sharing $K = 5$ channels (in this case computing the optimal solution is intractable). It can be seen in Fig. 3 that NBRF significantly outperforms the random channel allocation again. Only $100 - 200$ iterations (i.e., a total number of $100 - 200$ packet broadcasting due to Step 8 in Table II, which typically takes less than a second for WiFi packets for instance) are required to achieve more than $60\% - 70\%$ performance gain over a random allocation in terms of average rate.

VI. Conclusion

The distributed optimization problem over multiple collision channels shared by spatially distributed users was considered. We examined both the non-cooperative and cooperative settings. Under the non-cooperative setting, we developed a

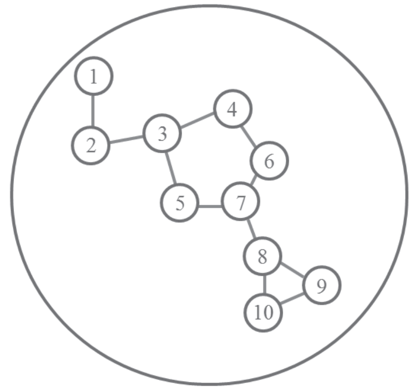

Fig. 1. An illustration of a small connected network with 10 users spatially distributed in a circle area of radius 10 meters. The users share 2 channels. Each pair of users with distance less than 2 meters (represented by an edge) cause mutual interference when transmitting simultaneously over the same channel.

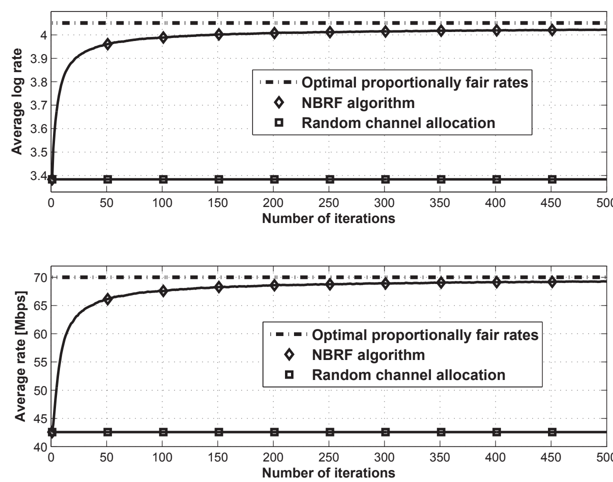

Fig. 2. Average sum-log rate and average rate as a function of the number of iterations. A wireless network containing 10 users and 2 channels.

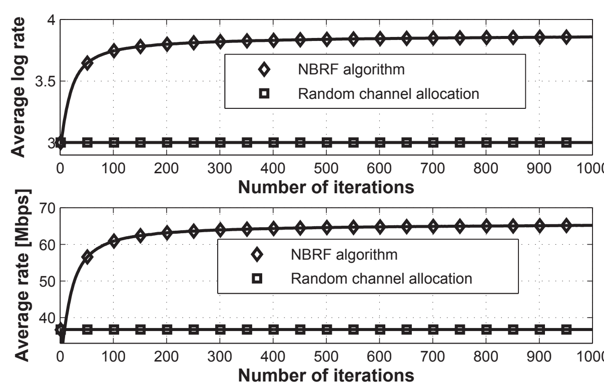

Fig. 3. Average sum-log rate and average rate as a function of the number of iterations. A wireless network containing 50 users and 5 channels.

distributed learning algorithm for the distributed rate maximization problem, in which each user maximizes its own rate irrespective of other user utilities. Convergence was proved using the theory of best-response potential games. Under the cooperative setting, we developed a distributed cooperative learning algorithm to achieve the global proportional fairness in the networks. While direct computation of the optimal solution is impractical in large-scale networks, we showed that the proposed distributed algorithm converges to the global optimum with high probability as time increases. Simulation results demonstrated strong performance of the algorithms.

Future research directions are convergence time analysis of the NBRF algorithm, and analyzing the performance of NBRF under malicious/malfunctioning nodes.

