Simultaneous Connections Routing in W-S-W Elastic Optical Switches with Limited Number of Connection Rates

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***Abstract*—The three-stage switching fabric of wavelength-space-wavelength architecture for elastic optical switches is considered in the paper. It serves connections which can occupy different spectrum width. The upper bound for rearrangeable condition for such switching fabric which serves a limited number of connection rates is derived and proved. The control algorithm based on matrix decomposition is also proposed. For the switching fabric of capacity 2×2 serving only two connection rates, necessary and sufficient conditions are derived and proved. The required number of frequency slot units in interstage links is much lower than in the strict-sense nonblocking switching fabrics.**

***Index Terms*—Elastic optical networks, elastic optical switching nodes, interconnection networks, rearrangeable nonblocking conditions.**

I. Introduction

The Elastic Optical Network (EON), called also the flexible optical networks, is a new kind of telecommunication network which is considered as a promising alternative approach for future high-speed network design [1]–[3]. With the growth of Internet traffic, efficient and cost-effective bandwidth usage is now an important issue. It becomes even more important when traffic in networks changes from static to dynamic, where connections are often set up and disconnected. In EON, spectrum is allocated to a lightpath according to bandwidth requirements of a client. This allows a flexible and efficient use of spectrum resources. Bandwidth assigned to an optical channel depends on the required transmission data rate, distance to be covered, path quality, wavelength spacing between channels, and/or the modulation scheme used [2]–[5]. To allocate bandwidth efficiently, the whole optical spectrum is divided into narrow slots and a different number of slots are assigned to optical connections. The minimum portion of spectrum is often called a *frequency slot unit* (FSU). A spectrum assigned to one connection is called a *frequency slot* and may use m FSUs. One important constrain is that the allocated FSUs must be adjacent. A connection which uses such m FSUs is called an m*-slot connection*.

Connections served in EON must be also served by switching nodes. Several architectures of elastic optical switching nodes were proposed in literature [6]–[9], short survey can be found in [10]. One of these switching fabric architectures is the W-S-W (wavelength-space-wavelength) switching fabric, considered in [11], which is called WSW1 [9]. This architecture will be considered further in this paper.

Up till now, strict-sense nonblocking (SSNB) conditions (necessary and sufficient) for WSW1 have been provided and proved in [11]. In SSNB switching fabrics, we can establish a connection from an idle set of FSUs in any input fiber to an idle set of FSUs in a requested output fiber regardless of how other connections are established. The problem is that such SSNB switching fabrics requires huge number of FSUs in interstage links, especially when the maximum number of FSUs which may be used by one connection is high.

Usually, SSNB switching fabrics require a large number of switching elements (crosspoints, centers stage switches, etc.). This number can be reduced by using rearrangements. In rearrangeably nonblocking (RNB) switching fabrics, we can also connect any pair of idle input and output, however, it may be necessary to move existing connections to alternate connecting paths [12], [13]. RNB switching fabrics are mostly used in packet switching, where packets in synchronously operated (slotted) networks arrive at all inputs at the same time. A model describing such requests is called a simultaneous connection model. RNB conditions for space-division three-stage Clos switching fabrics [14] were derived in [12], [15]. In [16], RNB conditions in time-division switching networks with single-rate connections were considered. In [17], RNB conditions for switching networks with multirate connections were given.

In this paper, we extend the RNB concept to elastic optical switching fabrics. We propose the matrix model for state representation and derive the RNB conditions for switching fabrics serving connections with two rates. We also propose the control algorithm, based on matrix decomposition, which allows to set up a set of compatible connections.

The rest of the paper is organized as follows. In Section II, the switching fabirc is presented and the problem is described in more detail. In Section III, the model used in the paper is reported. In Section IV, the control algorithm proposed in the paper is described. In Section V, sufficient conditions for rearrangeability of WSW1 switching fabrics are given, and in case of 2×2 networks serving two-rate connections necessary and sufficient conditions are derived and proved. Short comparison with SSNB switching fabrics is also provided. The paper ends with Conclusions.

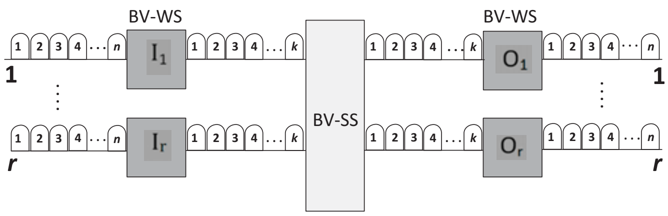

Fig. 1. The WSW1 switching fabric architecture.

