Convex Relaxation Algorithms for Energy-Infeasibility Tradeoff in Cognitive Radio Networks

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***Abstract*—In cognitive radio networks, uncontrolled access of secondary users degrades the performance of primary users and can even lead to system infeasibility, as the secondary users are allowed to transmit simultaneously on a shared spectrum. We study the feasibility of the total energy consumption minimization problem subjecting to power budget and Signal-to-Interference-plus-Noise Ratio (SINR) constraints. Finding the largest set of secondary users (i.e., the system capacity) that can be supported in the system is hard to solve due to the nonconvexity of the cardinality objective. We formulate this problem as a vector-cardinality optimization problem, and propose a convex relaxation that replaces the objective with a continuous and convex function. Motivated by the sum-of-infeasibilities heuristic, a joint power and admission control algorithm is proposed to compute the maximum number of secondary users that can be supported. Numerical results are presented to show that our algorithm is theoretically sound and practically implementable.**

***Index Terms*—Power control, admission control, optimization, feasibility, cognitive radio networks.**

I. Introduction

Cognitive radio networks are envisioned to provide high bandwidth to mobile users via dynamic spectrum access techniques and heterogeneous wireless architectures [1], [2]. Energy efficiency in cognitive radio networks is a growing focus as energy consumption and electromagnetic radiation increasingly become a global environmental concern [3]–[5]. In wireless networks, power control has traditionally been used to satisfy the Signal-to-Interference-plus-Noise Ratio (SINR) requirements of users and to minimize the total energy consumption [6]. Sorooshyari et al. developed an axiomatic framework for power control in cognitive radio networks in [7]. Since the seminal work by Foschini and Miljanic in [8] in designing power control algorithms for energy minimization subject to SINR constraints, it has been extended to consider power constraints, e.g., the constrained Distributed Power Control (DPC) algorithm in [9], when there is an individual power constraint for each user.

A key issue in this energy minimization problem is the infeasibility problem, i.e., when it is not possible to simultaneously meet the SINR constraints of all the secondary users. When there is infeasibility, existing power control algorithms, e.g., in [8], may not converge or may be unstable, e.g., using the DPC algorithm in [9], users may transmit at the maximum possible power and yet still cannot satisfy their SINR constraints that lead to undue interference. In [10], the authors proposed an energy-robustness tradeoff optimization to balance energy expenditure and robustness against outage. This problem is more severe in a cognitive radio network if the overwhelming interference to the primary users caused by the secondary users is uncontrolled. Thus, admission control is necessary to resolve the infeasibility issue in the energy minimization problem [11].

In cognitive radio networks, secondary users monitor the surrounding radio environment, dynamically adapt their transmission parameters, and opportunistically utilize the temporarily free spectrum resource licensed to primary users [12]. To simultaneously maximize the number of secondary users that can be supported and minimize the total energy consumption is generally hard to solve and in fact NP-hard [13]. Mathematically, it is equivalent to computing the maximum feasible set given an infeasible set of linear constraints [14]. In the power control literature, there has been extensive work on admission control to find the system capacity, i.e., the maximum feasible set. In [15], Ren et al. studied the impact of power allocation of secondary users with admission control. In [16], Mahdavi-Doost et al. developed a centralized gradual removal algorithm that removes users to increase the maximum achievable SINR in the system. In [17], Rasti et al. proposed a distributed temporarily removal algorithm, in which users stop transmission once their instantaneous power exceed certain threshold. Matskani et al. [13] and Mitliagkas et al. [18] proposed removing users based on convex relaxation to obtain an approximate solution to the system capacity.

Besides, Kang et al. in [19] proposed an optimal power allocation strategy for the secondary user under the primary user outage probability constraint. Huang et al. in [20] designed a distributed power control algorithm to maximize the throughput of secondary users and protect the primary user's quality-of-service. Anandkumar et al. in [21] proposed policies to obtain the optimal throughput transmission for secondary users. Phunchongharn et al. considered the channel gain uncertainty to design power control algorithms to tackle the system capacity violation in [22]. Halldorsson et al. gave

*The work in this paper was partially supported by grants from the Research Grants Council of Hong Kong Project No. RGC CityU 125212, Qualcomm Inc. and the Science, Technology and Innovation Commission of Shenzhen Municipality, Project No. JCYJ20120829161727318 on Green Communications in Small-cell Mobile Networks.

algorithms based on a novel linear programming formulation for capacity problems, with constant-factor performance guarantees for several capacity and throughput problems in [23]. Parsaeefard et al. proposed a robust distributed uplink power allocation algorithm for underlay cognitive radio networks to maximize the social utility of secondary users in [24].

The system capacity is intriguingly related to the amount of energy consumption in the network. Aggressive admission control unduly removes secondary users that leads to the network being under-utilized, albeit with a lower total energy consumption. On the other hand, a maximum system efficiency perspective requires supporting as many secondary users as possible albeit with a higher total energy consumption. This energy-infeasibility tradeoff entails an optimization of the system operating point to balance the system capacity and the energy consumption.

In contrast to the commonly used two-timescale approach (finding a maximum secondary user set first before minimizing the total energy consumption of all users in the set) in the literature, we propose a single timescale approach to jointly optimize this energy-infeasibility tradeoff that yields power control algorithm with low complexity. In particular, using convex relaxation and Lagrange duality, we propose an algorithm based on the sum-of-infeasibilities in optimization theory [25], which is also partially motivated by compressed sensing [14], to compute a (suboptimal) set of users that can be supported subject to a system constraint on the total energy consumption that can be tolerated.

Overall, the contributions in this paper are:

1) the formulation and relaxation of the feasibility problem as a vector-cardinality minimization problem,
2) a joint power and admission control algorithm in the form of a fixed-point algorithm that exhibits desirable convergence behavior.

