

# Maximal Distance Spectrum Assignment for Services Provisioning in Elastic Optical Networks

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**Abstract**—In elastic optical networks (EONs), a suitable guard-band should be inserted between the spectrum assigned to two lightpaths to prevent crosstalk if they share common fiber links. In the meantime, these guard-bands may have diverse sizes for different lightpath pairs due to the strength of the crosstalk among them, which is determined by many factors, such as the required bandwidth, the number of common fiber links, and the modulation level of each lightpath. However, in the presence of a high traffic load, not all communication requests can be satisfied at all times since the spectrum resource on fiber links is limited. In light of this, service providers need to prioritize communication requests and even reject some of the low-priority requests in cases of traffic blocking, especially in peak hours. Therefore, we formulate the maximal distance spectrum assignment (max-DSA) problem to investigate how to maximize the sum of weights of requests we can serve and give them spectrum assignments under the constraint of limited spectrum resources. At first, since max-DSA is  $\mathcal{NP}$ -hard, we propose an ILP model to get the optimal solution. Then, we give the upper and lower bounds of the optimal value of the max-DSA problem. To solve max-DSA efficiently in polynomial time, we propose a Vertex-Deletion Iteration (VDI) approximation algorithm. Our algorithm starts with building several initial feasible solutions and then improves the solutions iteratively until reaching the local optimality. Furthermore, we also prove that the max-DSA problem can be solved in polynomial time to optimality in some specific complete conflict graphs. The obtained numerical results have demonstrated that the VDI algorithm can find near-optimal solutions for max-DSA in various conflict graphs and under various limited spectral resources.

**Index Terms**—Elastic optical networks (EONs), maximal distance spectrum assignment (max-DSA), heuristic, iterative approximation.

## I. INTRODUCTION

RECENTLY, traffic requests in backbone networks grew rapidly, according to the Cisco Visual Networking Index [1]. For example, Global Internet Protocol (IP) traffic was expected to increase threefold from 2017 to 2022; busy hour internet traffic was expected to increase by a factor of 4.8, while average internet traffic was expected to increase by a factor of 3.7 [1]. To adapt to the fast growth of traffic requests, flexible-grid elastic optical networks (EONs) have been introduced to increase the flexibility of bandwidth allocation in the optical layer [2], [3]. Specifically, in EONs, the bandwidth-variable transponders (BV-Ts) and wavelength-selective switches (BV-WSS') provide several narrow-band (i.e., 12.5 GHz) and spectrally-consecutive frequency slots (FS) to build lightpaths and then transmit data over them

[4]. Hence, traffic requests from upper-layer networks can be served in EONs by utilizing just-enough bandwidths and the suitable bandwidth allocation granularity of an FS [5], [6].

Besides, if two routing lightpaths share one or more fiber links [7], [8], an appropriate guard-band should be inserted between their spectrum assignments to prevent inter-channel crosstalk [9]. Note that the size of the guard-band is required to be larger as the crosstalk level gets stronger, and several factors have an impact on the crosstalk level, e.g., the bandwidth, the number of common links and the modulation level [10]. Hence, different lightpath pairs require different guard-bands in EONs. For instance, suppose that there are three communication requests  $R_1$ ,  $R_2$  and  $R_3$  with bandwidth requirements 2, 4 and 3 FS, respectively in an EON. We show in Fig. 1 their spectrum assignments with blocks in different colors, and different-sized guard-bands with white blocks. Obviously, the sizes of guard-bands will occupy spectral resources and hence have a strong impact on the spectrum utilization in EONs [11], [12].

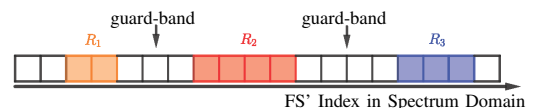


Fig. 1. Spectrum assignments with guard-bands in EONs.

Furthermore, the spectral resource of fiber links has already been fixed in the network planning phase. With the dramatically increasing traffic, it might not be sufficient to serve all the communication requests, especially during peak hours in the backbone networks. Although the distance spectrum assignment (DSA) problem considers the guard-band with diverse sizes [13], it was proposed for networking planning purposes. Note that the DSA aims to serve all communication requests while minimizing the total spectrum usage. However, the DSA is not suitable to handle the case when the EON is under-provisioned, where some requests should be rejected due to the lack of spectrum resources.

To fill this gap, we put forward a new problem, i.e., maximal distance spectrum assignment under limited spectral resources (max-DSA). We consider the service provisioning

problem where all the communication requests with routing lightpaths are given, each communication request has a weight, the spectral resource is given, and the guard-bands of the lightpath pairs are also known. With all the aforementioned information, max-DSA tries to maximize the sum of weights of communication requests we can serve. Without loss of generality, we also use request and its routing lightpath to represent communication request.

To the best of our knowledge, there is no theoretical study on the max-DSA problem in the literature. Furthermore, it is an extremely hard problem, at least as hard as the DSA problem [13], which is  $\mathcal{NP}$ -hard. Hence, we formulate the max-DSA problem and provide several significant and insightful theoretical results for future research. The contributions of our work can be summarized as follows:

- We provide for the first time a formal study of the max-DSA problem: put forward a generalized problem definition where the request weights can be heterogeneous, and propose an integer linear program (ILP) model to solve it exactly.
- We propose the upper and lower bounds of the optimal value of the max-DSA problem.
- To solve the max-DSA problem, we specifically build a conflict graph, where each vertex represents a request and an edge signifies the guard-band requirement between two requests. We prove that the Intermediate Spectrum Assignment (ISA) algorithm can obtain the optimal solution in some complete conflict graphs. In general, we propose a polynomial-time algorithm, namely Vertex-Deletion Iteration (VDI), to give a near-optimal solution of the max-DSA problem and study its performance on different types of conflict graphs. The VDI algorithm starts with building some initial feasible solutions and then improves the solutions iteratively until reaching local optimality.