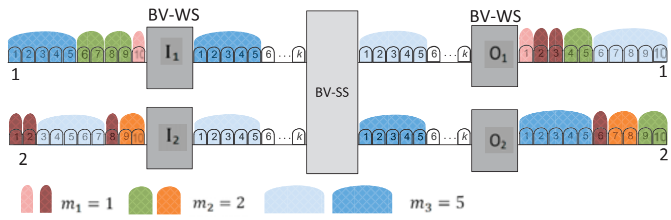

Fig. 2. The 2×2 WSW1 switching fabric with $\mathbb{C} = \{(I_1[1],\ O_2[1],\ 5); (I_1[6],\ O_1[4],\ 2); (I_1[8],\ O_2[9],\ 2); (I_1[10],\ O_1[1],\ 1); (I_2[1],\ O_1[2],\ 1); (I_2[2],\ O_2[6],\ 1); (I_2[3],\ O_1[6],\ 5); (I_2[8],\ O_1[3],\ 1); (I_2[9],\ O_2[7],\ 2)\}$.

II. Problem Statement

An architecture of the WSW1 switching fabric is shown in Fig. 1 [11]. It consists of r bandwidth-variable spectrum converting switches (BV-WSs) in the first and third stages, and one bandwidth-variable wavelength selective space switches (BV-SSs) of capacity $r \times r$ in the second stage. Each BV-WS in the first stage has one input fiber with n FSUs and one output fiber with k FSUs, while each BV-WS in the third stage has one input fiber with k FSUs and one output fiber with n FSUs. The internal architecture of BV-WSs and BV-SS can be found in [11]. BV-WSs in each stage are numbered from 1 to r, FSUs in input/output fibers – from 1 to n, and FSUs in interstage fibers – from 1 to k (see Fig. 1).

The WSW1 switching fabric serves m-slot connections. Usually, m is limited to a maximum value $m_{\max}$, i.e., not more than $m_{\max}$ FSUs can be occupied by a single connection. We assume that BV-WSs have full range conversion capability, i.e., an m-slot connection which uses a set of m adjacent FSUs in the input fiber can be switched to a set of any other m adjacent FSUs in the output fiber. Let us denote an m-slot connection from the input fiber of switch I_i to the output fiber of switch O_j by $(I_i,\ O_j,\ m)$. When it is important which FSUs are assigned to a connection, the number of the first FSU will be also provided. For instance, $(I_1[1],\ O_r[n{-}m{+}1],\ m)$ denotes an m-slot connection from the input fiber of switch I_1 to the output fiber of switch O_r, and it uses FSUs numbered from 1 to m in switch I_1 and from $n - m + 1$ to n in switch O_r.

When a new connection $(I_i,\ O_j,\ m)$ arrives to the switching fabric, a control algorithm must find a set of m adjacent FSUs in interstage links which can be used for this connection, and these must be FSUs with the same numbers in the interstage links from I_i and to O_j, since BV-SS has no spectrum conversion capabilities. When connection requests arrive to the switching fabric sequentially (one-at-a-time connection model), SSNB conditions were derived and proved in [11]. In this paper, the simultaneous connection model is considered. We assume that we have a set of compatible connection requests, i.e., connection requests occupy all FSUs in input and output fibers. Such set of connections is denoted by $\mathbb{C}$, and example of such set in 2×2 switching fabric with $n = 10$ is shown in Fig. 2. Nine connections of three types are to be set up: two 5-slot connections, three 2-slot connections, and four 1-slot connections. The problem now is, which FSUs in interstage links should be used by these connections, and how many FSUs are needed to set up all these connections, i.e., when the switching fabric is RNB. For instance, connections $(I_1[1],\ O_2[1],\ 5)$ and $(I_2[3],\ O_1[6],\ 5)$ are directed from different input switches to different output switches, so they are not in conflict in the BV-SS and they can be set up using the same FSUs numbered from 1 to 5 in interstage links (what is also shown in Fig. 2).

The problem of routing $\mathbb{C}$ is in some sense similar to the routing problem in the three-stage Clos network, however, the main differences are:

1) instead of finding connections which can be set up though one center stage switch, we have to find connections which can be set up using the same set of FSUs in interstage link,
2) connections occupy different number of FSUs which must be adjacent.

For these reasons, algorithms and solutions used for Clos networks cannot be directly used in the considered switching fabric. Proposition of the model which can be used in the WSW1 switching fabrics, a control algorithm, and RNB conditions are the subject of this paper.

III. Model Description

Let $\mathbb{C}$ is to be set up in the WSW1 switching fabric. Let also the number of connection rates be limited by z, i.e., there are only m_x-slot connections, $1 \leqslant x \leqslant z$. For instance, in the set of connection requests presented in Fig. 2, $z = 3$, $m_1 = 1$, $m_2 = 2$, and $m_3 = 5$. In contrary to the space-division three-stage Clos networks, for which one connection matrix was used to represent a set of connection requests, we use z connection matrices, denoted by H^{m_x}, each matrix represents connection requests of one connection rate. Matrix H^{m_x} represents m_x-slot connection requests and it is defined as follows:

$$\mathrm{H}^{m_x} = \begin{bmatrix} h_{11}^{m_x} & h_{12}^{m_x} & \dots & h_{1r}^{m_x} \\ h_{21}^{m_x} & h_{22}^{m_x} & \dots & h_{2r}^{m_x} \\ \vdots & \vdots & \ddots & \vdots \\ h_{r1}^{m_x} & h_{r2}^{m_x} & \dots & h_{rr}^{m_x} \end{bmatrix} \tag{1}$$

where $h_{ij}^{m_x}$ is equal to the number of m_x-slot connection requests from switch I_i to switch O_j. For instance, connection matrices for the set of connection requests in Fig. 2 are as follows:

$$\mathrm{H}^{m_1} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, \quad \mathrm{H}^{m_2} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad \mathrm{H}^{m_3} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}. \tag{2}$$

