Autonomous Uplink Intercell Interference Coordination in OFDMA-based Wireless Systems*

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***Abstract*—The inter-cell interference (ICI) problem in OFDMA-based wireless systems becomes a major impediment to attaining high rates particularly for cell-edge users in reuse-1 systems. Interference mitigation techniques attempt to combat ICI by using proper resource allocation schemes. Using centralized algorithms is not practical, particularly in HetNet environments, as these algorithms require intensive signaling, interference, and channel state information that may not always be practically available. In this paper, we present a framework for autonomous uplink ICIC in OFDMA-based wireless systems. The framework is based on imposing an overall interference limit for each cell. We use the proportional fair algorithm for resource block assignment and the power allocation problem is formulated as a maximization of the sum of the signal to leakage and noise ratio (SLNR) subject to the cell interference on other cells below a certain threshold. We propose a suboptimal closed form method, and an iterative allocation using Newton's method. These schemes do not need coordination between the cells where the resource allocation can completely be performed autonomously. Simulations show that relaxing the interference constraints improves the performance of the two proposed algorithms which exhibit better performance than the trivial equal power allocation. Comparison with a centralized scheme that uses global information shows good performance with acceptable degradation in the spectral efficiency which decreases as the interference limit increases.**

Keywords*—*Lagrangian; Complementary Slackness; Log-barrier method; Newton's method.

I. INTRODUCTION

In OFDMA-based systems, the available bandwidth is divided into a number of orthogonal subcarriers to mitigate the effect of frequency selective fading. In this paper we adopt, without loss of generality, LTE system definition of resource block (RB) comprised of a number of consecutive subcarriers (12 in LTE). A RB represents the minimum resource allocation unit. The orthogonality of RB assignment in each cell eliminates the intra-cell interference, however, the inter-cell interference (ICI) problem becomes a major impediment to attaining high rates particularly for cell-edge users in reuse-1 systems. The interference generated by terminals in the neighboring cells dramatically deteriorates the signal to interference and noise ratio (SINR) received at any base station (BS), and hence decreases the rates of the cell users, especially the cell edge users. Interference mitigation techniques attempt to combat ICI by using proper resource allocation schemes. A frame work for uplink power control to mitigate interference is proposed by Yates [1] using the definition of interference function. Using centralized algorithms is not practical, particularly in HetNet environments, as these algorithms require intensive signaling, interference, and channel state information that may not always be practically available.

A lot of research work has tackled the problem for downlink ICIC which can be classified as coordinated-distributed, semi-autonomous, and autonomous allocation schemes [2]. However, the uplink ICIC problem has been tackled but less extensively. In [3], Foschini and Miljanic prove the exponential convergence of a class of distributed uplink-downlink power control algorithms aiming to achieve a minimum SINR per user as long as the set of the required SINRs is feasible. Almost all proposed schemes aim to maximize some utility function subject to power and interference constraints. Due to the prohibitive complexity of the optimal solution, most schemes seek a heuristic suboptimal allocation that achieves near optimal results with much less complexity.

A. *Related Work*

In this subsection, we provide an overview of the existing research work that attempts to solve the uplink resource allocation problem. In [4], the authors propose an iterative algorithm to assign RBs using pairwise coordination between BSs taking into account the SC-FDMA nature of the uplink transmission in LTE. The BS assigns each user the RBs that maximize the total utility gradients of the cell users. The scheme allows/prevents two users in two neighboring cells to use the same RB according to a defined marginal utility function. This requires extensive coordination and the limit to which pairs are grouped is not defined.

In [5], the BSs exchange interference price messages to maximize the weighted sum rates of each cell. This scheme separates RB assignment from power allocation for simplicity. The problem of resource allocation is solved iteratively using the Karush-Kuhn-Tucker (KKT) equations or by Newton's method with some relaxations to reduce the complexity of the scheme. The authors also use the uplink-downlink duality to validate their scheme as a suboptimal scheme for uplink resource allocation. Similarly, in [6] a proposal for a semi-autonomous scheme that maximizes the

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weighed sum rates of each cell is provided. The paper proposes the concept of noise rise to reduce the interference caused by each cell to its neighbors and to reduce the exchange of messages between BSs. An iterative water-filling algorithm, whose complexity can be reduced by the use of binary search, is used to allocate resources. Authors also propose a constrained noise rise density algorithm by specifying a noise rise constraint for each RB in which only one user transmits per cell.

In [7] a five-steps-multi-sector-gradient scheme that needs infrequent exchange of interference cost messages between sectors is proposed. Each sector maximizes its utility function while considering the degradation that it causes to the utilities of the neighboring sectors. The resources allocated to each user are power, sub-band, and RBs in the sub-bands. To relax the required accuracy of interference estimation, the algorithm uses the average interference instead of the instantaneous interference.

