Constructing k-Connected k-Cover Set in Wireless Sensor Networks Based on Self-Pruning*

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Abstract. Density control is a promising approach to conserving system energy and extending lifetime of wireless sensor networks. Most of previous work in this field has focused on selecting a minimal subset of active sensor nodes for high efficiency while guaranteeing only 1-coverage (or plus 1-connectivity of the network). In this paper, we address the issue of constructing a k-connected k-cover set of a wireless sensor network for fault tolerance and balance efficiency. We propose a distributed, localized algorithm based on self-pruning for selecting active sensor nodes to form a k-connected k-cover set for the target region. The performance of the proposed algorithm is evaluated through numerical experiments.

1 Introduction

Because of advances in micro-sensors, wireless networking and embedded processing, wireless sensor networks (WSN) are becoming increasingly available for commercial and military applications, such as environmental monitoring, chemical attack detection, and battlefield surveillance, etc [1–3].

Energy is the most precious resource in wireless sensor networks due to the following factors. First, the sensor nodes are usually supported by batteries with limited capacity due to the extremely small dimensions. Second, it is usually hard to replace or recharge the batteries after deployment, either because the number of sensor nodes is very large or the deployment environment is hostile and dangerous (e.g. remote desert or battlefield). But on the other hand, the sensor networks are usually expected to operate several months or years once deployed. Therefore reducing energy consumption and extending network lifetime is one of the most critical challenges in the design of wireless sensor networks.

One promising approach to reducing energy consumption is density control, which only keeps a subset of sensors active and puts other sensors into low-powered sleep status. Most of previous researches on density control focus on

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only sensing coverage [4–8, 12]. If a sensor node's sensing area is completely included by its neighbors' sensing coverage, it is redundant and can be turned off safely. These papers don't consider the impact of coverage-scheduling on network connectivity. Some other researches [9–11] consider the coverage and connectivity requirement at the same time. That is, every point in the target region must be covered by at least one active sensor and the communication graph induced by active sensors must be connected. But only 1-coverage and 1-vertex connectivity can be guaranteed.

The k-coverage and k-connectivity properties are desirable in some critical applications. k-coverage and k-connectivity can enhance the robustness and fault-tolerance of the sensor network. Even if $k-1$ sensor nodes fail due to accidental damage or energy depletion, the target region is still completely covered and the communication network is still connected. Therefore the network can survive the failure of at most $k-1$ sensor nodes. And the k-coverage can improve the sensing accuracy. As the sensing function is often interfered with by noise signals, the sensing accuracy can be improved when each point is covered at least by k sensor nodes.When different sensor nodes report the sensed data back to the sink along different routes, the loss of event can be avoided. And in localization applications, the location of a target will be more accurate when it is detected by many sensors from different bearings. Also the k-connectivity can provide more routing flexibility, which is helpful to realize the load balancing of data traffic among sensor nodes.

The major contributions of this paper are as follows. First, we propose a general framework based on self-pruning to construct a k-connected k-cover set. The degree of coverage and connectivity can be flexibly specified in this framework according to application requirements and different algorithms that detect k-connectivity or k-coverage redundancy in a distributed, localized manner can be integrated into the proposed framework. Second, we propose a distributed, localized algorithm to detect whether a sensor node is k-coverage redundant based on order-k Voronoi diagram.

The rest of this paper is organized as follows. The problem addressed in this paper is formulated in section 2. And a general framework and distributed, localized algorithms are proposed in section 3. We present the experimental results in section 4 and end with conclusion remarks in section 5.

2 Problem Formulation

A point p is covered by a sensor node s_i if the distance between p and s_i is not larger than R_s, i.e., $d(s_i, p) \leq R_s$. A point p is k-covered if it is covered by k distinct active sensor nodes. An area R is completely k-covered by a sensor network if every point in R is k-covered by sensor nodes in the networks. Using omni-direction antenna, a sensor node s_i's communication range is a circle centered at s_i with radius R_c. Sensor nodes within s_i's communication rage are called s_i's communication neighbors, which s_i can directly communicate with.