## II. RELATED WORK

Previous studies proposed the concept of EONs to utilize resources flexibly in the optical layer and hence promoted interesting research about spectrum assignment [2] - [5], [7], [11] - [16]. In [2], bandwidth-variable transponders (BVTs) and bandwidth-variable crossconnects (BV-WXCs) have been proposed, which can be utilized to provide flexible and just-enough bandwidth allocation for traffic requests. Hence, compared with the traditional fixed-grid wavelength division multiplexing (WDM) networks, EONs can sufficiently and intelligently utilize the spectral resource to satisfy the requests.

In [5], the Routing and Spectrum Assignment (RSA) problem has been formally defined, which aims to find the suitable lightpath for each traffic request and then assign sufficient spectrum resource for each traffic request while minimizing the total spectrum resource used in EONs. They also prove the  $\mathcal{NP}$ -hardness of the RSA problem. In light of this, both an ILP model and two heuristic algorithms are proposed to solve the RSA problem. [14] and [17] studied the RSA problem with consideration of the multicast traffic in EONs, which is named RMSA. However, all the aforementioned studies

on RSA assume an identical-size guard-band to separate the lightpaths with common links, which is not realistic and will result in spectrum waste. Wu et al. [13] considered the RSA problem when guard-bands have various sizes and denoted it as Distance Spectrum Assignment (DSA) problem. This paper provides a thorough theoretical study of the DSA problem by giving the formal proof of its  $\mathcal{NP}$ -hardness, and the upper and lower bounds of the optimal DSA solution. Besides, a time-efficient heuristic algorithm is also proposed. All the discussions above are for networking planning purposes, assume that the spectral resource is sufficient, and aim to minimize the usage of the spectral resource to serve all requests in EONs. In this paper, we consider the a under-provisioned network. Hence, the spectral resource is limited, and we aim to maximize the total weight of requests we can serve.

Furthermore, we introduce the Maximal Routing and Wavelength Assignment (max-RWA) problem and the Minimal Routing and Wavelength Assignment (min-RWA) problem [18] in WDM networks to compare with the DSA and max-DSA problems, as shown in Table I. Note that the purpose of max-RWA problem is to maximize the number of requests that can be served with limited wavelengths in under-provisioned WDM networks. While the min-RWA problem aims to minimize the number of wavelengths needed to serve all requests in WDM networks for network planning purposes.

TABLE I  
COMPARISON BETWEEN MIN-RWA, MAX-RWA, DSA AND  
MAX-DSA

	Minimize Wavelengths /FS index	Maximize Requests
WDM	min-RWA	max-RWA
EON	DSA	max-DSA

Specifically, as shown in Table II, min-RWA and max-RWA can be solved by considering the classical coloring in the conflict graph [19], [20], while DSA and max-DSA can be solved by considering the fractional coloring [13]. Actually, DSA and max-DSA both have two differences compared with the fractional coloring: (1) Each vertex should be assigned consecutive colors in the DSA and max-DSA problems, while fractional coloring has no such constraint; (2) Any two adjacent color sets should keep a distance at least the size of its corresponding guard-band in the DSA and max-DSA problems, while they only need to be disjoint in the latter case.

As discussed above, the max-DSA problem is a new combinatorial optimization problem that has not yet been studied. The rest of this paper is organized as follows. Section III describes our model of the max-DSA problem and gives its ILP model. Section IV proposes the upper and lower bounds of the optimal value of max-DSA. Our algorithm is proposed in Section V, and its theoretical results are proved in Section VI, while its numerical results are shown in Section VII. Finally, Section VIII summarizes this paper.

TABLE II  
COMPARISON OF RELATIVE COLORING PROBLEMS

	Classical Coloring (min-RWA, max-RWA)	Fractional Coloring	DSA, max-DSA
Vertex Color	One color	A set of colors	A set of colors
Color Contiguity	N/A	no need	Required
Color Distance of Adjacent Vertices	Disjoint	Disjoint	Various positive integers

### III. MAXIMAL DISTANCE SPECTRUM ASSIGNMENT (MAX-DSA) PROBLEM

In this section, we construct a conflict graph to solve this problem, propose its ILP model, and prove its  $\mathcal{NP}$ -hardness.

#### A. Problem Description and Conflict Graphs

In this paper, the focus is on the provisioning over an existing EON. Specifically, we consider that all the requests with their routing paths are given, the spectral resource is given, and all sizes of guard-bands for each spectrally adjacent lightpath pair are known (given due to their crosstalk levels). Also, we give each request a weight to consider their profits. Then, max-DSA tries to maximize the sum of weights of requests we can serve under limited spectral resources.

To solve the max-DSA problem, we construct a conflict graph based on the above information as follows: 1) each vertex represents a request; the vertex width is its bandwidth demand in FS; and the vertex weight represents the weight of the request; 2) two vertices are connected if and only if there is a guard-band between the corresponding lightpath pairs; 3) the edge width represents the required guard-band size; 4) We give an constant integer  $C$  to represent the maximum spectral resource.

In order to realize spectrum assignments in the conflict graphs, we first introduce the following notations:

- $G(V, E)$ : The max-DSA conflict graph, where  $V$  represent the request set, and  $E$  the conflict edge set.
- $n$  or  $|V|$ : the number of vertices in  $V$ .
- $\mathbb{N}^+$ : the positive integer set representing the FS indices in the spectral domain.
- $v_i \in V$ : represent the  $i$ -th request.
- $w_{v_i}$ : the integer width representing bandwidth demand of request  $v_i$  in the number of FS.
- $L_{v_i}$ : the list of some integers representing the set of consecutive FS assigned to  $v_i$ .
- $L_{v_i}(a), L_{v_i}(b)$ : the smallest and biggest integer in  $L_{v_i}$ , respectively.
- $e$  or  $v_i v_j$ : the edge  $e$  or  $v_i v_j \in E$  connecting  $v_i$  and  $v_j$ , which means that the lightpaths of requests  $v_i$  and  $v_j$  share common fiber link(s).
- $w_e(w_{v_i v_j})$ : the positive integer width representing the least guard-band size between lightpaths  $v_i$  and  $v_j$ .
- $c_{v_i}$ : the weight of  $v_i$  representing the weight of the  $i$ -th request.