An iterative cooperative game theory approach is proposed in [8]. Every user plays in turn to select the subcarrier that reduces the interference it sees and the interference it produces using exhaustive search. An extra constraint is added to eliminate the dominant state in which each user transmits on a single subcarrier, which is not the best strategy from the viewpoint of spectral efficiency. Power is allocated to satisfy a predetermined SINR and maximum power constraint.

B. Summary of Contributions

This paper provides a new convex formulation of the resource allocation problem for autonomous ICIC in the uplink of OFDMA-based systems and proposes alternative solution techniques for it. We define the leakage power (which is quite similar to the noise rise in [6] but independent of the value of the noise power) as the total power leaked by the cell to its neighbors. We use the large-scale parameters (pathloss and shadowing) between the cell users and the neighboring BSs to calculate the leakage power as these parameters can be estimated through pilots in the downlink assuming TDD. The proposed power allocation does not utilize the exact channel gains between terminal i and the non-serving BSs as such information is usually not available in a real system. We consider the leakage power as a measure of the total interference produced by the cell.

The main thesis here is based on the notion of setting an overall interference limit that the terminals in a certain cell are allowed to leak/interfere to neighboring cells. If each cell limits its overall interference to neighbors, it will also get a similar treatment from the neighbors, and therefore, the overall interference that each cell sees will be limited. Instead of maximizing the terminals' uplink SINR which requires coordination between cells, we maximize the sum of the signal to noise and leakage ratios (SLNR) of the cell users subject to constraints on the amount of the total power leaked by the cell and on the maximum uplink power budget per user. This tends to have good effect on enhancing the SINR of the users since as each cell aims to minimize the amount of interference it produces, the interference seen by the cell itself decreases too due to the altruistic behavior of other cells.

The proposed power allocation depends on the channel gains between the cell users and their serving BS over different RBs, the large scale parameters with the neighboring BSs, the interference limit, and the maximum uplink power transmission per user. These parameters are autonomous and do not need to be exchanged between the cells. Thus each cell assigns RBs to its terminals, and then allocates the power to transmit on each RB autonomously as in single-cell scheduling because we do not need to know the resource allocation or the channel conditions in the other cells.

In this problem, we assume universal frequency reuse and we use the proportional fair (PF) algorithm for RB assignment to achieve long term fairness among users, and we mainly focus on power allocation. We propose a suboptimal scheme that provides a closed-form power allocation. We also reformulate the problem using logarithmic penalty functions for the constraints to get an iterative power allocation using Newton's method. The RB assignment is quite independent of the proposed power allocation schemes so any other RB assignment can be used instead of the PF. This resource allocation can be adaptive if BSs are allowed to exchange messages to indicate the interference level they see. This simple infrequent coordination allows the scheme to adapt to the dynamic loads in the network without considerable overhead or delay.

Simulation results show that the spectral efficiency of the system increases if we allow the cell to produce more interference. The suboptimal and the iterative power allocation schemes perform better than the trivial equal power allocation (EPA) especially at low interference limits. We also compare the performance with centralized utility maximization scheme based on DC programming [9]. They give acceptable performance compared with the centralized power allocation given that the scheme is based on local information and has much less computational complexity particularly the sub-optimal scheme which necessarily has a closed-form solution followed by a simple normalization algorithm.

The rest of this paper is organized as follows. Section II describes the system model and the convex optimization problem formulation. Section III discusses the different proposed solution techniques. Section IV evaluates the performance of these techniques, and finally, section V concludes the paper.

II. Formulation of the Optimization Problem

A. System Model

Consider an OFDMA-based cellular system consisting of a set of M BSs, let the set of served terminals be defined by I and the set of terminals served by BS $s \in M$ be defined by I_s. Let the function $|\,.\,|$ denotes the cardinality of the set, then we have $|I| = \sum_{s \in M} |I_s|$. Let the home/serving BS for user i be denoted by $s(i)$. Assume K to be the number of uplink RBs.