The paper is organized as follows: We introduce the system model in Section II. We first study a vector-cardinality formulation and its relaxation, and then propose a joint power and admission control algorithm based on the sum-of-infeasibilities in Section III. We evaluate the performance of our algorithm numerically and compare them to other baseline algorithms in Section IV. Finally, we conclude the paper in Section V.

The following notations are used in this paper: Boldface uppercase letters denote matrices, boldface lowercase letters denote column vectors and italics denote scalars. $\rho(\mathbf{A})$ denotes the Perron-Frobenius eigenvalue of a nonnegative matrix $\mathbf{A}$. The super-script $(\cdot)^\top$ denotes the transpose. $\|\cdot\|_0$ and $\|\cdot\|_1$ denote the ℓ_0 and ℓ_1 norm, respectively. $\mathbf{I}$ denotes the identity matrix. $e^{\mathbf{x}}$ and $\log \mathbf{x}$ denote $(e^{x_1}, \ldots, e^{x_n})^\top$ and $(\log x_1, \ldots, \log x_n)^\top$, respectively.

II. System model

In this section, we consider a cognitive radio network with a collection of primary users and secondary users. There are L_m primary users and L_s secondary users (transmitter-receiver pairs), communicating simultaneously over a common frequency-flat fading channel. The received SINR of the ith primary user and the jth secondary user in the transmission can be given in terms of the transmit power $\mathbf{p} = [\mathbf{p}^m; \mathbf{p}^s]$ as:

$$\mathsf{SINR}_i^m(\mathbf{p}) = \frac{G_{ii}^{mm} p_i^m}{\sum_{\substack{l=1 \\ l\neq i}}^{L_m} G_{il}^{mm} p_l^m + \sum_{j=1}^{L_s} G_{ij}^{ms} p_j^s + n_i^m}, \qquad (1)$$

and:

$$\mathsf{SINR}_j^s(\mathbf{p}) = \frac{G_{jj}^{ss} p_j^s}{\sum_{i=1}^{L_m} G_{ji}^{sm} p_i^m + \sum_{\substack{l=1 \\ l\neq j}}^{L_s} G_{jl}^{ss} p_l^s + n_j^s}, \qquad (2)$$

respectively, where the super-script m represents the primary user, the super-script s represents the secondary user, G_{ij}^{ms} is the channel gain from the jth secondary transmitter to the ith primary receiver, and n_i is the additive white Gaussian noise (AWGN) at the ith user.

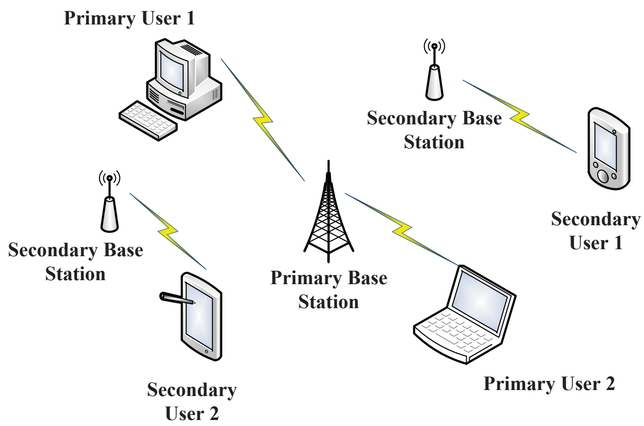

Fig. 1. Illustration of a cognitive radio network.

The optimization problem that minimizes the total energy consumption of both the primary and secondary users subject to power budget and SINR constraints is given by [6], [8]:

$$\begin{array}{ll} \text{minimize} & \sum_{i=1}^{L_m} p_i^m + \sum_{j=1}^{L_s} p_j^s \\ \text{subject to} & \mathsf{SINR}_i^m(\mathbf{p}) \geq \bar{\gamma}_i^m, \quad i = 1, \ldots, L_m, \\ & \mathsf{SINR}_j^s(\mathbf{p}) \geq \bar{\gamma}_j^s, \quad j = 1, \ldots, L_s, \\ & \mathbf{0} \leq \mathbf{p}^m \leq \bar{\mathbf{p}}^m, \\ & \mathbf{0} \leq \mathbf{p}^s \leq \bar{\mathbf{p}}^s, \\ \text{variables :} & \mathbf{p}^m, \mathbf{p}^s, \end{array} \qquad (3)$$

where $\bar{\mathbf{p}} = [\bar{\mathbf{p}}^m; \bar{\mathbf{p}}^s]$ is the upper bound of transmit power for all users and $\bar{\boldsymbol{\gamma}} = [\bar{\boldsymbol{\gamma}}^m; \bar{\boldsymbol{\gamma}}^s]$ is a given minimum SINR threshold vector, representing the quality-of-service requirement in the cognitive radio network.

To give a more compact representation, let us define the nonnegative vector:

$$\mathbf{v} = [\mathbf{v}^m; \mathbf{v}^s] = \left(\frac{n_1^m}{G_{11}^{mm}}, \cdots, \frac{n_{L_m}^m}{G_{L_m L_m}^{mm}}, \frac{n_1^s}{G_{11}^{ss}}, \cdots, \frac{n_{L_s}^s}{G_{L_s L_s}^{ss}} \right)^\top, \tag{4}$$

and the nonnegative matrix $\mathbf{F}$:

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}^{mm} & \mathbf{F}^{ms} \\ \mathbf{F}^{sm} & \mathbf{F}^{ss} \end{bmatrix}, \tag{5}$$

where $\mathbf{F}^{mm} \in \mathbb{R}^{L_m \times L_m}, \mathbf{F}^{ms} \in \mathbb{R}^{L_m \times L_s}, \mathbf{F}^{sm} \in \mathbb{R}^{L_s \times L_m}$ and $\mathbf{F}^{ss} \in \mathbb{R}^{L_s \times L_s}$, with entries:

$$F_{ij}^{ms} = \frac{G_{ij}^{ms}}{G_{ii}^{mm}}, \quad F_{ji}^{sm} = \frac{G_{ji}^{sm}}{G_{jj}^{ss}},$$

$$F_{li}^{mm} = \begin{cases} 0, & l = i, \\ \frac{G_{li}^{mm}}{G_{ll}^{mm}}, & l \neq i, \end{cases} \tag{6}$$

and:

$$F_{lj}^{ss} = \begin{cases} 0, & l = j, \\ \frac{G_{lj}^{ss}}{G_{ll}^{ss}}, & l \neq j. \end{cases} \tag{7}$$

Moreover, we assume that $\mathbf{F}$ is irreducible, i.e., each user has at least an interferer. Then, we can rewrite (3) as a linear program in matrix form [6]:

$$\begin{array}{ll} \text{minimize} & \mathbf{1}^\top \mathbf{p} \\ \text{subject to} & (\mathbf{I} - \mathrm{diag}(\bar{\boldsymbol{\gamma}})\mathbf{F})\mathbf{p} \geq \mathrm{diag}(\bar{\boldsymbol{\gamma}})\mathbf{v}, \\ & \mathbf{0} \leq \mathbf{p} \leq \bar{\mathbf{p}}, \\ \text{variables:} & \mathbf{p}. \end{array} \tag{8}$$

In general, (8) may or may not be feasible. It is well-known that a necessary (but not sufficient) condition for the feasibility of (8) is $\rho(\mathrm{diag}(\bar{\boldsymbol{\gamma}})\mathbf{F}) < 1$ [6], [8], [9], [26].

Suppose (8) is feasible. Then to solve (8), the following DPC algorithm has been proposed in [9]:

$$p_l(t+1) = \min \left\{ \frac{\bar{\gamma}_l p_l(t)}{\mathsf{SINR}_l(\mathbf{p}(t))}, \bar{p}_l \right\}, \quad l = 1, \ldots, L_m + L_s, \tag{9}$$

where $\mathsf{SINR}(\mathbf{p}) = [\mathsf{SINR}^m(\mathbf{p}); \mathsf{SINR}^s(\mathbf{p})]$. This algorithm converges to the optimal solution of (3) whenever (3) is feasible. Intuitively, the lth user increases its power if its $\mathsf{SINR}_l(\mathbf{p})$ is below $\bar{\gamma}_l$, and otherwise decreases it. However, when (3) is infeasible, (9) is a greedy algorithm that converges to a point, where some but not all of the users can satisfy their SINR thresholds. From a system efficiency perspective viewpoint, it is necessary to find the system capacity, i.e., the maximum number of users that can be supported. In a cognitive radio network, the overwhelming interference from the unlicensed secondary users can adversely affect the performance of the overall network. Thus, it is more interesting to study the impact of secondary users on the primary users. Therefore, we make the assumption that the system having only primary users is already feasible without any secondary user.

In the following, we first construct a feasible optimization problem for (3), i.e., a vector-cardinality minimization problem, whose convex relaxation is the sum-of-infeasibilities heuristic in optimization theory [25]. By exploiting the optimality conditions, we propose a fixed-point algorithm to solve this sum-of-infeasibilities problem in Section III. Figure 2 gives an overview of the key optimization problems solved in this paper.

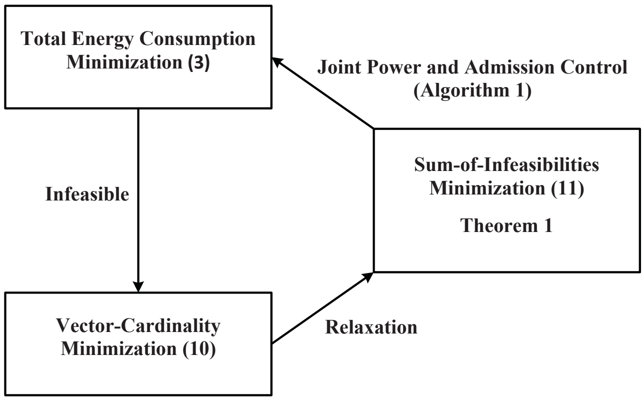

Fig. 2. Overview of the connection between the key optimization problems.

III. Relaxation by Sum-of-Infeasibilities

In this section, we first introduce a vector-cardinality problem and then adopt a convex relaxation technique by replacing the vector-cardinality with a sum-of-infeasibilities. In particular, we exploit the optimality conditions of this convex relaxation to design an iterative fixed-point algorithm.

A. Energy-Infeasibility Optimization Problem

Finding the largest set of users whose SINR thresholds can all be satisfied is a NP-hard combinatorial problem [17]. When the number of secondary users is large, it is not practical to examine all the combinations of the secondary users to select a feasible set with the maximum cardinality. In the following, we formulate another optimization problem related to (3) by adding auxiliary variables q_i^m and q_j^s to the right side of the SINR constraints for the ith primary user and the jth secondary user respectively. Letting $\mathbf{q} = [\mathbf{q}^m; \mathbf{q}^s]$, we consider the following (nonconvex) optimization problem:

$$\begin{array}{ll} \text{minimize} & \|\mathbf{q}\|_0 \\ \text{subject to} & \frac{\bar{\gamma}_i^m}{\mathsf{SINR}_i^m(\mathbf{p})} \leq 1 + q_i^m, \quad i = 1, \ldots, L_m, \\ & \frac{\bar{\gamma}_j^s}{\mathsf{SINR}_j^s(\mathbf{p})} \leq 1 + q_j^s, \quad j = 1, \ldots, L_s, \\ & \mathbf{0} \leq \mathbf{p} \leq \bar{\mathbf{p}}, \\ \text{variables:} & \mathbf{p}, \mathbf{q}, \end{array} \tag{10}$$

where $\mathbf{q}$ can be interpreted as the effect of SINR margins added to all SINR thresholds (that cushion and keep the SINR from falling below the SINR thresholds when there is fluctuation in the system operating point). For brevity, we also call $\mathbf{q}$ the SINR margin variable.