Definition 1. *(communication graph/path) Given a sensor network consisting of a set of sensor nodes,* $S = \{s_1, s_2, \ldots, s_n\}$*, the communication graph of the sensor network* $G_c = (V_c, E_c)$ *is an undirected graph, where* $V_c = S$ *and* $e_{ij} = (s_i, s_j) \in E_c$ *if* $d(s_i, s_j) \leq R_c$*. We say that the communication graph* G_c *is induced by* S*. A communication subgraph induced by a subset of sensor nodes* $S' \subseteq S$ *is the subgraph of* G_c *which only involves sensor nodes in* S'*. A communication path in the communication graph is a sequence of sensors where any two sequential sensors are communication neighbors. A communication graph* G_c *is connected if there is a communication path between any two vertices of* G_c*.*

Definition 2. *(k-connected k-cover set) Consider a sensor network consisting of a set of sensor nodes* $S = \{s_1, s_2, \ldots, s_n\}$ *deployed in a target region* R*. A subset of sensors* $S' \subseteq S$ *is said to be a k-connected k-cover set for* R *if:*

(1) R *is completely k-covered by* S'*, that is, every point in* R *is covered by at least k distinct sensor nodes in* S'*.*
(2) The communication graph induced by S' *is k-vertex connected.*

Minimal k-Connected k-Cover Set (MKCC) Problem: Given a sensor network consisting of a set of sensor nodes S deployed in a target region R, where S is a k-connected k-cover set for R when all sensor nodes are active. The minimal k-Connected k-Cover Set problem is to find a k-connected k-cover subset $S' \subseteq S$ with the minimal cardinality.

The **MKCC** problem is $\mathcal{NP}$-hard as it is a generalization of the minimal 1-connected 1-coverage problem, which is already known to be $\mathcal{NP}$-hard [9].

3 Distributed and Localized Algorithm Based on Self-Pruning

3.1 Basic Framework

The distributed, localized self-pruning algorithm is based on the following idea. A sensor node s_i can be safely turned off if its removal will not destroy the k-coverage and k-connectivity properties of the network. That is, the remaining sensor nodes after removing s_i from the sensor network still form a k-connected k-cover set for the target region. Sensor node s_i is not needed for k-connectivity if every pair of its one-hop neighbors has k alternate replacement communication paths not involving s_i. And sensor node s_i is not needed for k-coverage if each point in its coverage area is covered by at least k other sensors. When a sensor node satisfies both the above two conditions simultaneously, its removal will still preserve the k-connectivity and k-coverage characteristics of the sensor network. When several nodes rely on each other to satisfy the above two conditions, node priorities are used to resolve the cyclic dependency. And to limit the communication overhead in a reasonable level, each node makes its own decision based on neighborhood information only within l communication hops, where l is a small integer (about 2 or 3). Although the partial neighborhood information may generate incomplete communication graph and incorrect Voronoi diagram and thus cause more sensors than optimal to be active, the properties of k-connectivity and k-coverage are still guaranteed.

In this framework, the required connectivity degree and coverage level can be specified separately and arbitrarily according to application requirements. And also any algorithm for detecting k-connectivity redundancy and k-coverage redundancy in a distributed and localized manner can be integrated into this framework.

3.2 Algorithm Description

A. k-Connectivity Redundant Condition

A sensor node s_i is not needed for preserving the k-connectivity property of the sensor network S if it is k-connectivity redundant. We denote the set of remaining sensors after removing s_i from S by $S \backslash s_i$.

Definition 3. *(k-connectivity redundant) A sensor node s_i is k-connectivity redundant if the communication graph induced by $S \backslash s_i$ is still k-connected.*

k-Connectivity Redundant Condition: A sensor node s_i is k-connectivity redundant if for any two one-hop neighbors s_n and s_m of s_i, there are k node disjoint replacement paths connecting s_n and s_m via several intermediate nodes in $N_l(i)$ (if any) with lower priority than s_i, where $N_l(i)$ is node s_i's l-hop communication neighbors.

The node priority can be any combination of the remaining energy, node id, and random numbers. The only requirement is that the priority should be able to set up a total order among all sensor nodes so as to resolve the cyclic dependent relationship among neighbors. In paper [13], Wu et al. use a similar condition to construct a k-CDS for MANET.

B. k-Coverage Redundant Condition

A sensor node s_i is not needed for preserving the k-coverage property of the target region if it is k-coverage redundant.

Definition 4. *(k-coverage redundant) A sensor node s_i is k-coverage redundant if the target region is still completely k-covered by $S \backslash s_i$.*

The k-coverage redundancy of sensor node s_i is detected by utilizing the order-k Voronoi diagram.