Let the long-term (large-scale) channel gain between terminal i and BS-s, $s \in M$ and $i \in I$ be denoted by $g_{s,i}$, and let $h_{i,k}$ denotes the channel gain between terminal i and its serving BS $s(i)$ on RB-k. The value of $h_{i,k}$ reflects both large-scale channel gains and small-scale frequency-dependent fading component due to multi-path and frequency selectivity due to the variation of the channel response at the different RB's. Let $\sigma_{s,k}^2$ be the noise power on RB-k at the BS-s. Furthermore, let the indicator variables $\alpha_{i,k} \in \{0,1\}$ be equal to '1' if RB-k is allocated to terminal i and '0' otherwise, and $p_{i,k}$ be the uplink transmission power of terminal i on RB-k. Using this notation the resource block and power assignments are captured by the matrices $A_{|I|\times K} = [\alpha_{i,k}]$ and $P_{|I|\times K} = [p_{i,k}]$. Note that $\alpha_{i,k} \in \{0,1\}$ is ignored in the subsequent formulations, since if $\alpha_{i,k} = 0$ this can automatically be reflected in $p_{i,k} = 0$.

B. SLNR-based Optimization Problem Formulation

We define the overall autonomous uplink ICIC problem as in (1). U_s is the utility function of BS-s, T is the interference limit, P_{max} is the maximum uplink power allocated per terminal, and p_{mask} is the maximum power allocated per terminal on an RB basis. The first constraint is the overall interference constraint contributed by terminals in BS-s to all other uplink transmissions in the system, whereas the second constraint is the power mask constraint per RB and the last constraint is the total uplink-power-transmission constraint per terminal. Additional minimum rate constraints can be defined on the overall rate of each terminal i.

$$\begin{gathered}\text{Maximize } U_s \ \forall s \in M \\ \text{Subject to} \\ F_s = \sum_{n\in M, n\neq s} \sum_{i \in I_s} g_{n,i} \sum_{k=1}^{K} p_{i,k} \leq T \\ 0 \leq p_{i,k} \leq p_{mask} \\ \sum_{k=1}^{K} p_{i,k} \leq P_{max} \\ \forall i \in I_s, k = 1,2,\ldots,K\end{gathered} \tag{1}$$

An interesting formulation that removes the need for exchanging or measuring of mutual interference levels or interference costs among the neighboring BSs is to exploit SLNR to guide the uplink power allocation problem. SLNR is the ratio of the received signal strength to the sum of the noise and total leaked interference to other cells. It is always desirable to have a high SLNR as this typically translates to a high SINR. However, the exact relationship between SINR and SLNR is usually not explicitly available. Equation (2) shows the SLNR of terminal i transmitting with power $p_{i,k}$ on RB-k at BS $s = s(i)$.

$$\zeta_{i,k} = \frac{p_{i,k} h_{i,k}}{\sigma_{s,k}^2 + p_{i,k} \sum_{n\in M, n\neq s} g_{n,i}} = \frac{p_{i,k} h_{i,k}}{\sigma_{s,k}^2 + p_{i,k} G_{s,i}} \tag{2}$$

The term $p_{i,k} g_{n,i}$ represents the power received at BS-n due to terminal i transmitting with power $p_{i,k}$ over RB-k. This term represents an amount of power "leaked" to BS-n, which could affect the SINR at that BS. Therefore, the lower this value the lower the expected contribution to the interference at the neighbors of BS-s and the better the quality of their links. We call the quantity in (2) the individual SLNR. An alternative formulation, the coupled SLNR, appears in (3) if we include the overall leakage resulting from the transmissions of all terminals in a given cell in the denominator.

$$\begin{aligned}\zeta'_{i,k} &= \frac{p_{i,k} h_{i,k}}{\sigma_{s,k}^2 + \sum_{j\in I_s} \sum_{l=1}^{K} p_{j,l} \sum_{n\in M, n\neq s} g_{n,j}} \\ &= \frac{p_{i,k} h_{i,k}}{\sigma_{s,k}^2 + \sum_{j\in I_s} G_{s,j} \sum_{l=1}^{K} p_{j,l}}\end{aligned} \tag{3}$$

We set the objective function as the maximization of the sum of SLNR (or alternatively the minimization of the negation of the SLNR) for all terminals over all RB's for each cell subject to total cell leakage power below threshold T and the overall uplink power for each terminal below the maximum power P_{max}. Equation (4) shows the formulation of the optimization problem we seek to solve. We ignore the power mask constraint per RB and focus only on the total power allocated per user. This problem is guaranteed to be convex when using the individual SLNR since the second derivative is always positive semi-definite [10], however convexity is not guaranteed in the case of coupled SLNR

$$\begin{gathered}\text{Minimize} \\ U_s = -\sum_{i\in I_s} \sum_{k=1}^{K} \zeta_{i,k} \ \ \forall s \in M \\ \text{Subject to} \\ F_s = \sum_{i\in I_s} \sum_{n\in M, n\neq s} g_{n,i} \sum_{k=1}^{K} p_{i,k} \leq T \\ \sum_{k=1}^{K} p_{i,k} \leq P_{max} \ \ \forall i \in I_s\end{gathered} \tag{4}$$