- $C$ : the constant integer representing the maximum spectral resource.
- $B$ : a large constant integer.

Then, the spectrum assignments in the conflict graph should satisfy the following constraints:

- **Bandwidth Requirement Constraint:** Every request should be assigned an sufficient number of FS equal to its bandwidth demand. That is:

$$|L_{v_i}| = w_{v_i}, \forall v_i \in V \quad (1)$$

- **Spectrum Continuity Constraint:** All the fiber links a request utilizes should be assigned the same FS. This constraint has been satisfied since all the lightpaths have been routed and represented by vertices in the conflict graph.
- **Spectrum Contiguity Constraint:** Every request should be assigned a set of consecutive FS, i.e.,  $L_{v_i} = \{L_{v_i}(a), L_{v_i}(a) + 1, \dots, L_{v_i}(b)\}$  and all the integers are positive.
- **Spectrum Set Distance Constraint:** If two lightpaths are spectrally adjacent, their FS sets should have a distance at least the size of their guard-band. So we have the following inequality:

$$d(L_{v_i}, L_{v_j}) \geq w_{v_i v_j} \quad (2)$$

where

$$d(L_{v_i}, L_{v_j}) = \min\{|s - t| - 1 : s \in L_{v_i}, t \in L_{v_j}\}.$$

We say that a spectrum assignment is *proper* if it satisfies the above four constraints. Then the max-DSA problem can be solved by finding the maximum-weight vertex-induced subgraph with proper spectrum assignment in the conflict graph under the given spectral resource. We say that the feasible (optimal) solution in conflict graph is a vertex set or a vertex-induced subgraph. Obviously, if the conflict graph is disconnected, we can solve the max-DSA problem for every connected component of the conflict graph separately. Also, if there is a request whose width exceeds the given spectral resource, then we cannot serve it anyway. Hence, we ignore these two cases in this paper.

Table III, Fig. 2 and Fig. 3 show how to construct the conflict graph and solve max-DSA in the case when each request has the same weight. As in Table III, there are four requests with their bandwidth demands and routing paths, also shown in a 4-node ring topology in Fig. 2. We use the number of common links to represent their sizes of guard-bands (the size of a guard-band has a positive correlation to its crosstalk level [10]). For example, if the routing paths of  $R_1$  and  $R_2$  share exactly one common link B-A, then their guard-band is 1 FS. Hence, the conflict graph is illustrated in Fig. 3(a), where the number in each cycle is the bandwidth demand of each request, and the number near each edge is the guard-band size. Also, we assume here a limited spectral resource  $C = 10$ . Based on Fig. 3(a), the optimal solution is marked in red braces with their spectrum assignment in Fig. 3(b).

#### B. max-DSA Model and Integer Linear Program

As shown in the above illustrations, we can solve the max-DSA problem in the corresponding conflict graph. Since

TABLE III  
INFORMATION ON LIGHTPATHS

	Bandwidth Demand	Route
Request $R_1$	3 FS	B-A-D
Request $R_2$	2 FS	C-B-A
Request $R_3$	3 FS	A-D-C-B
Request $R_4$	1 FS	C-B-A-D

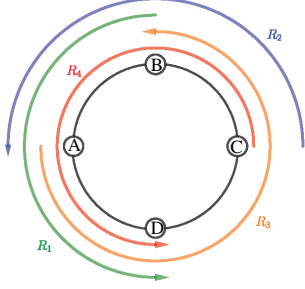


Fig. 2. A 4-node ring topology about four lightpaths in Table III.

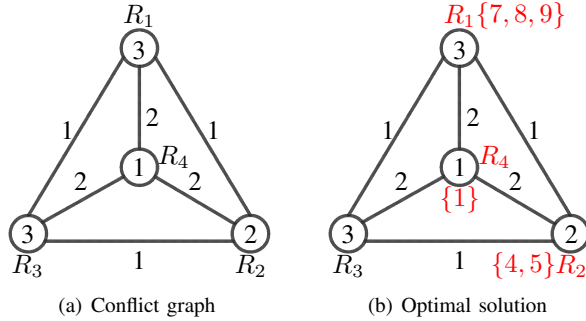


Fig. 3. The conflict graph for Table III and Fig. 2, and an optimal solution

the classical Maximum-k-Colorable-Induce-Subgraph (MkCS) problem [21] can be reduced to max-DSA, it is easy to see that the max-DSA problem is  $\mathcal{NP}$ -hard. Then an ILP model to obtain the optimal solution to the max-DSA problem is given below.

*Decision Variables:*

- $x_i^a, x_i^b$ : Integer variable representing the value of  $L_{v_i}(a), L_{v_i}(b)$ , respectively.
- $I_i$ : Binary variable, and  $I_i = 1$  if  $x_i^b \leq C$ ,  $I_i = 0$  if  $x_i^b > C$  for all  $i = 1, \dots, n$ .
- $O_{v_i v_j}$ : Binary variable, and  $O_{v_i v_j} = 0$  if  $x_i^b < x_j^b$ ,  $O_{v_i v_j} = 1$  otherwise.

*Objective Function:*

$$\max \sum_i c_{v_i} * I_i \quad (3)$$

subject to:

$$x_i^b - x_i^a = w_{v_i} - 1 \quad (4)$$

$$x_i^b - x_j^a + w_{v_i v_j} + 1 \leq B * O_{v_i v_j} \quad (5)$$

$$O_{v_i v_j} + O_{v_j v_i} = 1 \quad (6)$$

$$x_i^b \leq C * I_i + B * (1 - I_i) \quad (7)$$

$$x_i^a, x_i^b \in \mathbb{N}^+ \quad (8)$$

$$I_i \in \{0, 1\}, O_{v_i v_j} \in \{0, 1\} \quad (9)$$

$$v_i \in V, v_i v_j \in E \quad (10)$$

Constraint (4) is the bandwidth requirement. Constraints (5) and (6) are the spectrum set distance requirement. Constraint (7) chooses some vertices satisfying the limited spectral resource.