The set of matrices H^{m_x} has the following properties: for each row i

$$\sum_{j=1}^{r} \left\{ \sum_{x=1}^{z} \left(h_{ij}^{m_x} \cdot m_x \right) \right\} = n, \tag{3}$$

and for each column j

$$\sum_{i=1}^{r}\Big\{\sum_{x=1}^{z}\left(h_{ij}^{m_x}\cdot m_x\right)\Big\}=n. \tag{4}$$

Equation (3) says, that the sum of FSUs used by all connection requests from one input fiber are equal to n. This is true since in a set of compatible connection requests all FSUs in each input fiber are occupied. Equation (4) provide the same condition for the output fiber. It says that FSUs used by connection requests to one output fiber must be also equal to n, which is also true for the set of compatible connection requests.

IV. Control Algorithm

The problem of assigning FSUs to a particular connection requests will be solved by using the matrix decomposition algorithm. It is known that the square matrix H can be decomposed into n permutation matrices when the sum of elements in each row and each column is equal to n [12], [15]. In our case, however, this condition is not true. Therefore, some modifications are needed. Let us introduce some more terms and notation:

- $a_i^{m_x}$ denotes the number of m_x-slot connection requests at input i: $a_i^{m_x}=\sum_{j=1}^{r}h_{ij}^{m_x}$,
- $b_j^{m_x}$ denotes the number of m_x-slot connection requests at output j: $b_j^{m_x}=\sum_{i=1}^{r}h_{ij}^{m_x}$,
- $a_{\max}^{m_x}$ denotes the maximum number of m_x-slot connection requests at one input: $a_{\max}^{m_x}=\max\limits_{1\leqslant i\leqslant r}\{a_i^{m_x}\}$,
- $a_{\min}^{m_x}$ denotes the minimum number of m_x-slot connection requests at one input: $a_{\min}^{m_x}=\min\limits_{1\leqslant i\leqslant r}\{a_i^{m_x}\}$
- $b_{\max}^{m_x}$ denotes the maximum number of m_x-slot connection requests at one output: $b_{\max}^{m_x}=\max\limits_{1\leqslant j\leqslant r}\left\{b_j^{m_x}\right\}$
- $b_{\min}^{m_x}$ denotes the minimum number of m_x-slot connection requests at one output: $b_{\min}^{m_x}=\min\limits_{1\leqslant j\leqslant r}\left\{b_j^{m_x}\right\}$
- $c_{\max}^{m_x}=\max\left\{a_{\max}^{m_x};b_{\max}^{m_x}\right\}$
- $c_{\min}^{m_x}=\min\left\{a_{\min}^{m_x};b_{\min}^{m_x}\right\}$

The decomposition algorithm is presented in Algorithm 1. Decomposition of each matrix can be executed separately one after another, or may be run in parallel. The number of connection rates and the number of FSUs in each connection rate are the network parameters and they should be known at the stage of switching fabric design. The set of compatible connection requests is the input data for the algorithm. For each connection rate, H^{m_x} is calculated in the first step. In the next step, $a_i^{m_x}$ and $b_j^{m_x}$ are calculated for each row and column, respectively, and if these values are equal, the algorithm starts the decomposition process. If these numbers are not equal, "dummy" connection requests are added such that these sums will be equal to $\max\{a^{m_x};b^{m_x}\}$. Matrix H^{m_x} is then decomposed into $\max\{a^{m_x};b^{m_x}\}$ permutation matrices P^{m_x} using any of known algorithms [18]–[20], and finally, from some matrices elements representing "dummy" connection requests have to be removed. Decomposition of one matrix is realized in $\mathrm{O}(nr^4)$ time [12], so the time complexity of the proposed algorithm is not greater than $\mathrm{O}(znr^4)$ – we have to decompose z matrices.