III. Solution of the Optimization Problem

A. Lagrange Optimal Solution

In this subsection, we solve the KKT equations of the problem in (4) to reach an expression of the optimal power allocation. The Lagrangian of the optimization problem formulated in section II is shown in (5), where μ and λ_i are the Lagrange multipliers corresponding to the interference constraint and the maximum power constraint of terminal i respectively.

$$L(\vec{P},\mu,\vec{\lambda}) = -\sum_{i\in I_s}\sum_{k=1}^{K}\frac{p_{i,k}h_{i,k}}{\sigma_{s,k}^2 + p_{i,k}G_{s,i}} + \mu\left[\sum_{i\in I_s}G_{s,i}\sum_{k=1}^{K}p_{i,k} - T\right] + \sum_{i\in I_s}\lambda_i\left[\sum_{k=1}^{K}p_{i,k} - P_{max}\right] = f_o(\vec{P}) + \mu\tilde{f}(\vec{P}) + \textstyle\sum_{i\in I_s}\lambda_i f_i(\vec{P}) \quad (5)$$

The KKT conditions of the problem can be written as follows:

$$\mu^*\tilde{f}(\vec{P}) = 0, \quad (6.a)$$

$$\lambda_i^* f_i(\vec{P}) = 0, i\in I_s, \quad (6.b)$$

$$\nabla f_o(\vec{P}) + \mu^*\nabla\tilde{f}(\vec{P}) + \sum_{i\in I_s}\lambda_i^*\nabla f_i(\vec{P}) = 0, \quad (6.c)$$

$$\mu^* \geq 0, \lambda_i^* \geq 0.$$

Substituting with the first derivatives in (6.c), we get an expression of the power allocated to terminal i on RB-k as follows:

$$p_{i,k} = \frac{1}{G_{s,i}}\left[\sqrt{\frac{\sigma_{s,k}^2 h_{i,k}}{\mu^* G_{s,i} + \lambda_i^*}} - \sigma_{s,k}^2\right], \quad k = 1,2,\dots,K,\ \ i\in I_s. \quad (7)$$

The expression in (7) is a function of the optimal Lagrange multipliers μ^* and λ_i^*. Substituting with this power allocation in the complementary slackness conditions in (6.a), (6.b) yields the following two equations

$$\mu^*\left(\sum_{i\in I_s}G_{s,i}\sum_{k=1}^{K}\frac{1}{G_{s,i}}\left[\sqrt{\frac{\sigma_{s,k}^2 h_{i,k}}{\mu^* G_{s,i} + \lambda_i^*}} - \sigma_{s,k}^2\right] - T\right) = 0$$

$$\lambda_i^*\left(\sum_{k=1}^{K}\frac{1}{G_{s,i}}\left[\sqrt{\frac{\sigma_{s,k}^2 h_{i,k}}{\mu^* G_{s,i} + \lambda_i^*}} - \sigma_{s,k}^2\right] - p_{max}\right) = 0 \quad (8)$$

These equation should be solved iteratively to get numerical values for the optimal Lagrange multipliers. This approach does not provide a closed form of the power allocation because (8) does not give closed form for the Lagrange multipliers. So, we relax one of the complementary slackness conditions to have a closed form suboptimal power allocation in the next subsection.

B. Suboptimal Scheme

Assume the total interference constraint is set to equality at the optimal power allocation, whereas all power constraints are set to strict inequality. This is a reasonable assumption for practical situations as in most cases we encountered the interference limits constraint is usually more strict than total power constrain per terminal. This leads to a solution with $\mu^* \neq 0$ and $\lambda_i^* = 0\ \forall i \in I_s$. The two equations in (8) will reduce to the following equation:

$$\sum_{i\in I_s}\sum_{k=1}^{K}\left[\sqrt{\frac{\sigma_{s,k}^2 h_{i,k}}{\mu^*}} - \sigma_{s,k}^2\right] = T \quad (9)$$

Equation (9) is a single equation in a single unknown. This equation gives a closed form solution for the optimal Lagrange multiplier. Hence, the optimal power allocated to terminal i on RB k has a closed form as shown in (10).

$$\mu^* = \left(\frac{\sum_{i\in I_s}\sum_{k=1}^{K}\sqrt{\frac{\sigma_{s,k}^2 h_{i,k}}{G_{s,i}}}}{T + |I_s|\sum_{k=1}^{K}\sigma_{s,k}^2}\right)^2, \quad p_{i,k} = \max\left(\frac{1}{G_{s,i}}\left[\sqrt{\frac{\sigma_{s,k}^2 h_{i,k}}{\mu^* G_{s,i}}} - \sigma_{s,k}^2\right], 0\right), \quad (10)$$

$k = 1,2,\dots,K, i\in I_s$.