IV. UPPER AND LOWER BOUNDS OF THE OPTIMAL VALUE OF MAX-DSA

In this section, we give the upper and lower bounds of the optimal value of the max-DSA problem. We use several terminologies of graph theory [22] to bound the optimal value of max-DSA and give some definitions as below.

- $\alpha(G)$ : the cardinality of the maximum independent set of  $G$ .
- $opt(G)$ : the optimal value of the max-DSA problem, that is, the largest number of requests can be served.
- $w_e^i, w_v^i$ : the  $i$ -smallest width of edge and vertex in  $G$ , respectively.
- $m_e$ :  $m_e = w_e + w_{v_i} + w_{v_j}$ , where  $e = v_i v_j$ .

Now we give the lower bound and upper bound of the optimal value of the max-DSA problem.

*Theorem 1:* If every vertex has weight one, then  $\alpha(G) \leq opt(G) \leq \max\{s * \alpha(G) : \sum_{i=1}^{s-1} w_e^i + \sum_{i=1}^s w_v^i \leq C\}$ .

*Proof:* Obviously, the maximum independent set is a feasible solution of max-DSA, hence  $\alpha(G) \leq opt(G)$ .

Now we prove the upper bound. Let  $s$  be the maximum value such that  $\sum_{i=1}^{s-1} w_e^i + \sum_{i=1}^s w_v^i \leq C$ . Suppose that  $G'$  is an optimal solution of max-DSA of the conflict graph  $G$ , and we give every vertex of  $G'$  a proper assignment such that  $L_v(b) \leq C$  for all  $v \in V(G')$ . We use Algorithm 1 to partite  $V(G')$  as Fig. 4 showing. Let  $u_i \in V'_i$  be the vertex with the smallest width in  $V'_i$ , and  $e_i$  be the edge between  $V'_i$  and  $V'_{i+1}$  with the smallest width. By the Algorithm 1, the first vertex added to  $V'_i$  has a neighbor in  $V'_{i-1}$ , then all the integers of its list is bigger than its neighbor's in  $V'_{i-1}$ . Hence the biggest value of  $L_v(b)$  is at least  $\sum_{i=1}^{k-1} w_{e_i} + \sum_{i=1}^k w_{u_i} \leq C$ . By the  $s$  we choose and the definitions of  $w_v^i, w_e^i, w_{e_i}, w_{u_i}$ , we get  $k \leq s$ . Obviously,  $|V'_i| \leq \alpha(G)$ , then we get  $|V(G')| \leq s * \alpha(G)$ . ■

Suppose that every vertex has weight one. It can be shown that two bounds in Theorem 1 are tight when  $m_e > C$  for all  $e \in E(G)$ , and the conflict graph is a complete graph with the same edge width, respectively.

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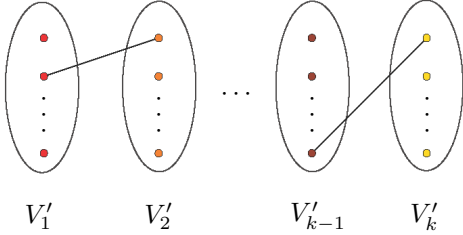
**Algorithm 1:** A Partition of  $V(G')$ 

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**Input :**  $G'$  and its proper assignment**Output:** A partition of  $V(G')$ 

```
1  $V \leftarrow V(G')$ ;
2  $i \leftarrow 1$ ;
3  $V'_1 \leftarrow \emptyset$ ;
4 while  $V \neq \emptyset$  do
5   Choose a vertex  $v \in V$  with the smallest  $L_v(a)$ ;
6   if  $V'_i \cup \{v\}$  is independent then
7      $\mid$  add  $v$  to  $V'_i$ ;
8   else
9      $\mid$   $i \leftarrow i + 1$ ;  $V'_i \leftarrow \emptyset$ ;
10     $\mid$  add  $v$  to  $V'_i$ ;
11  delete  $v$  from  $V$ ;
12  $k \leftarrow i$ ;
13 Return  $\{V'_1, V'_2, \dots, V'_k\}$ 
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Fig. 4. The partition of  $V(G')$  according to Algorithm 1.

## V. TIME-EFFICIENT APPROXIMATION ALGORITHM FOR MAX-DSA

In this section, we first give several algorithms to obtain vertex sequences and the spectrum assignments under these vertex sequences. Based on these algorithms, we give a time-efficient iterative approximation algorithm for max-DSA.

Note that the max-DSA problem is to prioritize vertices and give some of them the proper spectrum assignments under limited spectral resources. We first give some methods to order vertices. One method to give the initial vertex sequence is to use four indexes here to evaluate which vertex has high priority. The first three indexes are  $w_v, m_v, d(v)$ , where  $m_v = \sum_{v \in e} m_e / d(v)$  and  $d(v)$  is the degree of  $v, \forall v \in V(G)$ . Because the vertex with a small value of  $w_v, m_v, d(v)$  is easy to give the spectrum assignment with a small  $L_v(b)$ , it will have a small impact on the spectrum assignments of other vertices under the limited spectral resource. The last index is  $c_v$ , which is the weight of  $v$  (the weight of the request). As the argument above shows, we give the first vertex sequence such that  $b_v = \lambda_1 * w_v + \lambda_2 * d(v) + \lambda_3 * m_v - \lambda_4 * c_v$  is ascending, where  $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0.25$ . Another method to order vertices is to use a graph theory method to decompose the graph into some subgraphs and order the vertices in each subgraph (i.e. block decomposition). Note that a block in the graph is its maximal 2-connected vertex-induced subgraph. For example, the graph shown in Fig. 5 is decomposed into three subgraphs,

induced by brown, orange and green edges, respectively.

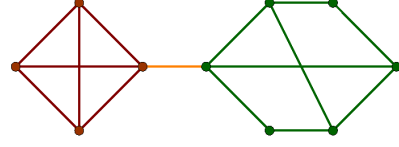


Fig. 5. An example of block decomposition

We denote this method as Block Decomposition (BD) described in Algorithm 2. In lines 1 and 2, we decompose  $G$  into  $B$  2-connected subgraphs, and order the vertices in each block. We use  $i$  to record the  $i$ -th sequence. We run lines 4-11 if there is a block that cannot be marked. After the while-loop in lines 4-11, we will get vertex sequences  $U_1, \dots, U_B$ . Now we calculate the time complexity of the BD algorithm. The block decomposition is based on the DFS [22] algorithm, whose time complexity is  $\mathcal{O}(|E(G)|) \leq \mathcal{O}(|V^2(G)|)$ , and the complexity of ordering the vertices is at most  $\mathcal{O}(|V^2(G)|)$ . Besides, the BD algorithm has two while-loops, whose time complexity is  $\mathcal{O}(B^2) \leq \mathcal{O}(|V^2(G)|)$ . Hence, the time complexity of the BD algorithm is  $\mathcal{O}(|V^2(G)|)$ .

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**Algorithm 2:** BD to get initial sequences

---

**Input :** A conflict graph $G(V, E, \{w_{v_i}\}, \{w_e\}, \{c_{v_i}\}, C)$ **Output:** Some sequences of vertices

