Spectrum Management in Elastic Optical Networks: Perspectives of Topology, Traffic and Routing

Haitao Wu, Fen Zhou, *Senior Member, IEEE*, Zuqing Zhu, *Senior Member, IEEE*, Yaojun Chen

***Abstract*—Elastic Optical Network (EON) has been considered as a promising optical networking technology to architect the next-generation backbone networks. The spectrum management in EONs is directly determined by the Routing and Spectrum Assignment (RSA). Generally, the RSA is solved by routing the requests with lightpaths first and then assigning spectrum resources to the lightpaths to optimize the spectrum usage. Thus, the spectrum assignment explicitly determines the spectrum usage. Besides, the network topology, traffic distribution and routing scheme implicitly impact the spectrum usage. However, few related work involves this implicit impact. In this paper, we aim to provide a thoroughly theoretical analysis on the impact of the three key factors on the spectrum usage. To this end, two theoretical chains are proposed: (1) The optimal spectrum usage can be measured by the chromatic number of the conflict graph, which is positively correlated to the intersecting probability, *i.e.*, the smaller the intersecting probability, the smaller the optimal spectrum usage; (2) The intersecting probability is determined by the network topology, traffic distribution and routing scheme via a quadratic programming parameterized with a matrix of conflict coefficients. The effectiveness of our theoretical analysis has been validated by extensive numerical results.**

***Index Terms*—Elastic Optical Networks (EONs), Spectrum Management, Network Topology, Traffic Distribution, Routing, Conflict Coefficients.**

I. Introduction

RECENTLY, various types of connection requests proliferate rapidly in the backbone networks. Since the traffic bandwidths are growing exponentially while the spectrum resources in the optical layer are not unlimited, highly-efficient and flexible optical networking technologies have attracted intensive research interests [1]. Specifically, due to fact that the flexibility of traditional Wavelength Division Multiplexing (WDM) networks is restricted by the fixed grids, the flexible-grid Elastic Optical Networks (EONs) have been considered as promising replacements to architect the next-generation backbone networks [2]. In an EON, the spectrum resources on each fiber link are divided into narrow-band (*i.e.*, 12.5 GHz or less) Frequency Slices (FSs), with which the EON can allocate just enough bandwidths to satisfy each request adaptively [3]. Therefore, the spectrum utilization can be effectively improved in EONs in contrast to the WDM networks.

The spectrum management is one of the fundamental problems for service provisioning in EONs, which is directly affected by the Routing and Spectrum Assignment (RSA) [4], *i.e.*, how to route a request from its source to its destination by a lightpath, and then assign a block of available FSs on it. The RSA problem has been proven to be $\mathcal{NP}$-hard in [4]. Similar to Routing and Wavelength Assignment (RWA) problem in WDM networks [5], two variants of the RSA problem have been studied in the literature: max-RSA and min-RSA. For the former problem, the focus is on the provisioning over an EON under limited spectrum resource, and the objective is to maximize the number of requests that can be served. The latter one has a planning concern and its objective is to minimize the required spectrum usage to serve all the requests. In this paper, we focus on the latter one with planning concern.

Although the spectrum assignment is the direct determinant, the network topology, traffic distributions and routing scheme have implicitly impacts on the spectrum management. Once a set of requests with a certain traffic distribution arrive in an EON, they should be routed on lightpaths and a spectrum assignment algorithms should be used to obtain the optimal spectrum usage. From another perspective, the optimal spectrum usage is the lower bound on the final spectrum usage that can not be reduced no matter how the spectrum assignment optimizes. The optimal spectrum usage, *i.e.*, the lower bound on the final spectrum usage, apparently stems from three other factors: network topology, traffic distributions and routing scheme. While the RSA problem has been extensively studied [6]–[9] by heuristics, Integer Linear Programming (ILP) and graph theory, currently few related works are involved in this topic. **The grasp of this implicit impact mechanism of the network topology, traffic distribution and routing scheme can help to adjust these three factors to optimize spectrum usage so as to boost the performance of EONs.**

By making the connection between the chromatic number and intersecting probability, we deduce two theoretical chains to rigorously analyze the impact of those three key factors on the spectrum usage in EONs. **The theoretical chain 1**: The optimal spectrum usage is positively correlated to the lightpath intersecting probability of any two requests. **The theoretical chain 2**: The network topology, traffic distribution, together with routing scheme, through a bridge named Global Optimal Formulation (GOF), determine the intersecting probability. The main contributions of this work are summarized as follows:

H. Wu is with the Department of Mathematics, Nanjing University, Nanjing 210093, China, and he is also with the CERI-LIA at the University of Avignon, France. (email: haitao.wu@alumni.univ-avignon.fr).

F. Zhou is with the LISITE lab of the Institut Superieur d'Electronique de Paris. He is also with the CERI-LIA at the University of Avignon, France. (emails: fen.zhou@{isep, univ-avignon}.fr).

Z. Zhu is with the School of Information Science and Technology, University of Science and Technology of China, Hefei, Anhui 230027, China. (e-mail: zqzhu@ieee.org).