Algorithm 1: Decomposition algorithm

Data: $\mathbb{C}$
Result: a set of permutation matrices

```
for x = 1 to z do
    Calculate H^{m_x};
    In H^{m_x}, add "dummy" connection requests such that
        a_i^{m_x} = b_i^{m_x} = max{a^{m_x}; b^{m_x}};
    Decompose H^{m_x} matrix into max{a^{m_x}; b^{m_x}}
        permutation matrices P_i^{m_x};
    Remove "dummy" connection requests from P_i^{m_x};
end
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How the algorithm works we show using example from Fig. 2. Matrices H^{m_x} are given by equations (2). For H^{m_3} we have $a_1^{m_3}=a_2^{m_3}=b_1^{m_3}=b_2^{m_3}=1$, and H^{m_3} is $\mathrm{P}_1^{m_3}$, i.e.,

$$\mathrm{H}^{m_3}=\mathrm{P}_1^{m_3}=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}. \tag{5}$$

As a result, connections $(I_1[1],\ O_2[1],\ 5)$ and $(I_2[3],\ O_1[6],\ 5)$ will be realized through FSUs from 1 to 5 of interstage links. For H^{m_2} we have: $a_1^{m_2}=b_2^{m_2}=2$ and $a_2^{m_2}=b_1^{m_2}=1$ (see equation (2)). In order to decompose this matrix, we added one "dummy" connection in position $h_{21}^{m_2}$ (positions, where such connections are added, are marked by a gray circle), so we get:

$$\mathrm{H}^{m_2}=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}. \tag{6}$$

This matrix can be decomposed into two permutation matrices:

$$\mathrm{P}_1^{m_2}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\quad \mathrm{P}_2^{m_2}=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\Longrightarrow\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}, \tag{7}$$

and in $\mathrm{P}_2^{m_2}$ we have to remove the "dummy" connection. As the result, $\mathrm{P}_1^{m_2}$ corresponds to connections $(I_1[6],\ O_1[4],\ 2)$ and $(I_2[9],\ O_2[7],\ 2)$ which will be set up through FSUs 6 and 7, and $\mathrm{P}_2^{m_2}$ – to connection $(I_1[8],\ O_2[9],\ 2)$ with assigned FSUs 8 and 9. Finally, in H^{m_1} we have: $a_1^{m_1}=b_2^{m_1}=1$ and $a_2^{m_1}=b_1^{m_1}=3$. This time we have to add two "dummy" connections in position $h_{12}^{m_1}$ and we have:

$$\mathrm{H}^{m_1}=\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}, \tag{8}$$

and this matrix can be decomposed into three permutation matrices:

$$\mathrm{P}_1^{m_1}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\quad \mathrm{P}_2^{m_1}=\mathrm{P}_3^{m_1}=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}. \tag{9}$$

This means that connections $(I_1[10],\ O_1[1],\ 1)$ and $(I_2[2],\ O_2[6],\ 1)$ will be set up using FSU 10 in interstage links, connection $(I_2[1],\ O_1[2],\ 1)$ – using FSU 11 in interstage links, and connection $(I_2[8],\ O_1[3],\ 1)$ – using FSU 12 in interstage links. The switching fabric of Fig. 2 with marked connections is shown in Fig. 3.

In the considered example, the required number of FSUs in interstage links is 12 ($k=12$). How many FSUs are sufficient to have WSW1 switching fabric rearrangeably nonblocking will be derived in the next Section.

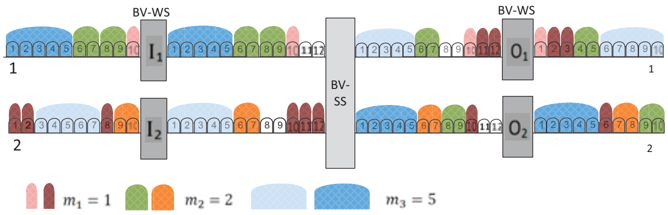

Fig. 3. The 2×2 WSW1 switching fabric of Fig. 2 with $\mathbb{C}$ set up through 12 FSUs according to Algorithm 1.

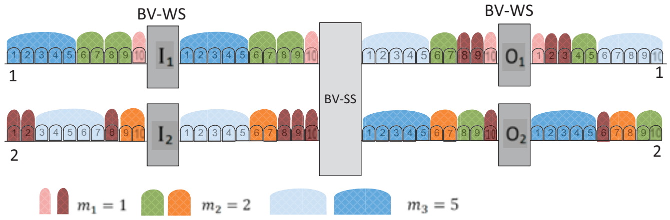

Fig. 4. The 2×2 WSW1 switching fabric of Fig. 2 with $\mathbb{C}$ set up through 10 FSUs.

V. Rearrangeable Conditions

First, we prove the sufficient condition for RNB of WSW1 in case $m \in \{m_x\}$, $1 \leqslant x \leqslant z$ and $r \geqslant 2$.

***Theorem** 1:* The WSW1 switching fabric presented in Fig. 1 is RNB for m-slot connections, $m \in \{m_x\}$ and $1 \leqslant x \leqslant z$, if:

$$k \geqslant \sum_{x=1}^{z} \left(\left\lfloor \frac{n}{m_x} \right\rfloor \cdot m_x \right). \tag{10}$$