```
1 Give  $G$  a block decomposition;
2 Calculate  $b_v$  in each block and order the vertices in
  each block such that  $b_v$  is ascending;
3  $i \leftarrow 0$ ;
4 while there is a block not be marked do
5    $\mid$   $i \leftarrow i + 1$ ;  $U_i \leftarrow \emptyset$ ;
6    $\mid$  mark one of the unmarked block;
7    $\mid$  choose this block as the first block;
8    $\mid$  Add the vertex order in this block to  $U_i$ ;
9    $\mid$  while there is a block not yet be chosen do
10     $\mid$   $\mid$  choose a block that has not yet been chosen
11     $\mid$   $\mid$  and contains a vertex in  $U_i$ , denote as  $v'$ ;
12     $\mid$   $\mid$  add the vertex order in this block to  $U_i$ , except
13     $\mid$   $\mid$   $v'$ ;
14 return  $U_1, \dots, U_B$ 
```

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Second, we use Intermediate Spectrum Assignment (ISA) algorithm, described in Algorithm 3 to give high-priority vertices the spectrum assignments with a certain vertex sequence under the limited spectral resource. For convenience, we suppose that the vertex sequence is  $(v_1, \dots, v_n)$ . We say that  $L_{v_i}$  is an *intermediate* assignment if  $L_{v_i}$  satisfies the Spectrum Set Distance Constraint to  $L_{v_1}, \dots, L_{v_{i-1}}$  and  $\min\{L_{v_j}(b) : j < i\} \leq L_{v_i}(a), L_{v_i}(b) \leq \max\{L_{v_j}(a) : j < i\}$  (i.e.  $L_{v_i}$  can be asserted to the middle of the before assignments).

In lines 1-2, we first add  $v_1$  to  $U$  and give it a spectrum assignment. We use  $i$  to record which vertex we consider

in each while-loop in lines 4-16. In lines 5-6, we use  $s_1$  ( $s_2$ ) to calculate the minimum  $L_{v_i}(a)$  (maximum  $L_{v_i}(b)$ ) in the current loop under the bandwidth requirement constraint, spectrum contiguity constraint, and spectrum set distance constraint, but maybe smaller than 1 (larger than  $C$ ). In lines 7-15, we consider three possible cases of the assignment of  $L_{v_i}$ . The calculations of  $s_1$ ,  $s_2$  need to consider all the edges. The determination about whether there is an intermediate assignment of  $v_i$  and the choice of the smallest assignment  $L_{v_i}$  need to consider all the before assignments  $L_{v_1}, \dots, L_{v_{i-1}}$ . Hence, its time complexity is at most  $\mathcal{O}(|V^2(G)|)$ .

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**Algorithm 3:** Intermediate Spectrum Assignment under certain sequence (ISA)

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**Input :** A conflict graph  $G(V, E, \{w_{v_i}\}, \{w_e\}, \{c_{v_i}\}, C)$  with a given vertex sequence  $(v_1, \dots, v_n)$ .

**Output:** An induced subgraph  $G[U]$  of  $G$  with its assignment strategy

- 1  $U \leftarrow \{v_1\};$
- 2  $L_{v_1} \leftarrow \{1, \dots, w_{v_1}\};$
- 3  $i \leftarrow 2;$
- 4 **while**  $i \leq n$  **do**
- 5      $s_1 \leftarrow \min\{L_{v_j}(a) - w_{v_i v_j} - w_{v_i} : v_i v_j \in E(G) \text{ and } v_j \in U\};$
- 6      $s_2 \leftarrow \max\{L_{v_j}(b) + w_{v_i v_j} + w_{v_i} : v_i v_j \in E(G) \text{ and } v_j \in U\};$
- 7     **if**  $s_1 \geq 1$  **then**
- 8         **add**  $v_i$  to  $U;$
- 9          $L_{v_i} \leftarrow \{1, \dots, w_{v_i}\};$
- 10    **else if**  $v_i$  has an intermediate assignment **then**
- 11         **add**  $v_i$  to  $U;$
- 12         **choose** the smallest  $L_{v_i}$  as its assignment;
- 13    **else if**  $s_2 \leq C$  **then**
- 14         **add**  $v_i$  to  $U;$
- 15          $L_{v_i} \leftarrow \{s_2 - w_{v_i} + 1, \dots, s_2\};$
- 16      $i \leftarrow i + 1;$
- 17 **return**  $G[U]$  and  $\{L_v : v \in U\}$

---