Y. Chen is with Department of Mathematics, Nanjing University, Nanjing 210093, China. (email: yaojunc@nju.edu.cn).

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- We give the upper and lower bounds of the optimal spectrum usage by analyzing the chromatic number of the conflict graph, which is a non-trivial extension of the counterpart in the WDM networks.
- By leveraging random graph theory, we provide an analytical approach on how to connect the chromatic number of conflict graph with the intersecting probability. Meanwhile, we propose a conflict matrix consisting of conflict coefficient, which is a conditional intersecting probability determined by the network topology and traffic distribution. Then, the optimal routing scheme can be obtained by introducing a quadratic programming named Global Optimal Formulation (GOF), which in turn determines the intersecting probability.
- Within the proposed theoretical analysis, we evaluate three realistic EONs under two traffic distributions. Extensive simulations verify the effectivenesses of our theoretical deductions.

The remaining of this paper is organized as follows. Sec. II introduces the related work and our motivation. We present the spectrum management problem and the conflict graph in Sec. III. Then, we reveal the relation between the optimal spectrum usage and the chromatic number of conflict graph in Sec. IV. In Sec. V, we introduce the connection between the intersecting probability and the chromatic number, and propose the conflict coefficients and the quadratic programming GOF. Within the proposed theoretical analysis, we evaluate three EONs under two traffic distributions in Sec. VI. Extensive simulations under different scenarios are conducted in Sec. VII. Finally, Sec. VIII summarizes this paper.

II. Related Work and Motivation

The spectrum management in EONs directly affected by the RSA problem, which is $\mathcal{NP}$-hard [4] and can be naturally separated into two subproblems [9]: lightpath routing and spectrum assignment. In the lightpath routing, a request should be routed on an appropriate lightpath to connect its source to its destination. The spectrum assignment, relatively analogous to the graph coloring problem [10], is to allocate available contiguous and continued FSs on the lightpath. Although the spectrum assignment directly decides the final spectrum usage, the routing scheme is also critical to the final spectrum usage. Meanwhile, the network topology and traffic distribution should also be taken in to account when planning the routing scheme. Since under different circumstances, the strategy of the routing scheme should be adaptive. Many routing schemes have been proposed [6]–[8]. These routing schemes are executed in a relatively common way: (1) First pre-calculate a set of candidate paths for each source-destination pair; (2) then select the "best" path to the request based on some heuristic principles such as "the least congested" [6] and "the smallest load" [8]. A comprehensive survey can be found in [9].

Although many theoretical works [4, 11, 12] on the RSA problem have been proposed by tools such as ILP or mixed ILP. However, currently there is no theoretical work to interpret the implicit impact of the network topology, traffic distribution and routing scheme on the spectrum usage. Meanwhile, the ultimate goal of this research is to determine the optimal routing scheme while taking into account the network topology and traffic distribution. To the best of our knowledge, there has been no previous work on this topic. In this work, by leveraging random graph theory, we give a theoretical explanation about the implicit impact of the three factors on the spectrum usage.

TABLE I
Notations

$G(V,E)$	EON, where V is the node set and E is the directed link set.
$\mathcal{D}$	The traffic distribution, which specially refers to the distribution of occurrence probabilities of source-destination pairs.
w_{sd}	The occurrence probability of source-destination pair (s,d) determined by the traffic distribution $\mathcal{D}$.
$\mathcal{R}$	The set of connection requests in $G(V,E)$.
n	$=\lvert\mathcal{R}\rvert$, the number of connection requests.
$R_i(s_i,d_i)$	$R_i \in \mathcal{R}$ represents the i-th request, where $s_i, d_i \in V$ are the source and destination nodes respectively generated by $\mathcal{D}$.
$\mathbb{N}^+$	The set of positive natural numbers representing the FS index set in the spectrum domain lying in each fiber link $e \in E$.
α/β	$\alpha, \beta \in \mathbb{N}^+$ the upper/lower bound of the bandwidths required by requests.
R_i^w	The number of contiguous FSs (bandwidth requirement) required by R_i, which is in the range of $[\alpha,\beta]$.
$\mathcal{P}_i$	The set of all the directed paths from s_i to d_i in $G(V,E)$.
P_i	$P_i \in \mathcal{P}_i$ is the directed path on which R_i is routed.
W_i	The set of contiguous FSs assigned to R_i.
R_i^b	$R_i^b \in \mathbb{N}^+$ is the start-index of W_i.
R_i^a	$R_i^a \in \mathbb{N}^+$ is the end-index of W_i.
GB	$GB \in \mathbb{N}^+$ is the number of FSs required for the guard-band.
MUFI	$=\max_{s\in(\cup W_i)}(s)$, is the maximum used FS index.
$\hat{G}(\hat{V},\hat{E})$	The conflict graph (weighted undirected graph), where $\hat{V}$ is the vertex set corresponding to $\mathcal{R}$, and $\hat{E}$ is the edge set.
$\hat{v}_i/\hat{v}_i^w$	$\hat{v}_i \in \hat{V}$ corresponds to R_i, and its vertex weight $\hat{v}_i^w = R_i^w$.
$W_{\hat{v}_i}$	The set of contiguous FSs assigned to $\hat{v}_i$.
$\hat{v}_i^b/\hat{v}_i^a$	$\hat{v}_i^b/\hat{v}_i^a \in \mathbb{N}^+$ is the start/end-index of $W_{\hat{v}_i}$.
$\hat{v}^w_{I(i)}/\hat{v}^w_{D(i)}$	The i-th biggest/smallest vertex weight in $\hat{V}$ *i.e.*, $\hat{v}^w_{I(1)} \geq \hat{v}^w_{I(2)} \geq ... \geq \hat{v}^w_{I(n)}$; $\hat{v}^w_{D(1)} \leq \hat{v}^w_{D(2)} \leq ... \leq \hat{v}^w_{D(n)}$.
$opt(\hat{G})$	The optimal spectrum assignment for $\hat{G}(\hat{V},\hat{E})$.
$\lvert opt(\hat{G})\rvert$	The MUFI of $opt(\hat{G})$, which is the optimal one.
$\chi(\hat{G})$	The chromatic number of $\hat{G}$.
p	The intersecting probability of any two requests.
K	The number of candidate paths for a source-destination pair.
p_i	The probability for a request to be routed on the i-th shortest candidate path.
θ_{ij}	The conflict coefficient that represents the intersecting probability of any two requests under the condition that one is routed on the i-th candidate path and the other on the j-th path of their own source-destination pairs.
CM	$=[\theta_{ij}]$, the real symmetric Conflict Matrix composed of θ_{ij}.