Furthermore, to make sure the maximum power constraint is satisfied for each terminal i, we define the quantity $\vartheta_i = \sum_{k=1}^{K}p_{i,k} - P_{max}$,

then the final power allocation becomes

$$\check{p}_{i,k} = \begin{cases} p_{i,k} & \vartheta_i < 0. \\ p_{i,k} - \frac{\vartheta_i}{\sum_{k=1}^{K}p_{i,k}}p_{i,k} - \varepsilon_i & \vartheta_i > 0. \\ p_{i,k} - \varepsilon_i & \vartheta_i = 0. \end{cases} \quad (11)$$

where $\varepsilon_i \to 0$.

In this case, the amount of power subtracted from terminal i on RB-k depends on the ratio of the power allocated to this RB to the total power allocated to the terminal. Alternatively, the RBs with small amount of allocated power should have either bad channels to the serving BS or high interference to the neighbors. Therefore, an iterative algorithm can be used to remove the power allocated to these bad RBs until the power constraints are met. This suboptimal scheme allocates zero power to some terminal over some RB if the first argument of the maximum function in (10) is negative. This behavior definitely affects the amount of interference generated by each cell, so we apply an interference normalization besides the former power normalization to make sure the interference constraint is satisfied.

This suboptimal power allocation does not need coordination between terminals in the same cell. The BS needs to broadcast the optimal value of μ^* and assign RBs to the cell user every slot, and then each user can calculate its transmission power on every RB from (10) and normalizes its total power to satisfy the maximum power constraint. Interference normalization can also be done independently by each user to normalize the interference. The lack of coordination may result in small violation in the interference limit constraint.

C. Solution using Penalty Function Approach

We reformulate the problem by assigning a logarithmic penalty function for each constraint. This approach converts the inequality constrained optimization problem in (4) to the following unconstrained problem in (12) where both η^T and $\eta_i^p \geq 0$ and are chosen as small as possible to approximate the indicator functions of constraint violations.

$$\text{Minimize}$$

$$\begin{aligned} f(\vec{P}) = &-\sum_{i\in I_s}\sum_{k=1}^{K}\frac{p_{i,k}h_{i,k}}{\sigma_{s,k}^2+p_{i,k}G_{s,i}} \\ &-\eta^T \log\left(T-\sum_{n\in M,n\neq s}\sum_{i\in I_s} g_{n,i}\sum_{k=1}^{K}p_{i,k}\right) \\ &-\sum_{i\in I_s}\eta_i^p \log\left(P_{max} \quad -\sum_{k=1}^{K}p_{i,k}\right) \\ = &\, f_0(\vec{P})+f_T(\vec{P})+\sum_{i\in I_s} f_{c,i}(\vec{P}) \end{aligned} \tag{12}$$

The optimization problem in (12) is convex and can be solved using a multitude of methods such as Newton's method. For the Newton's method, we need to evaluate the first derivative (gradient) and second derivatives (Hessian) of the objective function w.r.t $p_{i,k}$. The gradient of the objective function is

$$\nabla f(\vec{P}) = \nabla f_0(\vec{P}) + \nabla f_T(\vec{P}) + \sum_{i\in I_s}\nabla f_{c,i}(\vec{P}), \tag{13}$$

where,

$$\frac{\partial f_0(\vec{P})}{\partial p_{i,k}} = \frac{-\sigma_{s,k}^2 h_{i,k}}{\left[\sigma_{s,k}^2+G_{s,i}p_{i,k}\right]^2} \tag{14.a}$$

$$\frac{\partial f_T(\vec{P})}{\partial p_{i,k}} = \frac{\eta^T G_{s,i}}{\left[T-\sum_{i\in I}\sum_{k=1}^{K}G_{s,i}p_{i,k}\right]} \tag{14.b}$$

$$\frac{\partial f_{c,i}(\vec{P})}{\partial p_{j,k}} = \frac{\eta_i^P \delta_{i,j}}{\left[p_{max}-\sum_{k=1}^{K}p_{i,k}\right]} \tag{14.c}$$

$$\text{where } \delta_{i,j} = \begin{cases}1, i=j\\ 0, i\neq j\end{cases}$$

The Hessian of the objective function is given as follows

$$H = H_{f_0}+H_{f_T}+H_{\sum f_{c,i}(\vec{P})}, \tag{15}$$

where

$$\frac{\partial^2 f_0(\vec{P})}{\partial p_{j,l}\partial p_{i,k}} = \begin{cases}\dfrac{2\,\sigma_{s,k}^2 h_{i,k}G_{s,i}}{\left[\sigma_{s,k}^2+G_{s,i}p_{i,k}\right]^3}, j=i, l=k\\ 0, \text{otherwise}\end{cases} \tag{16.a}$$