Based on the BD and ISA algorithms, we give the Vertex-Deletion Iteration (VDI) algorithm described in Algorithm 4 for the max-DSA problem. In lines 1-2, we give the vertex sequence  $S_0$  where  $b_v$  is ascending. In line 3, we give the sequences  $S_1, \dots, S_B$  according to the BD algorithm. In line 4, we give the spectrum assignment by the ISA algorithm with vertex sequences  $S_0, \dots, S_B$ , respectively. Based on these initial results, we run lines 5-11. In lines 5-7, if  $\max\{|U|\} = n - 1$ , we reverse such a sequence that its output of the ISA algorithm satisfies  $|U| = n - 1$ , and run ISA algorithm with this sequence. In lines 8-11, if  $\max\{|U|\} < n - 1$ , we go into the for-loop in lines 9-11. We delete one vertex with the largest value of  $b_v$  in  $G$  or in blocks and get two subgraphs in each loop, and repeat lines 1,2,4 or lines 3-4, respectively. In line 12, we select the largest  $G[U]$  of all outputs of the ISA algorithm and its spectrum assignment. Note that, the largest  $G[U]$  is such vertex-induced subgraph with the largest sum of

weights. The VDI algorithm uses the BD algorithm at most  $n$  times and the ISA algorithm at most  $n^2$  times. So, the time complexity of the VDI algorithm is  $\mathcal{O}(|V^4(G)|)$ .

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**Algorithm 4:** Vertex-Deletion Iteration (VDI) with BD and ISA algorithms

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**Input :** A conflict graph  $G(V, E, \{w_{v_i}\}, \{w_e\}, \{c_{v_i}\}, C)$

**Output:** A solution  $G[U]$  and the spectrum assignments.

- 1 **calculate**  $b_v$  for all vertices;
- 2 **give** a sequence  $S_0$  of all vertices where  $b_v$  is ascending;
- 3 **run** BD algorithm and we get  $B$  sequences of all vertices  $S_1, \dots, S_B;$
- 4 **run** ISA algorithm with each sequence above, respectively;
- 5 **if**  $\max\{|U|\} = n - 1$  **then**
- 6     **let**  $S'$  be such sequence that its output of ISA algorithm satisfying  $|U| = n - 1;$
- 7     **run** SA algorithm again with the sequence that is the reverse of  $S';$
- 8 **if**  $\max\{|U|\} < n - 1$  **then**
- 9     **for**  $i = 1 : n - \max\{|U|\} - 1$  **do**
- 10         **abtain** two vertex-delete subgraphs by deleting  $v$  with the largest value of  $b_v$  in  $G$  and in the blocks, respectively;
- 11         **repeat** steps 1,2,4 and 3-4 with these two subgraphs, respectively;
- 12 **return** the largest  $G[U]$  of all outputs of ISA algorithm and its spectrum assignments

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## VI. ALGORITHM ANALYZES

In this section, we analyze the performance of our algorithms in complete conflict graphs. We prove here that if the conflict graph is a complete graph under some other constraints, we can find the optimal vertex sequence in polynomial time and then use the ISA algorithm to give the optimal solution.

*Theorem 2:* If the conflict graph is a complete graph with the same vertex weight and the same edge width, then we can choose a sequence of vertices in polynomial time such that the ISA algorithm gives an optimal solution.

*Proof:* Order the vertices as  $(v_{j_1}, \dots, v_{j_n})$  such that the widths of vertices are ascending. It is obviously that the induced subgraph  $G[\{v_{j_1}, \dots, v_{j_i}\}]$  is the optimal solution, where  $\sum_{k=1}^i w_{v_{j_k}} + \sum_{k=1}^{i-1} w_{v_{j_k} v_{j_{k+1}}} \leq C$  and  $i$  is as large as possible, and ISA algorithm can give the optimal solution. Since we can obtain this sequence in polynomial time, we see that the conclusion holds. ■

## VII. NUMERICAL RESULTS

In this section, we give the performance of the VDI algorithm. Since max-DSA is a new model to select high-priority vertices and give them the spectrum assignment, there

is no heuristic algorithm for comparison. Hence, we use ISA the algorithm with vertex sequence  $S_0$  to be the benchmark algorithm, which we denote as the AS0 algorithm. All the optimal solutions are given by the ILP model solved by Python 3.8 with the module *docplex.mp.model*. The approximate solutions of the AS0 and VDI algorithms are also solved in Python 3.8 with the module *networkx*. We run 30 independent simulations on each conflict graph and average the results to ensure sufficient statistical accuracy. All our simulations were conducted on a computer with an Intel(R) Core(TM) i5-1035G1 CPU 1.00GHz and 16GB of RAM.