III. Network Model and Problem Description

In this section, we present the network model considered in this paper and the formulation of the RSA. Some necessary notations are summarized in Table I.

A. Network Model

1) Network Topology: We use a directed graph $G(V,E)$ to represent the topology of an EON, where V and E denote the sets of nodes and directed fiber links respectively. A set of FSs lies on each directed fiber link.

2) Traffic Distribution: A request R in the EON $G(V,E)$ is a source-destination pair $(s \in V, d \in V)$ together with a bandwidth requirement R^w. Given an EON $G(V,E)$, there are $|V| \times (|V|-1)$ kinds of source-destination pairs. In this paper, we treat each request R as a random variable whose (s,d) is generated from a traffic distribution $\mathcal{D}$. The $\mathcal{D}$ specially refers to the distribution of occurrence probabilities of the $|V| \times (|V|-1)$ source-destination pairs. For example, assuming that request R is generated from the traffic distribution $\mathcal{D}$ in Table II, the (s,d) of the R has 45% probability to be (v_1,v_3) or (v_3,v_1) and 1% to be others, which implies that v_1 and v_3 are two important data center nodes in the EON.

TABLE II
TRAFFIC DISTRIBUTION $\mathcal{D}$

Node Pair $(s,\ d)$	Occurrence Probability	Node Pair $(s,\ d)$	Occurrence Probability
(v_1,v_2)	$w_{v_1v_2}=1\%$	(v_2,v_1)	$w_{v_2v_1}=1\%$
(v_1,v_3)	$w_{v_1v_3}=45\%$	(v_3,v_1)	$w_{v_3v_1}=45\%$
(v_1,v_4)	$w_{v_1v_4}=1\%$	(v_4,v_1)	$w_{v_4v_1}=1\%$
(v_2,v_3)	$w_{v_2v_3}=1\%$	(v_3,v_2)	$w_{v_3v_2}=1\%$
(v_2,v_4)	$w_{v_2v_4}=1\%$	(v_4,v_2)	$w_{v_4v_2}=1\%$
(v_3,v_4)	$w_{v_3v_4}=1\%$	(v_4,v_3)	$w_{v_4v_3}=1\%$

In a backbone EON $G(V,E)$, the traffic distribution $\mathcal{D}$ usually has some statistical characteristics [13] in a period and can be approached by some statistical methods. In this paper, we assume that there is a set of n requests $\mathcal{R}=\{R_1,R_2,...,R_n\}$ to be served, and each $R_i(s_i,d_i)$ is generated from a given traffic distribution $\mathcal{D}$.

3) Routing Scheme: In general, the number of paths connecting a source-destination pair is exponential in an EON. It is impossible to enumerate all possible paths. Thus, a practical way is to pre-compute a set of K (K is a preset constant) shortest candidate paths for each source-destination pair by some K-shortest path algorithms, and select one from these candidate paths to connect the source-destination pair of the request [6]–[8]. The most important thing for a routing scheme is how to appropriately dispense these K candidate paths to these source-destination pairs of requests. We use p_i to denote the percentage of requests which are routed on the i-th shortest candidate paths of their own source-destination pairs, and thus $\sum_{i=1}^{K} p_i = 1$. From the perspective of probability, since each request $R(s,d)$ is treated as a random variable in this paper, each p_i can also be interpreted as the probability of the request routed on the i-th shortest candidate path (Since $\sum_{i=1}^{K} p_i = 1$, this interpretation is well-defined). The proportion array $(p_1,p_2,...,p_K)$ is determined by the routing scheme. Thus, in this paper, the routing scheme is represented by the array $(p_1,p_2,...,p_K)$. For example, the array $(1,0,...,0)$ represents the routing scheme that select the first shortest candidate paths to connect all source-destination pairs of requests, which corresponds to the shortest-path routing scheme in [9].