Proof: Let $\mathbb{C}$ denote a set of compatible connections. We have z connection rates, $1 \leqslant x \leqslant z$, ordered in the ascending order: $m_1 < m_2 < \ldots < m_z$. A set of m_x-slot connections in $\mathbb{C}$ is represented by H^{m_x}. According to Algorithm 1, H^{m_x} can be decomposed into $c_{\max}^{m_x}$ permutation matrices $\mathrm{P}_i^{m_x}$. One $\mathrm{P}_i^{m_x}$ represents a set of m_x-slot connections which can be set up using the same m_x FSUs in interstage links. The total number of FSUs occupied by these connections is equal to $m_x \cdot c_{\max}^{m_x}$. This means that the upper bound for the required number of FSUs in each interstage link is:

$$k \geqslant \sum_{x=1}^{z} \left(m_x \cdot c_{\max}^{m_x} \right). \tag{11}$$

The value of $c_{\max}^{m_x}$ differs in different $\mathbb{C}$s. Since it represents the maximum number of m_x-slot connections in one of inputs or outputs, and the number of such connections in one input/output will never be greater than $\lfloor n/m_x \rfloor$, we can conclude that the following inequality is true:

$$c_{\max}^{m_x} \leqslant \left\lfloor \frac{n}{m_x} \right\rfloor. \tag{12}$$

From (11) and (12) we have:

$$k \geqslant \sum_{x=1}^{z} \left(m_x \cdot c_{\max}^{m_x} \right) \geqslant \sum_{x=1}^{z} \left(\left\lfloor \frac{n}{m_x} \right\rfloor \cdot m_x \right). \tag{13}$$

∎

The condition presented in *Theorem 1* is the upper bound. The lower bound is at least $k \geqslant n$, since we need at least n FSUs in interstage links to set connections which occupy n FSUs in input fibers. The upper bound assumes, that each set of m_x-slot connections is set up through separate FSUs in interstage links. This upper bound can be reduced when some of FSUs assigned for connections of one size, say m_{x_1}, can be used by connections of another rate, say m_{x_2}, $m_{x_2} < m_{x_1}$. For instance, in the example presented in Fig. 3 connection $(I_1[8],\ O_2[9],\ 2)$ which is represented by matrix $\mathrm{P}_2^{m_2}$ uses FSUs 8 and 9 in interstage links. This means that these FSUs are occupied in the interstage link from switch I_1 and to switch O_2, and these FSUs can be used by connections from switch I_2 to switch O_1 (in $\mathrm{P}_2^{m_2}$, $h_{21}^{m_2} = 0$). We can use these FSUs for setting up 1-slot connections represented by permutation matrices $\mathrm{P}_2^{m_1}$ and $\mathrm{P}_3^{m_1}$. The final FSUs assignment after the before mentioned changes is presented in Fig. 4. The total number of FSUs in interstage links is now reduced from 12 to 10 in interstage links.

The number of permutation matrices which, in general case, can be merged together, and connections represented by them can be set up using the same FSUs, is currently unknown. We will investigate this problem in the future. Up till now we were able to reduce the upper bound only for the special case, when $r = z = 2$, and $\frac{n}{m_1}$, $\frac{n}{m_2}$, and $\frac{m_2}{m_1}$ are integers.

***Theorem** 2:* The WSW1 switching fabric presented in Fig. 1 with $r = 2$ is rearangeably nonblocking for m-slot connections, where $m \in \{m_1; m_2\}$, $m_1 < m_2$, $\frac{n}{m_1}$, $\frac{n}{m_2}$, and $\frac{m_2}{m_1}$ are integers, is RNB if and only if:

$$k \geqslant n. \tag{14}$$

Proof: Necessity is obvious, since we need at least n FSUs in interstage links to serve all connections which occupy n FSUs in each input fiber. The sufficiency can be proved by showing, that no such $\mathbb{C}$ exists which requires more than n FSUs. In general, $\mathbb{C}$ is represented by connection matrices:

$$\mathrm{H}^{m_1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad \mathrm{H}^{m_2} = \begin{bmatrix} e & f \\ g & h \end{bmatrix}. \tag{15}$$