Note that (3) is feasible if and only if the optimal value of (10) is zero. Note that (10) is always feasible. We have $q_l^\star > 0$ if the SINR threshold of the lth user cannot be achieved. Intuitively, a feasible set of users for (3) can be obtained by removing all the users satisfying $q_l^\star > 0$ at the optimality of (10). However, (10) is still a computationally hard problem due to the nonsmooth and nonconvex objective function. Therefore, we consider the following optimization problem by replacing the objective function of (10) with the sum of $\mathbf{q}$, i.e., the sum-of-infeasibilities heuristic (cf. Chapter 11.4 in [25]), given by:

$$\begin{array}{ll} \text{minimize} & \mathbf{1}^\top \mathbf{q} \\ \text{subject to} & \frac{\bar{\gamma}_i^m}{\mathsf{SINR}_i^m(\mathbf{p})} \leq 1 + q_i^m, \quad i = 1, \ldots, L_m, \\ & \frac{\bar{\gamma}_j^s}{\mathsf{SINR}_j^s(\mathbf{p})} \leq 1 + q_j^s, \quad j = 1, \ldots, L_s, \\ & \mathbf{0} \leq \mathbf{p} \leq \bar{\mathbf{p}}, \\ & \mathbf{q} \geq \mathbf{0}, \\ \text{variables :} & \mathbf{p}, \mathbf{q}. \end{array} \tag{11}$$

Let the optimal $\mathbf{p}$ in (11) be denoted by $\mathbf{p}^\star$.

Lemma 1: The optimal value of (11) is smaller than the optimal value of (10).

Remark 1: For the nonnegative SINR margin variable, we have $1 - q_l \leq 1/(1 + q_l)$. Then, the objective function of (11) satisfies:

$$\begin{aligned} \sum_{i=1}^{L_m} q_i^m + \sum_{j=1}^{L_s} q_j^s \geq \; & \sum_{i=1}^{L_m} \left(1 - \frac{\mathsf{SINR}_i^m(\mathbf{p})}{\bar{\gamma}_i^m}\right) \\ & + \sum_{j=1}^{L_s} \left(1 - \frac{\mathsf{SINR}_j^s(\mathbf{p})}{\bar{\gamma}_j^s}\right). \end{aligned} \tag{12}$$

The equality in (12) is tight if (3) is feasible, i.e., $\mathbf{q}^\star = \mathbf{0}$. Otherwise, minimizing the left side of (12) has the effect of minimizing the differences between the SINR thresholds and the actual SINRs of all users.

Although (11) is still nonconvex, we can transform it to a convex problem by using a logarithmic transformation on the transmit power, i.e., $\tilde{\mathbf{p}} = \log \mathbf{p}$. Then, we obtain the following equivalent convex optimization problem:

$$\begin{array}{ll} \text{minimize} & \mathbf{1}^\top \mathbf{q} \\ \text{subject to} & \log \bar{\gamma}_i^m - \log \mathsf{SINR}_i^m(e^{\tilde{\mathbf{p}}}) \leq \log(1 + q_i^m), \forall i, \\ & \log \bar{\gamma}_j^s - \log \mathsf{SINR}_j^s(e^{\tilde{\mathbf{p}}}) \leq \log(1 + q_j^s), \forall j, \\ & e^{\tilde{\mathbf{p}}} \leq \bar{\mathbf{p}}, \\ & \mathbf{q} \geq \mathbf{0}, \\ \text{variables :} & \tilde{\mathbf{p}}, \mathbf{q}. \end{array} \tag{13}$$

We denote the optimal solution of $\mathbf{q}$ in (13) by $\mathbf{q}^\star = (q_1^\star, \ldots, q_{L_m+L_s}^\star)^\top$. Note that the optimal $\tilde{\mathbf{p}}$ in (13), denoted by $\tilde{\mathbf{p}}^\star$, is related to $\mathbf{p}^\star$ in (11) by $\tilde{\mathbf{p}}^\star = \log \mathbf{p}^\star$. Now, we characterize (13) with the following results, and this facilitates the design of a fixed-point algorithm.

Lemma 2: If $(\tilde{\mathbf{p}}, \mathbf{q})$ is a feasible point of (13), we have:

$$\rho\left(\text{diag}\left(\frac{\bar{\boldsymbol{\gamma}}}{1 + \mathbf{q}}\right)\left(\mathbf{F} + \frac{1}{\bar{p}_l}\mathbf{v}\mathbf{e}_l^\top\right)\right) \leq 1, l = 1, \ldots, L_m + L_s. \tag{14}$$

Next, we use the optimality conditions of (13) to develop an iterative fixed-point algorithm.