References

[1] W. Yu, G. Ginis, and J. M. Cioffi, "Distributed multiuser power control for digital subscriber lines," *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 5, pp. 1105–1115, 2002.
[2] A. Leshem, E. Zehavi, and Y. Yaffe, "Multichannel opportunistic carrier sensing for stable channel access control in cognitive radio systems," *IEEE Journal on Selected Areas in Communications*, vol. 30, pp. 82–95, Jan. 2012.
[3] K. Cohen, A. Leshem, and E. Zehavi, "Game theoretic aspects of the multi-channel ALOHA protocol in cognitive radio networks," *IEEE Journal on Selected Areas in Comm.*, vol. 31, no. 11, pp. 2276 – 2288, 2013.
[4] I. Menache and N. Shimkin, "Rate-based equilibria in collision channels with fading," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 7, pp. 1070–1077, 2008.
[5] Y. Jin and G. Kesidis, "Equilibria of a noncooperative game for heterogeneous users of an ALOHA network," *IEEE Communications Letters*, vol. 6, no. 7, pp. 282–284, 2002.
[6] J. Papandriopoulos and J. S. Evans, "Low-complexity distributed algorithms for spectrum balancing in multi-user DSL networks," in *IEEE International Conference on Communications (ICC)*, vol. 7, pp. 3270–3275, 2006.
[7] M. Voorneveld, "Best-response potential games," *Economics letters*, vol. 66, no. 3, pp. 289–295, 2000.
[8] A. MacKenzie and S. Wicker, "Selfish users in ALOHA: a game-theoretic approach," *IEEE Vehicular Technology Conference*, vol. 3, pp. 1354–1357, 2001.
[9] T. Alpcan, T. Başar, R. Srikant, and E. Altman, "CDMA uplink power control as a noncooperative game," *Wireless Networks*, vol. 8, no. 6, pp. 659–670, 2002.
[10] Z. Han, Z. Ji, and K. R. Liu, "Fair multiuser channel allocation for OFDMA networks using Nash bargaining solutions and coalitions," *IEEE Transactions on Communications*, vol. 53, no. 8, pp. 1366–1376, 2005.
[11] E. Altman, T. Boulogne, R. El-Azouzi, T. Jiménez, and L. Wynter, "A survey on networking games in telecommunications," *Computers & Operations Research*, vol. 33, no. 2, pp. 286–311, 2006.
[12] A. Leshem and E. Zehavi, "Bargaining over the interference channel," in *IEEE International Symposium on Information Theory*, pp. 2225–2229, 2006.
[13] A. Ozdaglar and R. Srikant, "Incentives and pricing in communication networks," *Algorithmic Game Theory*, pp. 571–591, 2007.
[14] H. Boche, M. Schubert, N. Vucic, and S. Naik, "Non-symmetric Nash bargaining solution for resource allocation in wireless networks and connection to interference calculus," in *Proc. European Signal Processing Conference*, 2007.
[15] M. Nokleby, A. L. Swindlehurst, Y. Rong, and Y. Hua, "Cooperative power scheduling for wireless MIMO networks," in *IEEE Global Telecommunications Conference*, pp. 2982–2986, 2007.
[16] H. Inaltekin and S. B. Wicker, "The analysis of Nash equilibria of the one-shot random-access game for wireless networks and the behavior of selfish nodes," *IEEE/ACM Transactions on Networking*, vol. 16, no. 5, pp. 1094–1107, 2008.
[17] E. A. Jorswieck and E. G. Larsson, "The MISO interference channel from a game-theoretic perspective: A combination of selfishness and altruism achieves pareto optimality," in *IEEE International Conference on Acoustics, Speech and Signal Processing*, pp. 5364–5367, 2008.
[18] J. Gao, S. A. Vorobyov, and H. Jiang, "Game theoretic solutions for precoding strategies over the interference channel," in *IEEE Global Telecommunications Conference*, pp. 1–5, 2008.