Definition 5. *(order-k Voronoi diagram [14]) Given a set of distinct generator sites $P = \{p_1, p_2, \ldots, p_n\}$ in the 2D plane $\mathbb{R}^2$. The order-k Voronoi region associated with a subset $P_i^k = \{p_{i1}, p_{i2}, \ldots, p_{ik}\} \subset P$ is defined as:*

$$V\left(P_i^k\right) = \left\{ q \in \mathbb{R}^2 \middle| \max_{p_h} \left\{ d\left(q, p_h\right) | p_h \in P_i^k \right\} \leq \min_{p_j} \left\{ d\left(q, p_j\right) | p_j \in P \backslash P_i^k \right\} \right\}.$$

The set of order-k Voronoi regions, $V^{(k)} = \left\{ V_1^{(k)}, V_2^{(k)}, \ldots \right\}$, is called the order-k Voronoi diagram of $\mathbb{R}^2$ generated by P.

Fig.1 is an example of order-3 Voronoi diagram with 20 random generator sites. Sensor node s_i can calculate the order-k Voronoi diagram of the target region by taking its l-hop neighbors $N_l(i)$ as generator sites. We use $NOVD\,(l, k, i)$ to denote the resultant Voronoi diagram, $NOVV\,(l, k, i)$ to denote a Voronoi vertex of $NOVD\,(l, k, i)$, and $NOVIP\,(l, k, i)$ to denote an intersection point between an edge of the $NOVD\,(l, k, i)$ and the circumcircle of s_i's sensing disk. In Fig.2, suppose the circle represents sensor node s_7's (which

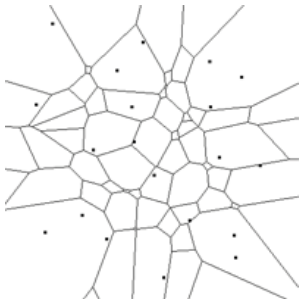


Fig. 1: Order-3 Voronoi Diagram with 20 random sites

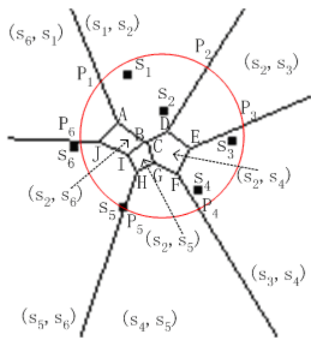


Fig. 2: Neighbor order-2 Voronoi diagram

is not shown in this figure) sensing area and assume its the 2-hop neighbor set is $N_2\,(7) = \{s_1, s_2, s_3, s_4, s_5, s_6\}$. Taking $N_2\,(7)$ as Voronoi sites, we can construct the neighbor order-2 Voronoi diagram $NOVD\,(2, 2, 7)$. Each Voronoi polygon is associated with a pair of sensor nodes (shown in bracket) and $NOVV\,(2, 2, 7) = \{A, B, C, D, E, F, G, H, I, J\}$ and $NOVIP\,(2, 2, 7) = \{P_1, P_2, P_3, P_4, P_5, P_6\}$.

Theorem 1. *A sensor node s_i is k-coverage redundant if and only if every $NOVV\,(l, k, i)$ vertex and every $NOVIP\,(l, k, i)$ point, which lies in s_i's sensing disk, is covered by all of the k corresponding Voronoi sites (sensor nodes in $N_l\,(i)$).*

Proof. (1) necessary condition. If sensor node s_i is k-coverage redundant, all $NOVV\,(l, k, i)$ vertices and $NOVIP\,(l, k, i)$ points in S_i are k-covered by other nodes. According to the definition of order-k Voronoi diagram, each of these points must be covered by its k closest sites, i.e., the corresponding nodes associated with the Voronoi polygon.

(2) sufficient condition. Sensor node s_i's sensing disk S_i is divided into several subareas by $NOVD\,(l, k, i)$. There are two types of subareas. One is the closed convex polygon involving only $NOVV\,(l, k, i)$ vertices. The other is a convex area involving not only $NOVV\,(l, k, i)$ vertices, but also $NOVIP\,(l, k, i)$ points.

Case 1. Consider the subarea involving only $NOVV\,(l, k, i)$ vertices. If all these $NOVV\,(l, k, i)$ vertices are covered by the k associated Voronoi sites, according to the convexity of the Voronoi region and sensor node's sensing area, the subarea formed by these $NOVV(l,k,i)$ vertices is covered k sensor nodes in $N_l(i)$.