$$\begin{aligned} H_{f_T} &= \vec{g}\vec{g}^T \text{ where} \\ \vec{g} &= \sqrt{\theta}[G_{s,1}\cdots G_{s,1}\; G_{s,2}\cdots G_{s,2}\cdots \\ &\qquad \cdots G_{s,|I_s|}\cdots G_{s,|I_s|}]^T \\ \theta &= \frac{\eta^T}{\left[T-\sum_{i\in I}\sum_{k=1}^{K}G_{s,i}p_{i,k}\right]^2} \end{aligned} \tag{16.b}$$

$$\begin{aligned} H_{\sum f_{c,i}(p)} &= \sum_{i=1}^{|I_s|} w_i\, w_i^T, \text{where} \\ w_i &= \left[0\cdots 0\,\sqrt{\varphi_i}\,\sqrt{\varphi_i}\cdots\sqrt{\varphi_i}\,0\cdots 0\right]^T \\ \varphi_i &= \frac{\eta_i^P}{\left[P_{max}-\sum_{k=1}^{K}p_{i,k}\right]^2} \end{aligned} \tag{16.c}$$

Newton's method requires a feasible initial solution (which is an initial power allocation that satisfies the power and interference constraints in this problem). All η's are taken very small so as not to disturb the main objective function, the negative sum of the cell's individual SLNRs, which is to be minimized. Each iteration, the power vector is updated by adding a Newton step ($\Delta\vec{P}$) to the old power vector as follows, n is the index of iteration, and f is the objective function.

$$\begin{aligned} \vec{P}^{n+1} &= \vec{P}^{n}+\Delta\vec{P}, \\ \Delta\vec{P} &= -(\nabla^2 f)^{-1}\nabla f. \end{aligned} \tag{17}$$

If any violation of constraints occurs, the Newton step is divided by a positive real number β until a feasible point is reached in the optimal direction (defined by the Newton direction). If a feasible point is not reached within a predetermined maximum number of iterations, the algorithm quits and considers the last achievable feasible point to be the optimal point. Generally, the optimization process terminates when the absolute difference between the values of the objective function in two successive iterations is less than some tolerance ϵ. The value of the tolerance is a tradeoff between the optimality of the solution and the number of iterations.

IV. PERFORMANCE EVALUATION

We compare the performance of the proposed schemes with that of a trivial EPA when the user's maximum power is calculated to satisfy the interference constraint statistically, and is divided equally among the assigned RBs. We also compare the results with an optimal centralized power allocation scheme [9] that maximizes the weighted sum rates of the terminals in the system.

Consider an OFDMA-based network of four BSs located at (500,500)m,(-500,500)m, (-500,-500)m, and (500,-500)m respectively. Twenty terminals are uniformly distributed over a 250m × 250m square area centered at the origin, and each terminal is served by the nearest BS. Each terminal and BS has a single Omni-directional antenna. The total number of RBs is 15, and each RB consists of 12 subcarriers. We consider the typical urban macro cell scenario using the WINNER II channel model. The noise power density is -174 dBm/Hz, the maximum power allocated per terminal is 24 dBm, and the total power leaked by each cell on the neighboring cells is constrained by an interference limit, which we vary in our simulations. The results are averaged over 30 runs each run has different user positions and 100 samples of channel variations.

Fig.1 shows the total spectral efficiency of the suboptimal, the optimal power allocation, EPA, and centralized scheme with PF RB assignment as function of interference limit. Fig. 2 shows the minimum and maximum spectral efficiency per user for the same setup. The proposed schemes exhibit better performance than EPA especially at low interference limit, when power should be limited. They also provide good performance compared with the centralized power allocation when the weight of each terminal is taken to be the inverse of its average rate (PF).

Fig.3 and Fig.4 show the spectral efficiency per user distribution in bits/sec/Hz at interference limits -90 dBm and -110 dBm respectively. As shown, when the interference limit is -90 dBm the EPA distribution is not very different from the other distributions because the terminals are allowed to transmit with maximum power. This is also clear in Fig.1 as the spectral efficiency is almost insensitive to variations in interference limit beyond -90 dBm. On the other hand, when the interference limit decreases, the EPA is incapable of achieving high spectral efficiencies per user unlike the other schemes which allows some terminals to achieve higher rates at the expense of other terminals which, in turn, increases the total spectral efficiency than its value in EPA. Fairness among terminals is achieved using the PF allocation algorithm.