Theorem 1: The optimal transmit power $\mathbf{p}^\star$ in (11), the $\mathbf{q}^\star$ in (13), and the dual variables $(\boldsymbol{\nu}^\star, \boldsymbol{\lambda}^\star)$ of (13) satisfy:

$$\mathbf{p}^\star = \text{diag}\left(\frac{\bar{\boldsymbol{\gamma}}}{1 + \mathbf{q}^\star}\right)(\mathbf{F}\mathbf{p}^\star + \mathbf{v}), \tag{15}$$

$$\nu_l^\star = p_l^\star \left(\sum_{i \neq l} \frac{G_{il}\nu_i^\star}{\sum_{j \neq i} G_{ij}p_j^\star + n_i} + \lambda_l^\star\right), \quad l = 1, \ldots, L_m + L_s, \tag{16}$$

$$\lambda_l^\star(p_l^\star - \bar{p}_l) = 0, \quad l = 1, \ldots, L_m + L_s, \tag{17}$$

and

$$q_l^\star = \max\{\nu_l^\star - 1, 0\}, \quad l = 1, \ldots, L_m + L_s, \tag{18}$$

where $\nu_l \in \mathbb{R}_+$, which is associated with the lth SINR constraint, can be interpreted as the admission price, and $\lambda_l \in \mathbb{R}_+$ is associated with the lth power constraint. Furthermore, by introducing an auxiliary variable $x_l^\star = \nu_l^\star / p_l^\star$ for each l, we can rewrite (16) as:

$$\mathbf{x}^\star = \mathbf{F}^\top \text{diag}\left(\frac{\bar{\boldsymbol{\gamma}}}{1 + \mathbf{q}^\star}\right)\mathbf{x}^\star + \boldsymbol{\lambda}^\star. \tag{19}$$

Remark 2: The dual variable $\lambda_l^\star = 0$ when the associated transmit power satisfies $p_l^\star < \bar{p}_l$ at optimality of (13). If the optimal value of (10) is greater than zero, the dual variables satisfy $\boldsymbol{\nu}^\star > \mathbf{0}$ and $\boldsymbol{\lambda}^\star \neq \mathbf{0}$. In general, $\mathbf{x}$ can be regarded as an auxiliary variable to assist the computation of the primal and dual variables in (13).

B. Sum-of-Infeasibilities Joint Power and Admission Control Algorithm

Now, we propose a fixed-point algorithm to compute the optimal solution of (13), and simultaneously remove secondary users iteratively through admission control to identify a subset of secondary users that is feasible in (3).

Algorithm 1: Sum-of-Infeasibilities Joint Power and Admission Control

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1) **Initialization:**
   - Initialize the set of supported secondary users $\mathcal{A}(0) = \{1, \ldots, L_s\}$.
2) **Update by each user l during the uplink time slot:**
   - Update the transmitter power $p_l(k+1)$ at the $(k+1)$th step for all users:

$$p_l(k+1) = \min\left\{\frac{\bar{\gamma}_l p_l(k)}{\max\{\nu_l(k), 1\}\mathsf{SINR}_l(\mathbf{p}(k))}, \bar{p}_l\right\}. \tag{20}$$

3) **Update by each user l during the downlink time slot: If $p_l < \bar{p}_l$**
   - Update the auxiliary variable $x_l(k+1)$:

$$x_l(k+1) = \sum_{j=1}^{L_m+L_s} \frac{F_{jl}\bar{\gamma}_j}{\max\{\nu_j(k), 1\}} x_j(k), \forall l. \tag{21}$$

- Update the admission price $\nu_l(k+1)$:

$$\nu_l(k+1) = x_l(k+1)p_l(k+1), \forall l. \quad (22)$$

else

- Update the admission price $\nu_l(k+1)$:

$$\nu_l(k+1) = \frac{\bar{\gamma}_l}{\mathsf{SINR}_l(\mathbf{p}(k+1))}, \forall l. \quad (23)$$

- Update the auxiliary variable $\mathbf{x}(k+1)$:

$$x_l(k+1) = \nu_l(k+1)/p_l(k+1), \forall l. \quad (24)$$

end

4) **Secondary user admission control:**
- After a predefined threshold T (i.e., the iteration number of inner loop (20)-(24)), let $q_l(k+1) = \max\{\nu_l(k+1)-1, 0\}$ for all users. If $\mathbf{1}^\top \mathbf{q}(k+1) > 0$, then switch off the worst secondary user j, where:

$$j = \arg\max_{l\in\mathcal{A}(k)} \nu_{l+L_m}(k+1), \quad (25)$$

- Update the set $\mathcal{A}(k+1) \leftarrow \mathcal{A}(k) - j$.

Theorem 2: If Algorithm 1 converges then it converges to a feasible set of supported secondary users for (3).

Remark 3: The computation of (21) and (25) can be made distributed by message passing. The limit point of $\lim_{k\to\infty} \mathbf{p}(k)$ solves (3), and $\lim_{k\to\infty} \mathbf{1}^\top \mathbf{q}(k) = 0$ implies that (3) is feasible. When $\lim_{k\to\infty} q_l(k) > 0$ for some l, the corresponding secondary users have to be removed to make (3) feasible.

Since the condition that $\mathsf{SINR}_l(\mathbf{p}^\star) = \frac{\bar{\gamma}_l}{1+q_l^\star}$, $q_l^\star = 0$ implies that the lth user can achieve its SINR threshold. Otherwise, $q_l^\star > 0$ implies that the lth user cannot reach its SINR threshold and it should be switched off. If we remove all the users that satisfy $q_l^\star > 0$, then (3) is guaranteed to be feasible. However, some users are unnecessarily removed since we have used the relaxation (11) instead of (10). To remove the least possible number of users, we introduce the idea of deflation into the admission control. Based on $\mathbf{q}^\star$, an educated guess to reduce the sum of the infeasibilities is to delete the worst secondary user corresponding to $\arg\max_{l\in\mathcal{A}(k)} q_l^\star$, which is equivalent to (25). This is implemented in Step 4. The unsupported secondary user is switched off not only to reduce its transmit power, but also to avoid adding interference to other users.

Furthermore, note that the SINR margin satisfies $\mathbf{q}^\star = 0$ after the system becomes feasible. Then, since the admission price satisfies $\boldsymbol{\nu}^\star \leq 1$ based on (18), the power update in (20) is equivalent to that in (9). Therefore, the total transmit power is minimized on the set of primary users and a subset of feasible secondary users.