Case 2. Consider the subarea of the second type. In this case, the boundary of the convex subarea includes an arc segment of s_i's coverage circumcircle C_i. Let's take Fig.3 as an example. Points VIP_1 and VIP_2 are the intersection points between C_i (solid circle) and two Voronoi edges. To cover these two $NOVIP(l,k,i)$ points, sensor node s_j must lie in the intersection area between circles C_1 and C_2 (dotted circle), where $C_1\,(C_2)$ is centered at $VIP_1\,(VIP_2)$ with radius R_s. For every point p on the arc segment between VIP_1 and VIP_2 (counterclockwise), $d(s_j,p) \leq R_s$. If all other $NOVV(l,k,i)$ vertices (e.g., A, B, and C) of this convex region are also covered by s_j, every point in this convex region will be covered by s_j. Similar to case 1, if all $NOVV(l,k,i)$ vertices and $NOVIP(l,k,i)$ points of the convex region are covered by each of the associated k closest sensor nodes, this convex subarea is surely k-covered even without s_i, which means that s_i is k-coverage redundant in this case. ■

To avoid that two neighboring sensor nodes turn off simultaneously thus leaves blind points in the target region, node priority is also used to prevent the cyclic dependent relationship as the k-connectivity redundant condition does.

k-Coverage Redundant Condition:

A sensor node s_i is k-coverage redundant if every $NOVV(l,k,i)$ vertex and every $NOVIP(l,k,i)$ point, which lies in s_i's sensing disk, is covered by the corresponding associated Voronoi sites (sensors) in $N_l(i)$ with lower priorities than s_i.

Fig.4 illustrates the k-coverage redundant condition on the basis of Fig.2. The shadowed circle is sensor node s_7's coverage area. If we take node id as node priority, node s_7 has the highest priority among its 2-hop neighbors. And we can see that, when P_6, J, I, H, P_5 points are covered by both s_5 and s_6, P_5, H, G, F, P_4 are covered by both s_4 and s_5, P_4, F, E, P_3 are covered by both s_3 and s_4, P_3, E, D, P_2 are covered by both s_2 and s_3, P_2, D, C, B, A, P_1 are covered by both s_1 and s_2, P_1, A, J, P_6 are covered by both s_1 and s_6, A, B, I, J are covered by both s_2 and s_6, B, C, G, H, I are covered by both s_2 and s_5, D, C, G, F, E are covered by both s_2 and s_4, then s_7 is 2-coverage redundant. If

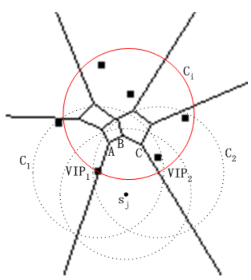


Fig. 3: Proof of Case 2

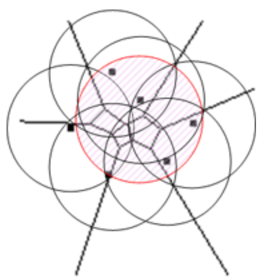


Fig. 4: Example of k-coverage redundant condition ($k = 2$)

a sensor node meets both the above two redundant conditions, it is safe to put the sensor node into low-powered sleep status immediately. Finally, all sensor nodes that don't satisfy the above two conditions remain active and form the k-connected k-cover set for the target region.

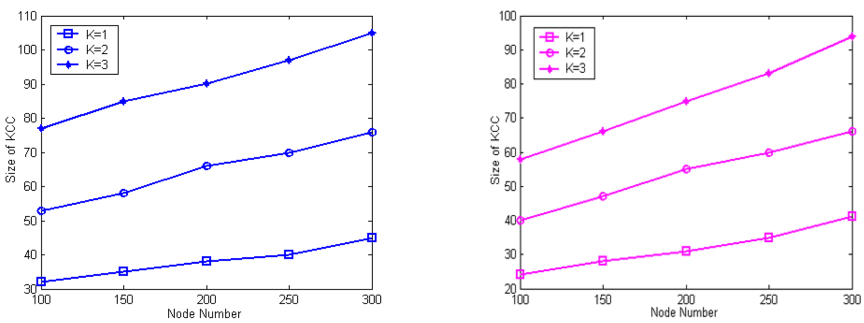
It has been shown that when $R_c \geq 2R_s$ the complete coverage of the target region implies connectivity of the network [11]. Further, it can be easily proved that the k-coverage implies k-connectivity if $R_c \geq 2R_s$. So in the case of $R_c \geq 2R_s$, the k-coverage redundant condition alone can construct a k-connected k-cover set for the target area.