B. Problem Description

In this paper, we study the spectrum management problem with a planning concern, *i.e.*, given a set of requests $\mathcal{R}$ in an EON $G(V,E)$, we intend to minimize the spectrum usage in the optical fibers. For each request $R_i(s_i,d_i)\in\mathcal{R}$, we need to establish a lightpath and assign enough bandwidths on it so as to forward the data of the request. More specially, the RSA problem consists in selecting a lightpath P_i from the set $\mathcal{P}_i$ for R_i and assigning just enough FS set W_i on this lightpath while satisfying the following constraints:

- **Bandwidth Requirement Constraint**. The number of FSs assigned to each request should no smaller than its bandwidth requirement, *i.e.*, the cardinality of W_i assigned to R_i must be equal to its weight:
$$|W_i| = R_i^w, \quad \forall R_i \in \mathcal{R}. \tag{1}$$
Without loss of generality, we make the following assumption in this paper.
***Hypothesis** 1:* The bandwidth requirement of each request is uniformly distributed in the range of $[\alpha,\beta]$, *i.e.*, $R_i^w \in [\alpha,\beta]\ \forall i$, where α and β are two constants, *e.g.*, $[\alpha,\beta]=[1,2]$ in [8] or $[1,3]$ in [14].
- **Spectrum Contiguity Constraint**. The FSs assigned to request R_i must be contiguous in the spectrum domain, *i.e.*, $\mathbb{N}^+$. Thus, W_i can be expressed as $\{R_i^b, R_i^b+1,...,R_i^a-1,R_i^a\}$. This is a physical layer constraint for all-optical communications.
- **Spectrum Continuity Constraint**. All the directed fiber links on the lightpath for R_i (*i.e.*, $e\in P_i$) should be assigned with the same set of contiguous FSs W_i.
- **Guard Band Constraint**. To mitigate mutual interference, when P_i and P_j share common some directed fiber link(s), the distance between W_i and W_j in the spectrum domain should be no less than GB:
$$distance(W_i,W_j) \geq GB, \quad \forall P_i \cap P_j \neq \emptyset, \tag{2}$$
where,
$$distance(W_i,W_j) = \min_{s\in W_i, t\in W_j} (|s-t|-1)\,.$$

For planning purposes, the RSA problem studied in this paper aims to minimize the **Maximum Used FS Index** (MUFI), which can be expressed by Eq. (3):

$$\min\left[\max_{s\in(\cup W_i)}(s)\right] \qquad (\textbf{RSA}). \tag{3}$$

Given an EON G, a traffic distribution $\mathcal{D}$ and a routing scheme $(p_1,p_2,...,p_K)$, the studied objective of this paper is to figure out how the three factors impact on the optimal MUFI, *i.e.*, the lower bound on the final spectrum usage as mentioned in the introduction.

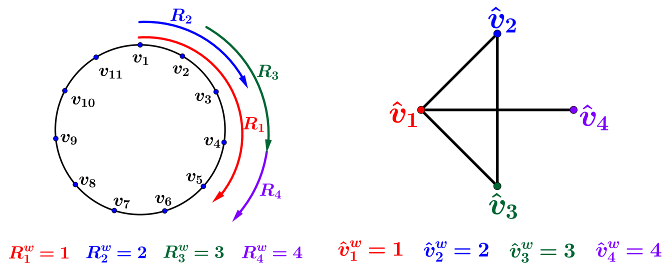
C. Conflict Graph

After the set of requests $\mathcal{R}=\{R_1,R_2,...,R_n\}$, generated from the traffic distribution $\mathcal{D}$, arrive at the EON $G(V,E)$ and are routed on their lightpaths by the routing scheme $(p_1,p_2,...,p_K)$, we can construct an auxiliary graph, conflict graph, which embodies the three factors' impact on the optimal MUFI. The formal definition is given as follows.

***Definition** 1:* The *conflict graph* [15] $\hat{G}(\hat{V},\hat{E})$ is such **a weighted undirected graph** whose vertex set $\hat{V}$ represents the set of requests, *i.e.*, $\mathcal{R}$. Any two vertices $\hat{v}_i,\hat{v}_j\in\hat{V}$ (representing R_i and R_j respectively) are connected by an edge $\hat{e}\in\hat{E}$, *i.e.*, they are adjacent in $\hat{G}$, if and only if P_i **intersects with** P_j, *i.e.*, $P_i\cap P_j\neq\emptyset$ **(at least one directed**

fiber link shared by P_i **and** P_j), where P_i and P_j are the lightpaths selected for R_i and R_j respectively. We denote by $\hat{v}_i^w$ the weight of vertex $\hat{v}_i$, and $\hat{v}_i^w = R_i^w$.

Besides, we can also define *proper spectrum assignment for the conflict graph*: Let $\hat{v}_i^b$, $\hat{v}_i^a$ and $W_{\hat{v}_i}$ have the same meanings as R_i^b, R_i^a and W_i respectively, and $|W_{\hat{v}_i}| = \hat{v}_i^w$. If $\hat{v}_i$ and $\hat{v}_j$ are adjacent in $\hat{G}$, then the distance between $W_{\hat{v}_i}$ and $W_{\hat{v}_j}$ should be no less than GB. Then, these $W_{\hat{v}_i}$ compose of a proper spectrum assignment for the conflict graph.



(a) Routed lightpaths. (b) Conflict graph.

Fig. 1. The conflict graph of the routed lightpaths.

Figure 1(b) showcases a 4-node conflict graph for the 4 requests (they are already routed on their lightpaths) in Fig. 1(a), where $\hat{v}_i$ corresponds to $R_i, \forall 1 \le i \le 4$.

According to the definition, obviously, any proper spectrum assignment for the conflict graph corresponds to a proper spectrum assignment for these lightpaths, vice versa. Thus, the optimal spectrum assignment $opt(\hat{G})$ for the conflict graph, which produces the minimum MUFI $|opt(\hat{G})|$, corresponds to the optimal spectrum assignment for these lightpaths. Therefore, the optimal MUFI is equal to $|opt(\hat{G})|$.