### A. Simulation Setup

To verify the performance of our algorithm, we both consider the theoretical and real situation of conflict graphs. In the theoretical situations, we consider different properties of conflict graphs, i.e. vertex number, edge number, spectral resource and complete conflict graphs. In the real situations, we consider three famous practical EON topologies, NSFNET, US Backbone and GERMANY [18]. We generate all the conflict graphs by using the functions of Python 3.8 with module *networkx*. All the vertex widths here are randomly chosen within  $[1, |V|]$  in both situations. All the edge widths here are randomly chosen within  $[1, |V|]$  in the theoretical situation. In all the conflict graphs below, we give two cases to simulate:

- (1) Each vertex has a weight of one.
- (2) Each vertex's weight is the same as its vertex width.

We run simulations in the first four theoretical situations and the last real situation as below.

- **Vertex number:** We give six random conflict graphs with  $|V| \in [10, 15]$ , as shown in Fig. 6. Each edge probability here is 50%. In each case, we set  $C = 40$ .
- **Edge number:** We give six random conflict graphs with  $|V| = 13$  and  $|E(G)| \in \{20, 30, 40, 50, 60, 70\}$ , as shown in Fig. 7. And in each case, we set  $C = 40$ .
- **Limited spectral resource  $C$ :** In each case, we give six random conflict graphs that  $|V| = 13$ , each edge probability is 50%, and set  $C \in \{10, 20, 30, 40, 50, 60\}$ .
- **Complete graphs:** In each case, we give six complete conflict graphs with  $|V| = 13$  and  $C \in \{10, 20, 30, 40, 50, 60\}$ .
- **NSFNET, US Backbone and GERMANY:** To consider the real situations, we perform simulations on three practical EON topologies : NSFNET, US Backbone and GERMANY [18]. We generate their conflict graphs as follows: (1) Randomly choose the source and destination of each request in the EON topology by uniform distribution. (2) The shortest path between any source-destination pair is used as the lightpath to route each request. (3) Each guard-band (edge width) between two requests is computed according to the number of common links in their lightpaths, which corresponds to the practical crosstalk level [10]. Following these principles, conflict graphs can be constructed in each case.

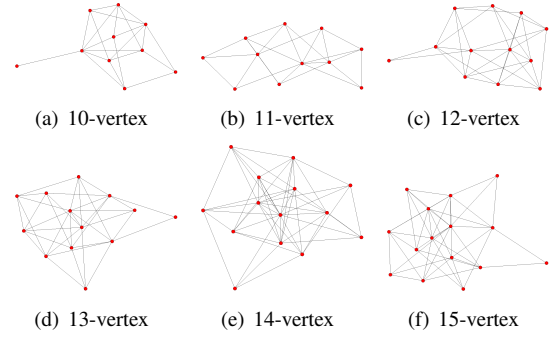


Fig. 6. Six random conflict graphs with 10-15 vertices.

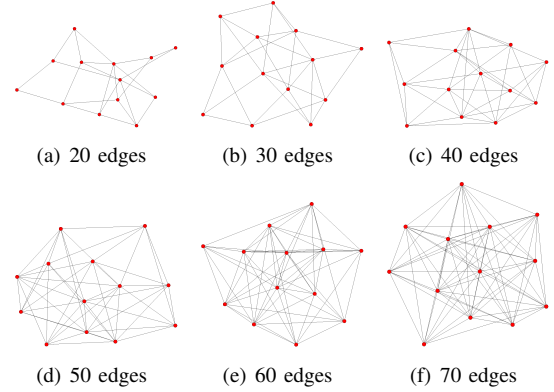


Fig. 7. Six random conflict graphs with 13 vertices and 20-70 edges.

### B. Simulation Results

1) *Vertex number:* Table IV presents the average results computed by AS0, VDI, and ILP respectively, for the six random conflict graphs in Fig. 6 in two cases. In Table IV, we can see that the improved solutions of VDI are better than those of AS0, especially in case (2). It means that VDI performs well when the weights of requests,  $c_v$ , are different.

TABLE IV  
NUMERICAL RESULTS FOR FIG. 6 IN CASE (1)/(2)

case (1)/(2)	(a)	(b)	(c)	(d)	(e)	(f)
<b>AS0</b>	7/40	8/52	9/55	9/74	9/70	9/60
<b>VDI</b>	9/53	9/60	11/76	10/85	10/89	10/92
<b>ILP</b>	10/55	10/62	11/76	11/85	11/89	11/92

2) *Edge number:* Fig. 8 and Fig. 9 show the simulation results on six random conflict graphs in Fig. 7 in two cases. The maximum sum of weights of requests solved by AS0, VDI, and ILP is indicated by the blue, red, and purple bars, respectively. The performance of VDI is better than AS0 and is almost optimal whenever the number of edges is high. We also see that the maximum sum of weights decreases when the number of edges increases. The approximation ratio of VDI decreases when the number of edges increases. Therefore, if a good routing algorithm can reduce more common links, then we can get a larger sum of weights of communication requests we can serve.