C. Discussion of Threshold T

The convergence of Algorithm 1 depends on the predefined stopping threshold T at Step 4. If T is large enough, Algorithm 1 removes the secondary users that cause infeasibility based on relatively stabilized admission price and may converge rather slowly. As T becomes smaller, Algorithm 1 converges faster but may prematurely remove more secondary users based on (yet to stabilize) admission price. In practice, it is observed that the users in the feasible subset can attain their required SINR even when the convergence time of Algorithm 1 is small. Hence, the choice of T reflects the aggressiveness of admission control and convergence. To understand this better, we use the outage probability, which is defined as the ratio of the number of removed users to the total number of secondary users, as a parameter to study the tradeoff between the outage probability and the convergence time of Algorithm 1 by choosing different T.

The total number of iterations of Algorithm 1 for convergence is affected by T which in turn affects the number of users eventually removed. From a practical perspective, the system should become feasible as soon as possible. Larger cognitive radio networks may have more secondary users and require a dynamically adaptable T. We describe a heuristic to adapt T. First, we empirically get an (a priori) outage probability r_o in the cognitive radio networks. Suppose we desire an expected convergence time of Algorithm 1, denoted as $\bar{T}$. Then, we use the threshold $T = \bar{T}/(L_s \times r_o)$ for admission control. We may have more than one secondary user satisfying (25). In this case, we remove secondary users by breaking ties uniformly at random.

IV. Numerical Examples

In this section, we provide experimental results to illustrate that our proposed algorithm outperforms other known alternatives in terms of the energy consumption and the system capacity.

Example 1: We compare our methods with the distributed power control algorithm with temporary removal and feasibility check (DFC) in [17]. Although the model in [17] is the special case for a single cell where the channel gain for each user is the same $G_{lj} = G_{jj}$, we use the same setup for the convenience of comparison. The AWGN at the receiver, i.e., $n = \sigma^2$, is assumed to be 5×10^{-15} W. The channel gain is adopted from the well-known model $G_{jj} = kd_j^{-4}$, where d_j is the distance between the jth transmitter and its receiver, and $k = 0.09$ is the attenuation factor that represents power variations due to path loss. The upper bounds of the transmit power for all users are the same, i.e., $\bar{p}_l = 1$ W for all l. There are 5 users indexed by 1 to 5 in a single-cell environment where the distance vector is $d = [300, 530, 740, 860, 910]^\top$ m, in which each element is the distance of the corresponding receiver from its transmitter. The SINR threshold vector is $\bar{\boldsymbol{\gamma}} = [0.40, 0.30, 0.35, 0.25, 0.25]^\top$, which is equivalent to $\bar{\boldsymbol{\gamma}} = [-4, -5.2, -4.6, -6, -6]^\top$dB. User 1 is the primary user while the others are the secondary users.

Figure 3 shows the same simulation result of DFC as [17], which sets $p_5 = 0$ to switch off User 5 so that the other users reach their SINR thresholds with the minimum total energy consumption as the system is infeasible. The optimal power is $\mathbf{p} = [0.0061, 0.0483, 0.2063, 0.2904, 0]^\top$. In addition, the performance of DFC depends on the initial point. Although

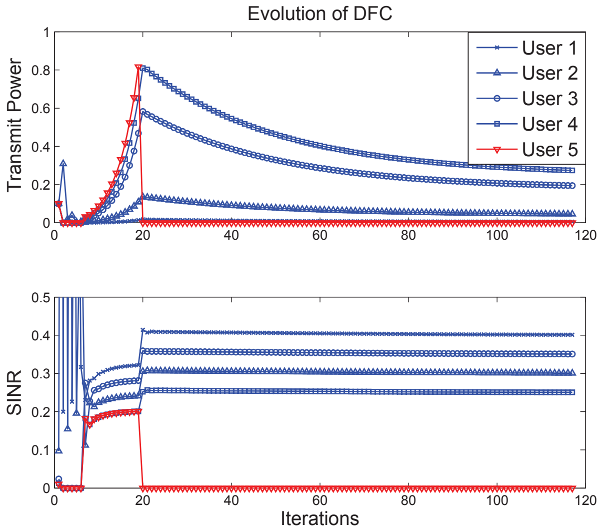

Fig. 3. The evolution of transmit power and SINR for DFC with proper initial point. The blue lines are 4 supported users. The red line is the removed secondary user.

DFC performs well when the initial point is chosen appropriately, the iteration may oscillate for other initial points. It is mentioned in [17] that such oscillation can be further removed through additional heuristics.

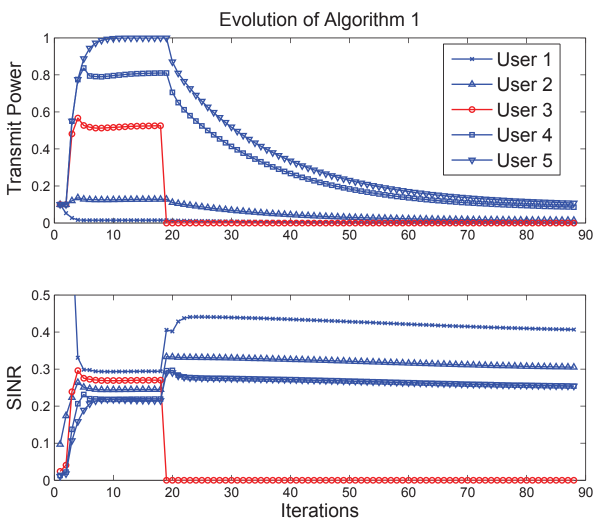

Fig. 4. The evolution of transmit power and SINR for Algorithm 1. The blue lines are 4 supported users. The red line is the removed worst secondary user.