IV. Optimal MUFI and Chromatic Number of Conflict Graph

In this section, we explore the relation between the $|opt(\hat{G})|$ and the chromatic number $\chi(\hat{G})$[1] of the conflict graph $\hat{G}$.

A. Optimal MUFI and Chromatic Number

In the WDM networks, the parallel relation is that the minimum number of wavelengths used on the conflict graph is equal to its chromatic number. However, in EONs, how to determine the $|opt(\hat{G})|$ for the conflict graph $\hat{G}$ has not been investigated yet. Thus, we address this issue by giving Theorem 1.

***Theorem** 1:* Let $\hat{G}(\hat{V}, \hat{E})$ be the conflict graph, then

$$(\chi(\hat{G})-1)\cdot GB + \sum_{i=1}^{\chi(\hat{G})} \hat{v}_{D(i)}^w \le |opt(\hat{G})| \le (\chi(\hat{G})-1)\cdot GB + \sum_{i=1}^{\chi(\hat{G})} \hat{v}_{I(i)}^w. \tag{4}$$

where $\hat{v}_{I(i)}^w$ and $\hat{v}_{D(i)}^w$ are the i-biggest and i-smallest vertex weight in $\hat{V}$ respectively, and $\chi(\hat{G})$ is the chromatic number of $\hat{G}$.

Proof. Due to space limitation, we give here a brief proof and the detailed version can be found in our report [16]. We prove the upper bound by finding a feasible spectrum assignment f whose MUFI is not bigger than it. This solution f can be obtained in such way: (1) Separating $\hat{V}$ into $\chi(\hat{G})$ disjoint independent sets[2] ; (2) assigning FSs for each independent set (The number of FSs assigned is equal to the biggest vertex weight in this set); (3) patching them up. It is easy to see the MUFI of f is not bigger than the upper bound.

Now, we prove the lower bound. Let $opt(\hat{G}) = \{W_{\hat{v}}, \forall \hat{v} \in \hat{V}\}$ be the optimal spectrum assignment. With respect to $opt(\hat{G})$, let $A = \{\hat{v}^a, \forall \hat{v} \in \hat{V}\}$ and $B = \{\hat{v}^b, \forall \hat{v} \in \hat{V}\}$ be the optimal end-index and start-index sets for all $W_{\hat{v}}$ in $opt(\hat{G})$ respectively. We then can separate $\hat{V}$ into different parts as follows:

First, let, *w.l.o.g.*, $\hat{v}_1$ be the vertex whose end-index $\hat{v}_1^a$ is the minimum in A, then we assert that $\mathcal{F}_1 = \{\hat{v} | \hat{v}^b \le \hat{v}_1^a + GB, \forall \hat{v} \in \hat{V}\}$ is an independent set of $\hat{G}$, *i.e.*, for any two vertices $\hat{v}_i, \hat{v}_j \in \mathcal{F}_1$, $\hat{v}_i$ is not adjacent to $\hat{v}_j$ in $\hat{G}$. We prove it by contradiction. Assuming $\hat{v}_i$ is adjacent to $\hat{v}_j$ in $\hat{G}$ and $W_{\hat{v}_i} = [\hat{v}_i^b, \hat{v}_i^a]$ and $W_{\hat{v}_j} = [\hat{v}_j^b, \hat{v}_j^a]$, we have $distance(W_{\hat{v}_i}, W_{\hat{v}_j}) \ge GB$. Thus, we, *w.l.o.g.*, can assume $\hat{v}_i^a + GB < \hat{v}_j^b$. However, according to $\mathcal{F}_1$, we have $\hat{v}_j^b \le \hat{v}_1^a + GB$ and $\hat{v}_i^a \ge \hat{v}_1^a$ (Since $\hat{v}_1^a$ is the minimum in all end-indices), which means $\hat{v}_j^b \le \hat{v}_i^a + GB$, a contradiction. Therefore $\mathcal{F}_1$ is an independent set of $\hat{G}$.

Next, let, *w.l.o.g.*, $\hat{v}_2$ be the vertex whose end-index is the minimum in $\hat{V} \backslash \mathcal{F}_1$, and similarly, we can assert $\mathcal{F}_2 = \{\hat{v} | \hat{v}^b \le \hat{v}_2^a + GB, \forall \hat{v} \in \hat{V} \setminus \mathcal{F}_1\}$ is an independent set of $\hat{G}$. After finite steps using the same technique, say k, we can separate $\hat{V}$ into k disjoint independent sets: $\mathcal{F}_1$, $\mathcal{F}_2$,..., $\mathcal{F}_k$. Besides, according to the principle of selecting $\mathcal{F}_i, \forall 1 \le i \le k$, we have $\hat{v}_i^a + GB < \hat{v}_{i+1}^b, \forall 1 \le i \le k$. Therefore, we have $(k-1)\cdot GB + \sum_{i=1}^k \hat{v}_i^w \le |opt(\hat{G})|$.

Finally, according to the definition of chromatic number that $\chi(G)$ is the minimum number of independent sets into which we can separate $\hat{V}$, so we immediately have $\chi(G) \le k$. Besides $\sum_{i=1}^k \hat{v}_{D(i)}^w \le \sum_{i=1}^k \hat{v}_i^w$, we hence have $(\chi(\hat{G})-1)\cdot GB + \sum_{i=1}^{\chi(\hat{G})} \hat{v}_{D(i)}^w \le (k-1)\cdot GB + \sum_{i=1}^k \hat{v}_{D(i)}^w \le (k-1)\cdot GB + \sum_{i=1}^k \hat{v}_i^w \le |opt(\hat{G})|$. ■

Theorem 1 reveals the relation between the chromatic number of the conflict graph $\hat{G}$ and the optimal MUFI $|opt(\hat{G})|$, which is a non-trivial generation from the RWA (Since in the RWA, the guard band constraint does not exists so $GB = 0$, and each vertex weight is the same as a unit wavelength.