4 Performance Evaluation

The target region is an area of 40×40 unit square. The sensing model and wireless communication model are presented in section 2. In our experiments, neighbor hop number l is 2 and node id is used as node priority. All results shown here are the average values over 50 runs.

Fig.5 shows how the size of KCC (number of active sensor nodes) constructed by the proposed self-pruning algorithm varies with the network size (deployed node number) when k is set to 1, 2 and 3 separately. We can see that the size of KCC is much smaller than that of the original network. Therefore the proposed algorithm can decrease the number of active sensor nodes and hence reduce the total energy consumption effectively, which is helpful to prolong the network lifetime. In both figures the size of KCC increases with the network size under all settings of k. We also notice that when $R_c = 2R_s$ the size of KCC is smaller than the corresponding size when $R_c = R_s$.

Fig.6 shows how the size of 2-connected 2-cover set varies with R_s when R_c is fixed to 10 units. We see that under different network size (150 and 250), the number of active sensor nodes decreases with the increase of R_s. In Fig.7,



(a)$R_s = 10, R_c = 10$ (b)$R_s = 10, R_c = 20$

Fig. 5: Size of KCC vs. network size

we compare the performance of the proposed self-pruning algorithm with the distributed version of the Greedy algorithm in [9] under different network size when $k = 1$ and $R_c = R_s = 10$. Although the Greedy algorithm can result in a slightly smaller active sensor node set, it must maintain global state information during its executing process and therefore it is prone to message loss. On the contrary, the proposed self-pruning algorithm only needs local neighborhood infomation and hence is more robust to message loss.

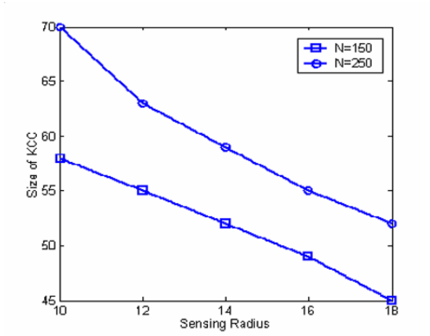


Fig. 6: Size of KCC vs.R_s ($k = 2, R_c = 10$)

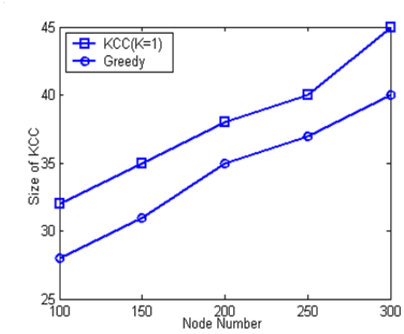


Fig. 7: Self-pruning ($k = 1, R_s = R_c = 10$) vs. Greedy

Table 1

Node Number	Original VCD	Origianl CD	KCC VCD	KCC CD	Success Ratio
100	3	4	2	2	100%
150	5	4	3	2	100%
200	9	5	3	2	100%
250	10	5	4	2	100%
300	10	6	4	2	100%

Table 1 shows the variation of the network Vertex Connectivity Degree (VCD) and the Coverage Degree (CD) before and after applying the self-pruning algorithm. The original vertex connectivity degree is computed when all sensor nodes are active using the max-flow min-cut algorithm. The coverage degree d means that each sensor node can cover its associated Voronoi vertices in the order-d Voronoi diagram while can't cover all of its Voronoi vertices in the order-$(d+1)$ Voronoi diagram. We consider the comparison when $k = 2, R_c = R_s = 10$. From Table 1 we can see that both the vertex connectivity degree and the coverage degree are reduced but still satisfy the specified requirement ($k = 2$). The success ratio is 100% under different network size.

5 Conclusions

In this paper we address the issue of constructing a minimal k-connected k-cover set (KCC) for a target region and propose a general framework for this problem. Different algorithms for detecting k-connectivity and k-coverage redundancy in a localized manner can be integrated into the self-pruning framework. And different connectivity and coverage requirements can be specified flexibly in our framework. We also propose a novel, distributed and localized algorithm to detect k-coverage redundancy of a sensor node based on order-k Voronoi diagram. Experimental results show that the proposed self-pruning algorithm can construct the k-connected k-cover set reliably and reduce the number of active sensor nodes whilst maintaining the k-connectivity and k-coverage properties of the original network, which is helpful to reduce system energy consumption and prolong the network lifespan.

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