[19] A. Leshem and E. Zehavi, "Cooperative game theory and the Gaussian interference channel," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 7, pp. 1078–1088, 2008.
[20] U. Candogan, I. Menache, A. Ozdaglar, and P. Parrilo, "Competitive scheduling in wireless collision channels with correlated channel state," *International Conference on Game Theory for Networks (GameNets)*, pp. 621–630, 2009.
[21] E. Altman, K. Avrachenkov, L. Cottatellucci, M. Debbah, G. He, and A. Suarez, "Operating point selection in multiple access rate regions," in *21st International Teletraffic Congress (ITC)*, pp. 1–8, IEEE, 2009.
[22] I. Menache and A. Ozdaglar, "Network games: Theory, models, and dynamics," *Synthesis Lectures on Communication Networks*, vol. 4, no. 1, pp. 1–159, 2011.
[23] K. Cohen and A. Leshem, "Distributed game theoretic optimization and management of multi-channel ALOHA networks," *submitted to IEEE/ACM Transactions on Networking, available at arXiv http://arxiv.org/abs/1312.7513*.
[24] I. Pountourakis and E. Sykas, "Analysis, stability and optimization of ALOHA-type protocols for multichannel networks," *Computer Communications*, vol. 15, no. 10, pp. 619–629, 1992.
[25] D. Shen and V. Li, "Stabilized multi-channel ALOHA for wireless OFDM networks," *The IEEE Global Telecommunications Conference (GLOBECOM)*, vol. 1, pp. 701–705, 2002.
[26] E. Altman, R. El Azouzi, and T. Jiménez, "Slotted aloha as a game with partial information," *Computer Networks*, vol. 45, no. 6, pp. 701–713, 2004.
[27] K. Bai and J. Zhang, "Opportunistic multichannel Aloha: distributed multiaccess control scheme for OFDMA wireless networks," *IEEE Transactions on Vehicular Technology*, vol. 55, no. 3, pp. 848–855, 2006.
[28] T. To and J. Choi, "On exploiting idle channels in opportunistic multichannel ALOHA," *IEEE Communications Letters*, vol. 14, no. 1, pp. 51–53, 2010.
[29] K. Kar, S. Sarkar, and L. Tassiulas, "Achieving proportional fairness using local information in aloha networks," *IEEE Transactions on Automatic Control*, vol. 49, no. 10, pp. 1858–1863, 2004.
[30] X. Wang and K. Kar, "Cross-layer rate optimization for proportional fairness in multihop wireless networks with random access," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 8, pp. 1548–1559, 2006.
[31] F. Baccelli, B. Blaszczyszyn, and P. Muhlethaler, "An Aloha protocol for multihop mobile wireless networks," *IEEE Transactions on Information Theory*, vol. 52, no. 2, pp. 421–436, 2006.
[32] F. Baccelli, B. Blaszczyszyn, and P. Muhlethaler, "Stochastic analysis of spatial and opportunistic Aloha," *IEEE Journal on Selected Areas in Communications*, vol. 27, no. 7, pp. 1105–1119, 2009.
[33] P. Gupta and A. L. Stolyar, "Throughput region of random-access networks of general topology," *IEEE Transactions on Information Theory*, vol. 58, no. 5, pp. 3016–3022, 2012.
[34] K. Cohen, A. Nedić, and R. Srikant, "Distributed learning algorithms for spectrum sharing in spatial random access networks," *Technical Report, available at http://www.ifp.illinois.edu/~yscohen/*.
[35] D. Monderer and L. Shapley, "Potential games," *Games and economic behavior*, vol. 14, pp. 124–143, 1996.
[36] L. E. Blume, "The statistical mechanics of strategic interaction," *Games and economic behavior*, vol. 5, no. 3, pp. 387–424, 1993.
[37] H. P. Young, "Individual strategy and social structure," *Princeton University Press*, 1998.
[38] B. Hajek, "Cooling schedules for optimal annealing," *Mathematics of operations research*, vol. 13, no. 2, pp. 311–329, 1988.