From properties (3) and (4) we receive following equations:

$$(a+b) \cdot m_1 + (e+f) \cdot m_2 = n, \tag{16}$$

$$(c+d) \cdot m_1 + (g+h) \cdot m_2 = n, \tag{17}$$

$$(a+c) \cdot m_1 + (e+g) \cdot m_2 = n, \tag{18}$$

$$(b+d) \cdot m_1 + (f+h) \cdot m_2 = n. \tag{19}$$

The number of permutation matrices representing connections of one connection rate, which cannot be set up through the same FSUs in interstage links with connections of another connection rate (i.e., they will contains, exactly one element 1 in each row and each column), is equal to $c_{\min}^{m_x}$. Let us assume that $c_{\min}^{m_1} = (a+b)$. This means that $(a+b) \leqslant (c+d)$ and, from equations (16) and (17), we have $(g+h) \leqslant (e+f)$, i.e., $c_{\min}^{m_2} = (g+h)$. As the result, H^{m_1} can be divided into $(a+b)$ permutation matrices and H^{m_2} into $(g+h)$ such matrices. After these decomposition, we get:

$$\mathrm{H}_1^{m_1} = \mathrm{H}^{m_1} - \sum_{i=1}^{a+b} \mathrm{P}_i^{m_1} = \begin{bmatrix} 0 & 0 \\ c' & d' \end{bmatrix} \tag{20}$$

$$\mathrm{H}_1^{m_2} = \mathrm{H}^{m_2} - \sum_{i=1}^{g+h} \mathrm{P}_i^{m_2} = \begin{bmatrix} e' & f' \\ 0 & 0 \end{bmatrix}, \tag{21}$$

where $(c' + d') = (c + d) - (a + b)$ and $(e' + f') = (e + f) - (g + h)$. The same relationships are true if sum of any other row or column is the minimum. It also means, that there is no situation when the minimum value in one matrix is for a row and in another matrix for a column. In general, we may have four cases after first part of decomposition, depending on which sum of elements, in row or column, is the minimum:

$$\text{case 1:} \quad \begin{bmatrix} 0 & 0 \\ c' & d' \end{bmatrix} \begin{bmatrix} e' & f' \\ 0 & 0 \end{bmatrix}, \qquad \text{case 2:} \quad \begin{bmatrix} a' & b' \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ g' & h' \end{bmatrix},$$

$$\text{case 3:} \quad \begin{bmatrix} 0 & b' \\ 0 & d' \end{bmatrix} \begin{bmatrix} e' & 0 \\ g' & 0 \end{bmatrix}, \qquad \text{case 4:} \quad \begin{bmatrix} a' & 0 \\ c' & 0 \end{bmatrix} \begin{bmatrix} 0 & f' \\ 0 & h' \end{bmatrix}.$$

Matrices $\mathrm{P}_i^{m_1}$ and $\mathrm{P}_i^{m_2}$ determines which FSUs will be used for setting up $a + b$ m_1-slot connections and $g + h$ m_2-slot connections, respectively, and in matrices $\mathrm{H}_1^{m_1}$ and $\mathrm{H}_1^{m_2}$ there are only connections not set up yet. These connections will use $n - (a + b) \cdot m_1 - (g + h) \cdot m_2$ FSUs, and according to properties (3) and (4), for matrices $\mathrm{H}_1^{m_1}$ and $\mathrm{H}_1^{m_2}$ we have:

$$(e' + f') \cdot m_2 = n - (a + b) \cdot m_1 - (g + h) \cdot m_2, \tag{22}$$

$$(c' + d') \cdot m_1 = n - (a + b) \cdot m_1 - (g + h) \cdot m_2, \tag{23}$$

$$c' \cdot m_1 + e' \cdot m_2 = n - (a + b) \cdot m_1 - (g + h) \cdot m_2, \tag{24}$$

$$d' \cdot m_1 + f' \cdot m_2 = n - (a + b) \cdot m_1 - (g + h) \cdot m_2. \tag{25}$$

From equations (22) and (23) we get:

$$(c' + d') \cdot m_1 = (e' + f') \cdot m_2, \tag{26}$$

and from equations (24) and (25) we get:

$$(c' - d') \cdot m_1 = (f' - e') \cdot m_2. \tag{27}$$

After some operations on equations (26) and (27) we come to the following dependencies:

$$c' = f' \cdot \frac{m_2}{m_1}, \tag{28}$$

$$d' = e' \cdot \frac{m_2}{m_1}. \tag{29}$$

From equations (28) and (29), we can conclude that to each matrix $\mathrm{P}_i^{m_2}$ obtained by decomposing $\mathrm{H}_1^{m_2}$ we can find $\frac{m_2}{m_1}$ matrices $\mathrm{P}_i^{m_1}$ such that a connection corresponding to $\mathrm{P}_i^{m_2}$ and connections corresponding to $\mathrm{P}_i^{m_1}$ can be set up through the same FSUs of different interstage links. We say that matrix $\mathrm{P}_i^{m_2}$ can be merged with matrices $\mathrm{P}_i^{m_1}$. For instance, let