Fig.5 shows the 10-percentile spectral efficiency. It is also evident that at high interference values there is not much difference between schemes, however at low interference limits the most unlucky 10% of terminals have a better chance of having higher rates in the EPA at the expense of the spectral efficiency of the system.

We also note from Fig.1, Fig.2, and Fig.5 that the performances of the proposed schemes saturate at high values of the interference limit. This is because the only effect of the interference limit in the proposed problem formulation is limiting the maximum power allocated to users. For this setup, all terminals are allowed to transmit with maximum power for high interference limits starting from -90dBm, consequently the performance is almost insensitive to variations in interference limit beyond this value.

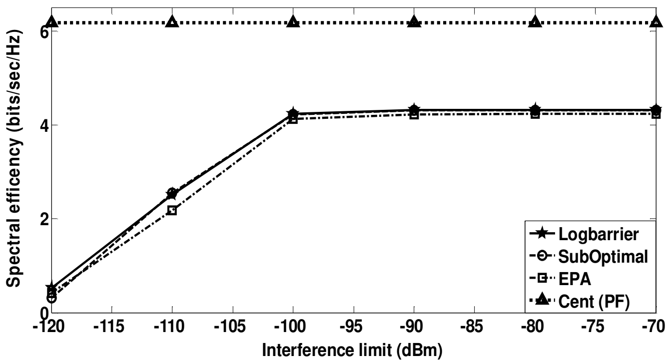

Figure 1. System spectral efficiency.

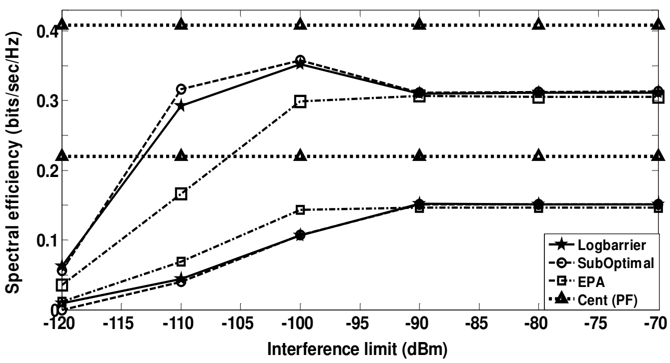

Figure 2. Maximum and minimum spectral efficiency per user.

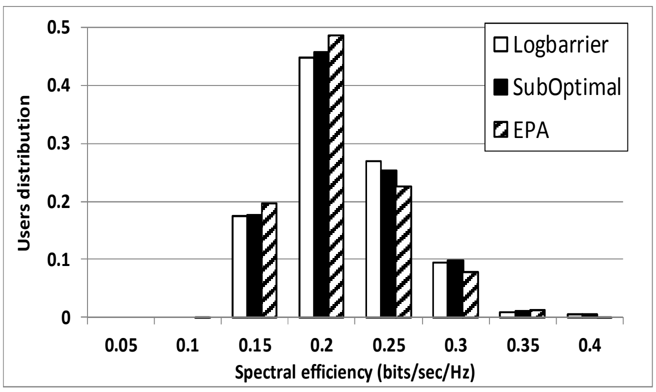

Figure 3. Users distribution at -90 dbm.

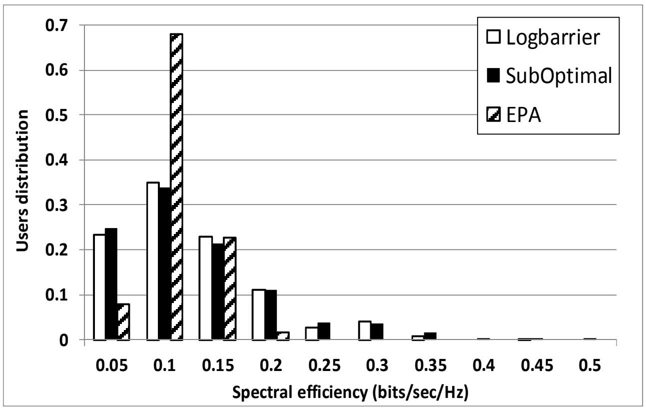

Figure 4. Users distribution at -110 dbm.

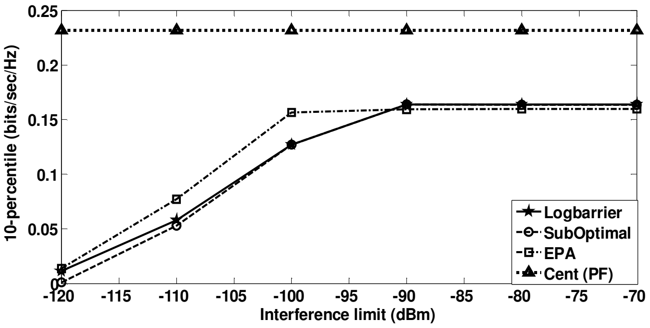

Figure 5. 10-percentile throughput.