Figure 4 shows that Algorithm 1 obtains the same feasible set in terms of the cardinality. Also, the solution of the power is $\mathbf{p} = [0.0015, 0.0121, 0, 0.0728, 0.0912]^\top$, and Algorithm 1 removes User 3 instead of User 5. Hence, our feasible set gives an energy saving of $\frac{0.5511-0.1776}{0.5511} \times 100\% = 67.8\%$. The main reason is that DFC temporally removes the user that first hits the upper bound of the user's individual power constraint, whereas, our method predicts the worst secondary user with an educated guess that exploits the SINR margin variable.

Table I shows the admission criteria of secondary users when Algorithm 1 obtains the optimal solution of (13). Although we can reduce the sum of power to 0.1731 by removing User 1 with $\mathbf{p} = [0, 0.0092, 0.0393, 0.0553, 0.0693]^\top$, we do not remove User 1 because it is the primary user. In addition, our results are the same as the results produced by the centralized algorithm in [16], which greedily removes the user that provides the highest marginal increase in the maximum achievable SINR once it is removed. Nevertheless, the centralized algorithm in [16] has a higher complexity as it tries to remove every user at each iteration based on the global information that has to be obtained in a centralized manner.

TABLE I
ADMISSION CRITERIA OF ALGORITHM 1

Alg. 1	$\mathbf{p}^\star$	$\mathbf{q}^\star$
User 1	0.0152	0.3590
User 2	0.1284	0.2233
User 3	0.5272	0.2954
User 4	0.8127	0.1406
User 5	1.0000	0.1668
$\sum$	2.4835	1.1852

Example 2: We compare Algorithm 1 with the widely used constrained DPC algorithm (9) for general networks with different channel gains $G_{lj} \neq G_{jj}$ of each user. There are 2 primary users and 8 secondary users in an infeasible cognitive network where the channel gains are generated randomly. The upper bounds of the power constraints and the SINR thresholds are the same for all l, that are $\bar{p}_l = 1$ W and $\bar{\gamma}_l = 0.5$, respectively.

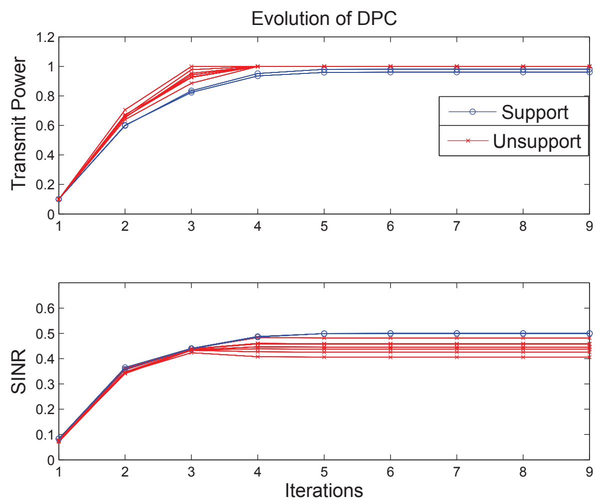

Fig. 5. The evolution of transmit power and SINR for DPC. The red lines are unsupported 8 users. The blue lines are supported 2 users.

Figure 5 shows the evolution of DPC where 8 unsupported users transmit at their maximum power level but do not achieve their SINR thresholds. As these unsupported users increase the

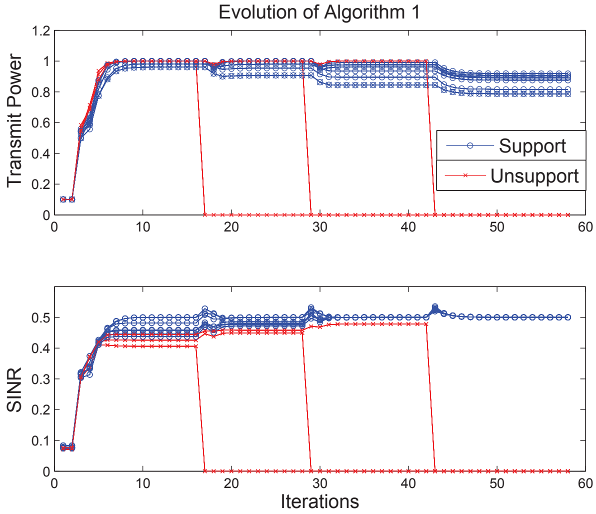

Fig. 6. The evolution of transmit power and SINR for Algorithm 1. The red lines are removed worst 3 secondary users. The blue lines are supported 7 users.

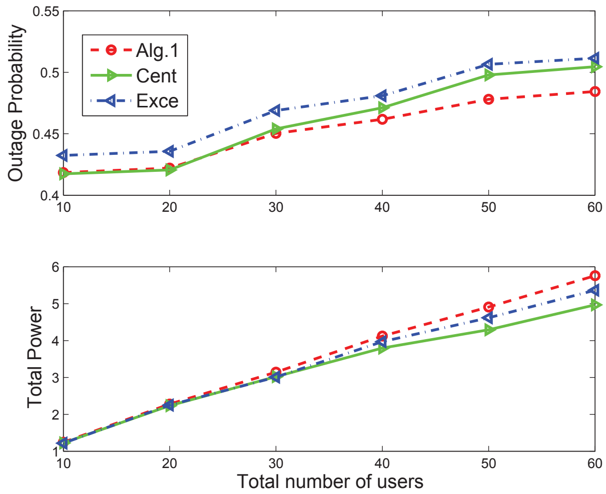

Fig. 7. Average outage probability and average total energy consumption versus total number of users. The lower bound of SINR thresholds are the same $\bar{\gamma}_l = -6$ dB for all l.

interference in the network, there are only 2 users achieving their SINR thresholds. Figure 6 shows the evolution of Algorithm 1 where there are 7 users supported by the network after we gradually remove the $\{5, 3, 10\}$-th users which are the secondary users. Meanwhile, the centralized algorithm in [16] gets the same approximate maximum feasible set by greedily removing the $\{5, 10, 3\}$-th users. Compared to DPC, our algorithm increases the system capacity from 20% to 70%.