From above discussions, we can see that the spectrum assignment, closely analogous to the graph coloring problem, is intractable, and the $|opt(\hat{G})|$ is bounded by the chromatic number of the conflict graph. Therefore, the three factors of the network topology, traffic distribution and routing scheme are critical for the final performance of the RSA, since they decide the property of the conflict graph $\hat{G}$. How to reduce the chromatic number $\chi(\hat{G})$, thereby the $|opt(\hat{G})|$, is pivotal.

[1]The chromatic number is the minimum number of colors needed to color $\hat{G}$ such that adjacent vertices do not share a same color.

[2]According to the definition of chromatic number, each monochrome part is an independent set, *i.e.*, no two vertices of which are adjacent.

V. Theoretical Chains of the Impact

In the previous section, we theoretically deduced the impact of the $\chi(\hat{G})$ on the $|opt(\hat{G})|$. Now, the important thing is to study how the network topology G, traffic distribution $\mathcal{D}$ and routing scheme $(p_1, p_2, ..., p_K)$ affect the $\chi(\hat{G})$.

To this end, we need some preparations in random graph theory. We first introduce the concept of intersecting probability and its relation with the chromatic number. Then, we deduce how the intersecting probability can be influenced by the network topology G, traffic distribution $\mathcal{D}$ and routing scheme $(p_1, p_2, ..., p_K)$. Thus, by the connection between the intersecting probability and the chromatic number, we finally figure out the impacts of the three factors on the optimal spectrum usage.

A. Intersecting Probability and Chromatic Number

***Definition** 2:* In the EON $G(V, E)$, the *intersecting probability* (denoted by p) of any two requests (say R_1 and R_2), generated from $\mathcal{D}$ and routed by $(p_1, p_2, ..., p_K)$, is the probability that their lightpaths P_1 and P_2 share at least one common fiber link, *i.e.*, $P_1 \cap P_2 \neq \emptyset$.

Given $\mathcal{R} = \{R_1, R_2, ..., R_n\}$, the vertex set of the conflict graph $\hat{G}(\hat{V}, \hat{E})$ is that $\hat{V} = \{\hat{v}_1, \hat{v}_2, ..., \hat{v}_n\}$. Thus, the edge set $\hat{E}$ determines the final $\chi(\hat{G})$. For any two vertices, say $\hat{v}_1, \hat{v}_2 \in \hat{V}$, the probability that they are adjacent in the conflict graph $\hat{G}$ is equal to the intersecting probability p. We introduce an important Lemma in random graph theory which reveals the relation between the intersecting probability p and the chromatic number $\chi(\hat{G})$.

***Lemma** 1:* [17] Let $|\hat{V}| = n$, p be the intersecting probability, and $\hat{G}$ be the conflict graph, then we have $\chi(\hat{G}) = (\frac{1}{2} + o(1)) \cdot \log(\frac{1}{1-p}) \cdot \frac{n}{\log n}$, where $o(1)$ is an infinitesimal of n.

From Lemma 1, we can see that there is a strongly positive correlation between the $\chi(\hat{G})$ and the intersection probability p. More specifically, the smaller the intersection probability p, the smaller the $\chi(\hat{G})$, which conforms to our intuition. Meanwhile, by Theorem 1, the smaller the chromatic number $\chi(\hat{G})$, the smaller the $|opt(\hat{G})|$. Therefore, we obtained the first important theoretical chain in Fig. 2.

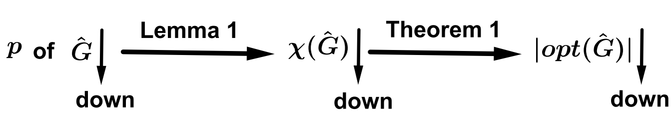


Fig. 2. Theoretical Chain 1

Now, the question is that how to determine the intersecting probability p. Obviously, the p depends on the three factors: 1 the network topology $G(V, E)$, 2 the traffic distribution $\mathcal{D}$ and 3 the routing scheme $(p_1, p_2, ..., p_K)$.

B. Conflict Coefficients

Here, we propose a new concept named **conflict coefficient** of an EON, which plays an important role for our next theoretical analysis.

***Definition** 3:* The *conflict coefficient* θ_{ij} is **the intersecting probability** of any two requests (say R_1 and R_2) generated from $\mathcal{D}$, **under the condition that** R_1 and R_2 are respectively routed on the i-th and j-th shortest paths of their own source-destination pairs in the EON G.

Thus, the θ_{ij} is only related to network topology $G(V, E)$ and traffic distribution $\mathcal{D}$. The θ_{ij} can be computed as follows. We first generate the i-th and j-th shortest paths for all $|V| \times (|V| - 1)$ source-destination pairs in $G(V, E)$. We construct a matrix $M^{\theta_{ij}}$ in Table III.

TABLE III
Matrix $M^{\theta_{ij}}$ of the conflict coefficient θ_{ij}.