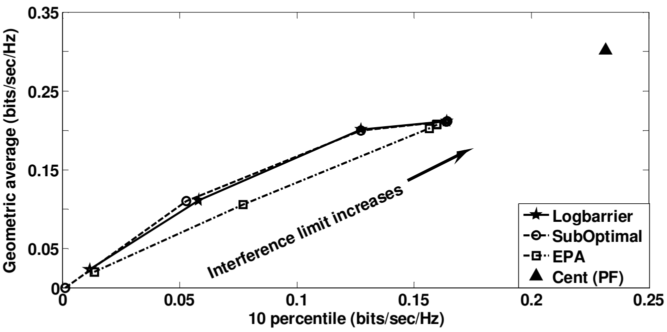

Figure 6. Geometric average Vs 10-percentile throughput.

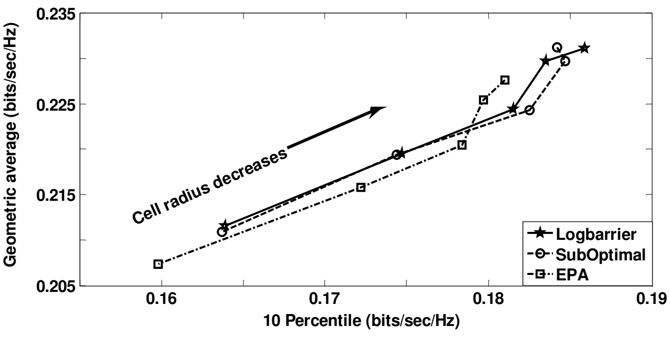

Figure 7. Geometric average Vs 10-percentile for different cell radii

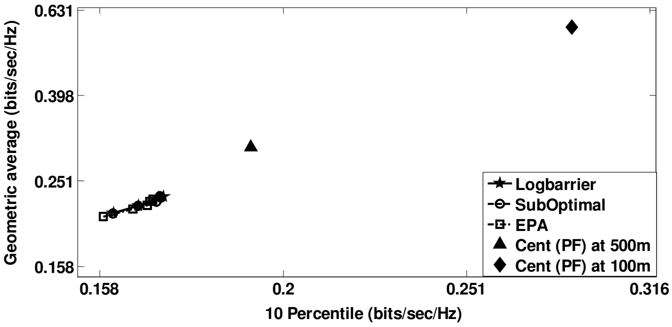

Figure 8. Comparison with centralized at different cell radii

Fig.6 shows the geometric average of the terminals' throughput in bits/sec/Hz versus the 10-percentile throughput in bits/sec/Hz. The geometric average is a good measure of rate utility of the terminals. Both the geometric average and the 10-percentile throughput increase as the interference limit increases. Their values almost saturate at -90 dBm interference limit. For the same 10-percentile throughput, the proposed autonomous schemes exhibit higher spectral efficiency than the EPA.

Fig.7 shows the geometric average versus the 10-percentile throughput for different cell radii (R), from 100m to 500m with step 100m, at interference limit -70 dBm. Users are uniformly distributed over a $0.5R \times 0.5R$ square area centered at the origin. The geometric average and the 10-percentile increase as the cell radius decreases. This is good indication of the resilience of the proposed schemes in high interference systems as in small cells which typically generate higher interference than large cells.

Fig. 8 compares the results shown in Fig.7 with the geometric average and 10 percentile throughput of the centralized algorithm at 500 m and 100 m cell radii. While the centralized optimization-based ICIC provides better performance this comes at a high cost of computations and signaling in the network whereas the proposed scheme solves the problem in a full autonomous manner and in the case of the semi-optimal scheme, has very low computation complexity as well.

V. CONCLUSIONS

This paper has proposed a closed form suboptimal autonomous power allocation in the uplink of OFDMA-based wireless systems. It has also proposed an iterative autonomous power allocation using Newton's method and the log-barrier method. Simulation results have shown that these schemes have better performance than the EPA especially at low interference constraints. They also have acceptable performance compared with centralized power allocation. These power allocation schemes can be adapted to dynamic loads in the network by using the overload indicator (OI) signal in the LTE standard. This signal is triggered when the SINR is less than a certain threshold. The OI should be a good measure to update the interference limit T. This work can be further extended by using a different interference limit for every RB or for every participating cell depending on cell type and required coverage.

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