Example 3: It is possible to obtain different maximum feasible sets from different algorithms, as the maximum feasible set may not be unique. Hence, we compare the system capacity obtained (equivalently the outage probability) and the energy consumption based on different algorithms. This example reports the Monte-Carlo (MC) average results for at least 300 MC runs. For each MC run, transmitter locations are uniformly drawn on a 2Km $\times$ 2Km square. For each transmitter location, a receiver location is drawn uniformly in a disc of radius 400 meters, excluding a radius of 10 meters. The primary users are randomly selected from all users and the remaining ones are secondary users. All upper bounds of transmit power are fixed as $\bar{p}_l = 1$ W. The channel gains are calculated by $G_{lj} = d_{lj}^{-4}$ where d_{lj} is the Euclidean distance between the jth transmitter and the lth receiver. The receiver noise is set as -60 dBm. In the figures, Alg. 1 is our proposed Algorithm 1 in Section III, Cent is the centralized removal algorithm in [16] and Exce is the algorithm where we use the heuristic that considers the removal metric [18] with the worst secondary user j:

$$j = \arg\max_{a \in \mathcal{A}} \sum_{l \neq a} G_{la} p_a^e + \sum_{l \neq a} G_{al} p_l^e, \qquad (26)$$

where $\mathcal{A}$ is the set of current secondary users in system and p_l^e is the excess transmission power needed for User l to attain its SINR threshold.

Figures 7 and 8 show that our Algorithm 1 outperforms

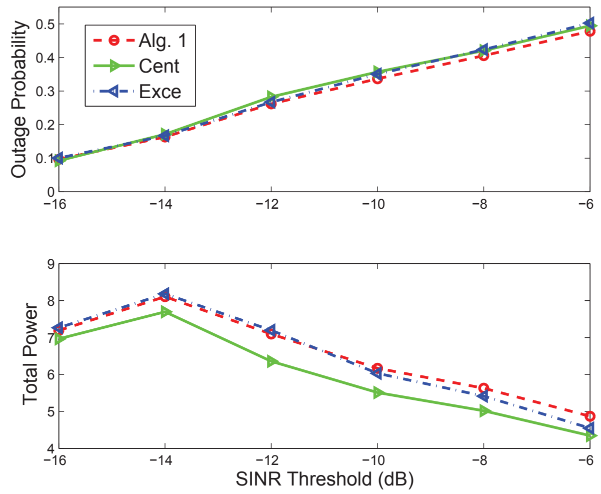

Fig. 8. Average outage probability and average total energy consumption versus different SINR thresholds. The total number of users is 50.

the centralized greedily removal algorithm in [16] and the algorithm that uses the removal heuristic in (26) for admission control. Although the centralized algorithm and the algorithm that uses the removal heuristic in (26) have an overall smaller total energy consumption than those obtained by Algorithm 1, this is due to the fact that they support fewer users thus yielding a lower system capacity. When the outage probabilities are the same, Algorithm 1 achieves smaller total energy consumption than the Exce algorithm. This demonstrates the value of optimizing the admission price as compared to the metric in (26). Figure 9 shows that the convergence time becomes longer with a larger T , while the outage probability tends to be smaller in the same case. When T is large enough, the convergence time and outage probability can be stabilized, which means that Algorithm 1 converges and the increase of T does not affect significantly the performance. In this example, we set $T = 16$ by letting the expected convergence time be $\bar{T} = 340$ and we empirically get a priori outage probability which is $r_o = 0.43$.

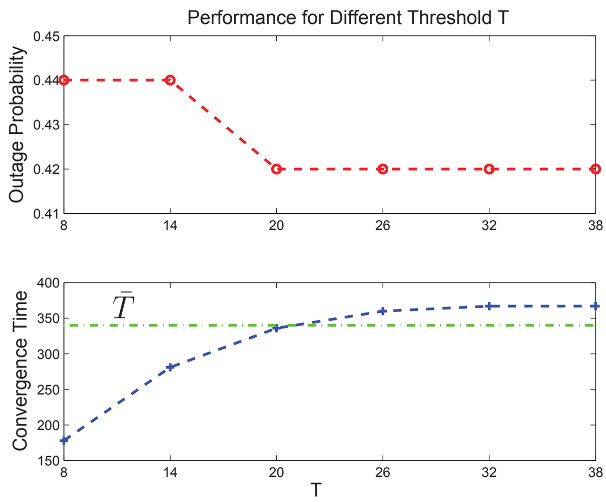

Fig. 9. Average outage probability and average convergence time for different threshold T in a 50 users case. The lower bound of all the SINR minimum thresholds are set to be the same $\bar{\gamma}_l = -8$ dB for all l.

V. Conclusion

In this paper, we studied the feasibility problem of energy minimization in cognitive radio networks subject to power and SINR constraints. We formulated this problem as a vector-cardinality optimization problem. We used a sum-of-infeasibilities heuristic to relax the vector-cardinality optimization problem and to design a joint power and admission control algorithm. Numerical evaluations showed that our proposed sum-of-infeasibilities joint power and control algorithm was computationally fast and converged to the equilibrium that was near-optimal in terms of maximizing the system capacity with minimal energy consumption. Our algorithm can also outperform existing admission control and power control algorithms.

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