3) *Limited spectral resource  $C$ :* Fig. 10 and Fig. 11 plot the simulation results on random graphs that  $|V| = 13$ , each edge probability is 50%, and set  $C \in \{10, 20, 30, 40, 50, 60\}$

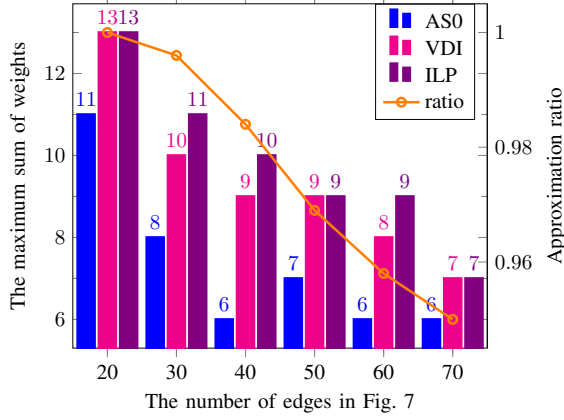


Fig. 8. Numerical results for Edge number scenario in case (1).

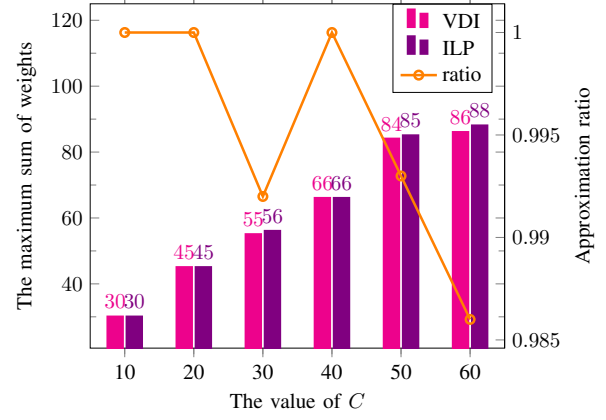


Fig. 11. Numerical results for Limited spectral resource scenario in case (2).

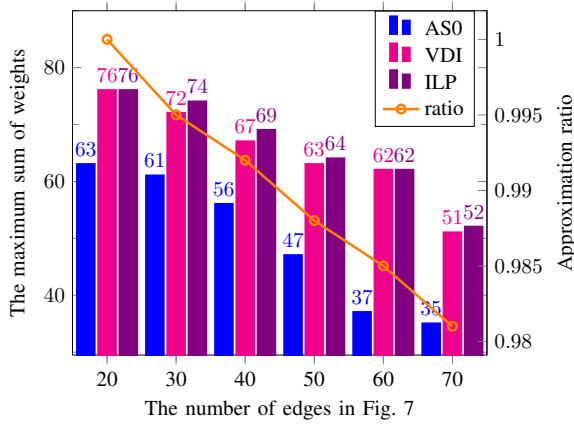


Fig. 9. Numerical results for Edge number scenario in the case (2).

4) *Random complete graphs*: Fig. 12 and Fig. 13 plot the simulation results on complete graphs that  $|V| = 13$  and  $C \in \{10, 20, 30, 40, 50, 60\}$  in two cases. We can see that if  $C \leq 40$ , then VDI can get an almost optimal solution. When  $C \geq 50$ , we just get the near-optimal solution. It means that when the conflict graph is complete and the spectral resource is small, the VDI algorithm can get the optimal solution. We also see that the approximation ratio of case (2) performs better than that of case (1), which means that VDI performs well in complete conflict graphs when the weights of requests are different.

in two cases. We can see that the more spectral resource  $C$  is, the larger the sum of weights we can get. In case (1), VDI can get the optimal solution when  $C \leq 30$ , but gets the near-optimal solution when  $C \geq 40$ . That means VDI can get the optimal solution when the limited spectral resources are small in case (1). In case (2), VDI gets almost optimal solutions. It means that VDI performs well when the weights of requests are different.

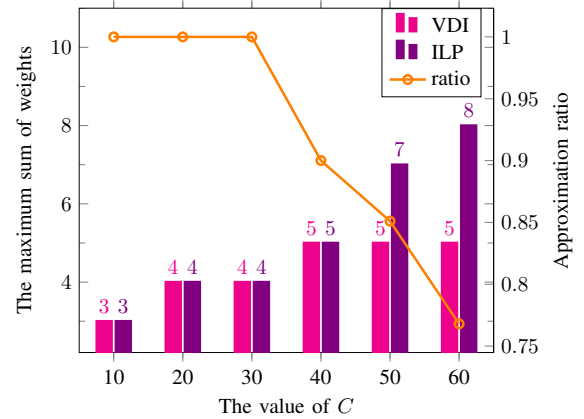


Fig. 12. Numerical results for complete graphs in case (1).

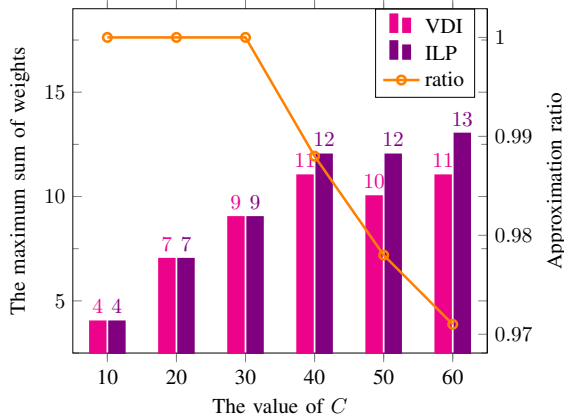


Fig. 10. Numerical results for Limited spectral resource scenario in case (1).

5) *NSFNET, US Backbone and GERMANY*: We evaluate the performance of the VDI algorithm in three famous EON topologies. We simulate them in case (1), with 30 requests and  $C \in \{25, 40, 55, 70, 85, 100\}$  in Tables V - VII. We can see that VDI performs well in the real EON topologies. Furthermore, the VDI can get the optimal or an almost optimal solution in US Backbone or GERMANY with all the value of spectral resource  $C$ , and performs well than in NSFNET. This can be interpreted as follows. Note that NSFNET is a small topology with 14 nodes, while US Backbone has 28 nodes, and GERMANY has 50 nodes. The larger the topology, the fewer common links all lightpath pairs will have. When the number of common links is small, the VDI algorithm can perform



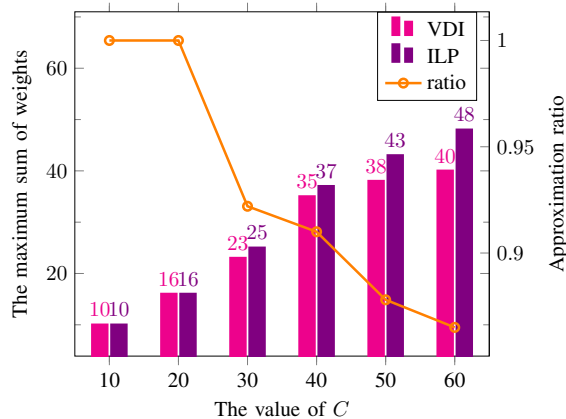


Fig. 13. Numerical results for complete graphs in case (2).

well.

TABLE V  
NUMERICAL RESULTS FOR NSFNET

C	25	40	55	70	85	100
<b>VDI</b>	14	17	23	25	26	28
<b>ILP</b>	14	18	23	27	28	30

TABLE VI  
NUMERICAL RESULTS FOR US BACKBONE

C	25	40	55	70	85	100
<b>VDI</b>	11	20	22	23	27	28
<b>ILP</b>	11	20	22	24	28	29

TABLE VII  
NUMERICAL RESULTS FOR GERMANY

C	25	40	55	70	85	100
<b>VDI</b>	18	22	25	28	29	29
<b>ILP</b>	18	22	26	28	29	30

Based on the above simulations, we can conclude that the VDI algorithm closely approximates the optimal solution for the max-DSA problem in both randomly generated scenarios and realistic EONs.

## VIII. CONCLUSIONS

In this paper, we study the max-DSA problem in EONs. First, we propose an ILP model to solve it optimally and give the upper and lower bounds of its optimal value. Then, we introduce the VDI algorithm to solve max-DSA efficiently and prove that its time complexity is  $\mathcal{O}(|V|^4)$ . We also give a polynomial-time algorithm, namely ISA, to get the optimal solution for complete conflict graphs under some constraints. Finally, we present some simulation results to demonstrate that the VDI algorithm can find near-optimal solutions for max-DSA in various conflict graphs.

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