		On the j-th path					
		.	.	(s_2, d_2) $w_{s_2 d_2}$	.	.	
	.	.	.	.	.	.	
On	.	.	.	.	.	.	
the	(s_1, d_1)	.	.	$w_{s_1 d_1} \times w_{s_2 d_2}$	.	.	$\|V\| \times (\|V\| - 1)$
i-th	$w_{s_1 d_1}$	.	.	.	.	.	
path	.	.	.	.	.	.	
	.	.	.	.	.	.	
		$\|V\| \times (\|V\| - 1)$					

The top row in Table III represents all the $|V| \times (|V| - 1)$ source-destination pairs, which are routed on their own j-th shortest paths, while the leftmost column represents the $|V| \times (|V| - 1)$ source-destination pairs, which are routed on their own i-th shortest paths. The weight w_{sd} for each source-destination pair (s, d) is its occurrence probability determined by the traffic distribution $\mathcal{D}$. If the i-th shortest path of (s_1, d_1) intersects with the j-th shortest path of (s_2, d_2) in $G(V, E)$, then this entry is $w_{s_1 d_1} \times w_{s_2 d_2}$, otherwise 0. According to Definition 3, conflict coefficient $\theta_{ij} = \sum M^{\theta_{ij}}$, where $\sum M^{\theta_{ij}}$ represents the sum of all entries in the matrix.

Following the same idea, we can compute all the conflict coefficients of an EON G under a traffic distribution $\mathcal{D}$, which compose **a real symmetric Conflict Matrix (CM)** as below.

$$CM = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} & \dots & \theta_{1K} \\ \theta_{21} & \theta_{22} & \theta_{23} & \dots & \theta_{2K} \\ \theta_{K1} & \theta_{K2} & \theta_{K3} & \dots & \theta_{KK} \end{bmatrix}$$

C. Global Optimal Formulation (GOF)

Under the routing scheme $(p_1, p_2, ..., p_K)$, for any two requests, say R_1 and R_2, the probability that the R_1 are routed on the i-th shortest candidate path and R_2 on the j-th shortest candidate path is $p_i p_j$. Combing with the conflict coefficient θ_{ij}, the conditional intersecting probability that any two requests, routed on i-th and j-th shortest candidate paths respectively, intersect is $\theta_{ij} p_i p_j$. Thus, the intersecting probability p is equal to the sum of all the conditional intersecting intersecting probabilities. We obtain a Global Optimal Formulation (GOF) as follows, which is a quadratic programming and determines the intersecting probability p:

$$p = \sum_{1 \leq i,j \leq K} \theta_{ij} p_i p_j \quad (\textbf{GOF}), \tag{5}$$

$$\textbf{s.t.} \quad \sum_{i=1}^{K} p_i = 1, \tag{6}$$

$$p_i \geq 0, \quad 1 \leq i \leq K. \tag{7}$$

Here, K is the number of pre-computed candidate paths for each source-destination pair, and $\theta_{ij}, \forall i, j$ are the conflict coefficients. In GOF, K is predetermined, and θ_{ij} are the parameters determined by the the network topology G and traffic distribution $\mathcal{D}$, while p_i is determined by the routing scheme. Thus, the complexity of the quadratic programming is with one constraint, K variables and K^2 parameters. By now, we summarize what we have obtained in Fig. 3 which is another important theoretic chain.

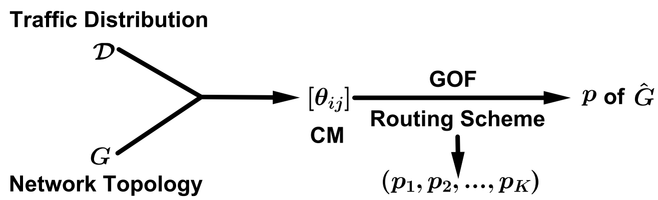


Fig. 3. Theoretical Chain 2

Combing Theoretic Chain 1 in Fig. 2 with Theoretic Chain 2 in Fig. 3, we finally figured out how the network topology G, traffic distribution $\mathcal{D}$ and routing scheme $(p_1, p_2, ..., p_K)$ impact on the optimal spectrum usage.

VI. CM Estimation and Optimal Routing Scheme in Realistic EONs

In this section, we estimate the conflict coefficients and solve the corresponding GOFs in three realistic EONs under two traffic distributions respectively. The three EONs are, as shown in Fig. 4, the Ring, NSFNET and NJ-LATA [18] respectively. The three EONs are of almost the same size in terms of the number of nodes. Thus, we shall also compare the three EONs from the perspective of intersecting probability. **Since in the Ring, there are only two candidate paths for each source-destination pair, to make a fair comparison and a simple and intuitive analysis, we set $K = 2$ for all EONs by pre-computing the first and second shortest paths for each source-destination pair.** Therefore, the intersecting probability p in GOF can be written as

$$p = \begin{bmatrix} p_1 & p_2 \end{bmatrix} \begin{bmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$

where, $p_1 + p_2 = 1$, or

$$p = \theta_{11} \times p_1^2 + 2\theta_{12} \times p_1 p_2 + \theta_{22} \times p_2^2. \tag{8}$$

In the following, we compute p under two traffic distribution scenarios respectively: uniform and weighted.

A. Uniform Traffic Distribution

In this subsection, we consider a uniform traffic distribution $\mathcal{D}$, which is a benchmark traffic distribution in many studies [8, 13], in the three EONs, *i.e.*, each source-destination pair occurs with the same occurrence probability $\frac{1}{|V| \times (|V|-1)}$. Following the computing method in Section V-B, we can get the corresponding conflict coefficients and GOFs of the three EONs under uniform distribution.