$$\mathrm{P}_{g+h+1}^{m_2} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{30}$$

which corresponds to $(I_1,\ O_2,\ m_2)$. This matrix can be merged with $\frac{m_2}{m_1}$ matrices

$$\mathrm{P}_{a+b+1}^{m_1} = \cdots = \mathrm{P}_{a+b+\frac{m_2}{m_1}}^{m_1} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \tag{31}$$

which correspond to $\frac{m_2}{m_1}$ connections $(I_2,\ O_1,\ m_1)$, and all these connections can use the same FSUs in interstage links. After the merging operation we will have $(c' + d') = \frac{m_2}{m_1} \cdot (e' + f')$ permutation matrices, each using m_1 FSUs. In total, connections represented by all permutation matrices use not more than:

$$(a + b) \cdot m_1 + (g + h) \cdot m_2 + (c' + d') \cdot m_1 = n \tag{32}$$

FSUs, i.e., $k \geqslant n$ is sufficient to set up any $\mathbb{C}$, i.e., the switching fabric is RNB. ■

Example 1. As an example, let us consider the 2×2 WSW1 switching fabric with $n = 16$, $z = 2$, $m_1 = 1$, $m_2 = 3$, and $\mathbb{C} = \{(I_1[1],\ O_1[1],\ 1);\ (I_1[2],\ O_1[2],\ 1);\ (I_1[3],\ O_1[3],\ 1);\ (I_1[4],\ O_2[1],\ 3);\ (I_1[7],\ O_2[4],\ 1);\ (I_1[8],\ O_2[5],\ 3);\ (I_1[11],\ O_2[8],\ 3);\ (I_1[14],\ O_2[11],\ 3);\ (I_2[1],\ O_1[7],\ 1);\ (I_2[2],\ O_1[8],\ 1);\ (I_2[3],\ O_1[9],\ 1);\ (I_2[4],\ O_1[10],\ 1);\ (I_2[5],\ O_1[11],\ 1);\ (I_2[6],\ O_1[12],\ 1);\ (I_2[7],\ O_1[13],\ 1);\ (I_2[8],\ O_1[4],\ 3);\ (I_2[11],\ O_1[14],\ 3);\ (I_2[14],\ O_2[14],\ 3)\}$. $\mathbb{C}$ is represented by matrices $\mathrm{H}^{m_1} = \begin{bmatrix} 3 & 1 \\ 7 & 0 \end{bmatrix}$ and $\mathrm{H}^{m_2} = \begin{bmatrix} 0 & 4 \\ 2 & 1 \end{bmatrix}$.

The number of permutation matrices for H^{m_1}, which after decomposition cannot be merged with other matrices, is equal to $c_{\min}^{m_1} = (b + d) = 1$. This means that $(b + d) \leqslant (a + c)$, and because of equations (18) and (19), we have $(e + g) \leqslant (f + h)$, so $c_{\min}^{m_2} = (e + g)$, and such is in this example, since $c_{\min}^{m_2} = (e + g) = 2$. Matrix H^{m_1} can be divided into $(b + d) = 1$ permutation matrix and H^{m_2} – into $(e + g) = 2$ permutation matrices. For H^{m_1}, $\mathrm{P}_1^{m_1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, we get $\mathrm{H}_1^{m_1} = \mathrm{H}^{m_1} - \mathrm{P}_1^{m_1} = \begin{bmatrix} 3 & 0 \\ 6 & 0 \end{bmatrix}$. For H^{m_2} the permutation matrices are $\mathrm{P}_1^{m_2} = \mathrm{P}_2^{m_2} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, and we get $\mathrm{H}_1^{m_2} = \mathrm{H}^{m_2} - \mathrm{P}_1^{m_2} - \mathrm{P}_2^{m_2} = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}$. We can notice that H^{m_1} and H^{m_2} correspond to case 4, i.e., matrices $\begin{bmatrix} a' & 0 \\ c' & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & f' \\ 0 & h' \end{bmatrix}$. We have $c' = 6$ and $f' = 2$ so equation (28) is true, and similarly, since $d' = 3$ and $e' = 1$, equation (29) is also true. Matrix $\mathrm{H}_1^{m_2}$ can be further decomposed into matrices $\mathrm{P}_3^{m_2} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\mathrm{P}_4^{m_2} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, and $\mathrm{P}_5^{m_2} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, and $\mathrm{H}_1^{m_1}$ – into matrices $\mathrm{P}_2^{m_1} = \mathrm{P}_3^{m_1} = \mathrm{P}_4^{m_1} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\mathrm{P}_5^{m_1} = \mathrm{P}_6^{m_1} = \mathrm{P}_7^{m_1} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, and $\mathrm{P}_8^{m_1} = \mathrm{P}_9^{m_1} = \mathrm{P}_{10}^{m_1} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$. Matrix $\mathrm{P}_3^{m_2}$, which corresponds to one 3-slot connection, can be merged with $\frac{m_2}{m_1} = 3$ matrices (they correspond to three 1-slot connections represented in $\mathrm{H}_1^{m_1}$), and these may be matrices $\mathrm{P}_5^{m_1}$, $\mathrm{P}_6^{m_1}$, and $\mathrm{P}_7^{m_1}$. Similarly, $\mathrm{P}_4^{m_2}$ can be merged with $\mathrm{P}_8^{m_1}$, $\mathrm{P}_9^{m_1}$, and $\mathrm{P}_{10}^{m_1}$, while $\mathrm{P}_5^{m_2}$ – with $\mathrm{P}_2^{m_1}$, $\mathrm{P}_3^{m_1}$, and $\mathrm{P}_4^{m_1}$. All connections represented by merged matrices can used the same FSUs in interstage links. After merging, connections will use $(c + d) \cdot m_1 + (e + g) \cdot m_2 + (a' + c') \cdot m_1 = n = 16$ in total, and not more FSUs will be needed. All permutation matrices, connections they represent, and assigned FSUs in interstage links, are listed in Tab. I.

From *Theorem 1* we can see that the upper bound for the number of FSUs in interstage links k depends on z, n, m_x, and do not depend on the number of input/output fibers r. To

TABLE I
ASSIGNMENT OF FSUs FOR CONNECTIONS IN EXAMPLE 1.

Perm. matrix	Merged perm. matr.	Connection	Merged Connection	Assigned FSUs
P_1^{m1}	—	$(I_1[7],\ O_2[4],\ 1)$	—	1
	—	$(I_2[1],\ O_1[7],\ 1)$		
P_1^{m2}	—	$(I_1[4],\ O_2[1],\ 3)$	—	2—4
	—	$(I_2[8],\ O_1[4],\ 3)$		
P_2^{m2}	—	$(I_1[8],\ O_2[5],\ 3)$	—	5—7
	—	$(I_2[11],\ O_1[14],\ 3)$		
P_3^{m2}		$(I_1[11],\ O_2[8],\ 3)$		8—10
	P_5^{m1}		$(I_2[2],\ O_1[8],\ 1)$	8
	P_6^{m1}		$(I_2[3],\ O_1[9],\ 1)$	9
	P_7^{m1}		$(I_2[4],\ O_1[10],\ 1)$	10
P_4^{m2}		$(I_1[14],\ O_2[11],\ 3)$		11—13
	P_8^{m1}		$(I_2[5],\ O_1[11],\ 1)$	11
	P_9^{m1}		$(I_2[6],\ O_1[12],\ 1)$	12
	P_{10}^{m1}		$(I_2[7],\ O_1[13],\ 1)$	13
P_5^{m2}		$(I_2[14],\ O_2[14],\ 3)$		14—16
	P_2^{m1}		$(I_1[1],\ O_1[1],\ 1)$	14
	P_3^{m1}		$(I_1[2],\ O_1[2],\ 1)$	15
	P_4^{m1}		$(I_1[3],\ O_1[3],\ 1)$	16

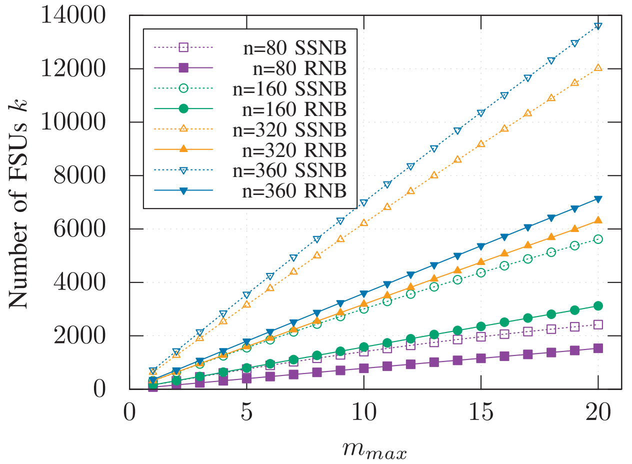

Fig. 5. Number of FSUs k versus $m_{\max}$ for selected n in SSNB and RNB WSW1 switching fabrics.

compare k in RNB and SSNB switching fabrics, we should use the same parameters in both networks, however, in SSNB, conditions are derived for m limited to certain value $m_{\max}$ and in RNB, we have a number of connection rates z. Only for $z = m_{\max}$ we have the same set of connection rates, i.e. $m_1 = 1, m_2 = 2, \ldots, m_z = m_{\max}$, and we can compare RNB with SSNB. In our comparison, we also limit $m_{\max}$ to not more than $n/2$, since when any m_x in RNB is greater than $n/2$, in each input link we may have only one such connection, which correspond to the space-division switching case, and all connections with $m_x > n/2$ can be represented by one connection matrix. Charts presenting the relationship between SSNB and RNB for selected values of n and $m_{\max} \leqslant 20$ are plotted in Fig. 5. When $m_{\max}$ grows, k also grows in both kind of switching fabrics, however, this number is always smaller in RNB for about 50%.

VI. CONCLUSIONS

We considered WSW1 switching fabrics for elastic optical switching nodes. For switching fabrics of capacities $r > 2$ we derived the upper bound for rearrangeablility. This upper bound may be improved, but for now we only did it for the case $r = 2$. For WSW1 with $r = 2$ $k = n$ is necessary and sufficient for the network to be RNB. When $r > 2$, we are able to show (this is not included in the paper), that at least $n + 1$ FSUs in interstage links are needed. The better upper bound in this case are the subject of future work.

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