1) The Ring under the Uniform Traffic Distribution: The CM in this case is

$$CM_{Ring}^{Uni} = \begin{bmatrix} 0.2328 & 0.4360 \\ 0.4360 & 0.5014 \end{bmatrix}$$

and we can thus get the intersecting probability via the GOF

$$p = 0.2328 \times p_1^2 + 0.8720 \times p_1 p_2 + 0.5014 \times p_2^2. \tag{9}$$

where, $p_1 + p_2 = 1$.

By minimizing Eq. (9), we get the minimum intersecting probability $p_{min} = 23.28\%$. This can be achieved by the optimal routing scheme: $p_1 = 1$ and $p_2 = 0$, *i.e.*, $(1, 0)$.

2) The NSFNET under the Uniform Distribution: The CM in this case is that

$$CM_{NSF}^{Uni} = \begin{bmatrix} 0.0979 & 0.1377 \\ 0.1377 & 0.2042 \end{bmatrix}$$

and thus the intersecting probability by the GOF is

$$p = 0.0979 \times p_1^2 + 0.2754 \times p_1 p_2 + 0.2042 \times p_2^2. \tag{10}$$

where, $p_1 + p_2 = 1$.

By minimizing Eq. (10), the minimum intersecting probability $p_{min} = 9.79\%$ and the optimal routing scheme is $p_1 = 1$ and $p_2 = 0$, *i.e.*, $(1, 0)$.

3) The NJ-LATA under the Uniform Distribution: The CM in this case is that

$$CM_{NJ}^{Uni} = \begin{bmatrix} 0.0901 & 0.0852 \\ 0.0852 & 0.1157 \end{bmatrix}$$

and thus the intersecting probability by the GOF is

$$p = 0.0901 \times p_1^2 + 0.1704 \times p_1 p_2 + 0.1157 \times p_2^2. \tag{11}$$

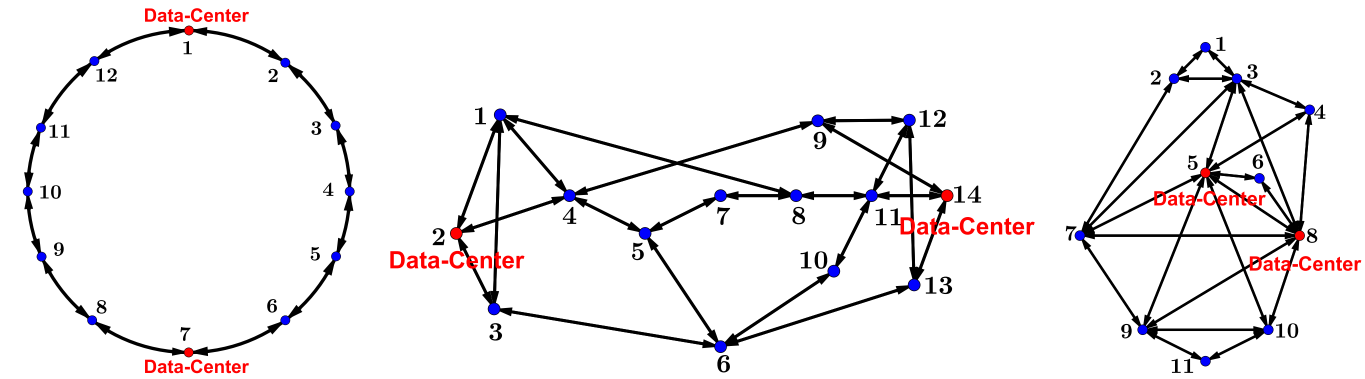
where, $p_1 + p_2 = 1$.

By minimizing Eq. (11), we have the minimum intersecting probability $p_{min} = 8.94\%$ and the optimal routing scheme: $p_1 = 0.8621$, $p_2 = 0.1379$, *i.e.*, $(0.8621, 0.1379)$.

B. Weighted Traffic Distribution

Nowadays, EONs begin to support new networking capabilities and demanding network services such as data centers and cloud. Hence, the Optical Cross-Connect (OXC) in EONs connected to data-centers will produce a large amount of traffic among them, for instance data migration and content provisioning. This kind of traffic contributes to the majority of the total traffic [19].

In this subsection, we assume that there are two data-center nodes in the three considered EONs for simplicity. Both the two data-center nodes have the same big occurrence probability (45% in this paper) to be involved in a request (as source or destination), while the other EON nodes equally



(a) Ring (12 nodes and 12 bidirectional fiber links)

(b) NSFNET (14 nodes and 22 bidirectional fiber links)

(c) NJ-LATA (11 nodes and 23 bidirectional fiber links)

Fig. 4. Three realistic EON topologies.

share the remaining 10% possibility. It should be noted the value of the occurrence probability given here is just for illustrative purpose, which can be in fact arbitrary. We call this distribution as the weighted traffic distribution in this paper.

1) The Ring under the Weighted Traffic Distribution:

We assume that the nodes 1 and 7 in the Ring are the two data-center nodes. Thus, we have its CM

$$CM_{Ring}^{Con} = \begin{bmatrix} 0.3829 & 0.1766 \\ 0.1766 & 0.5000 \end{bmatrix}$$

and its intersecting probability

$$p = 0.3829 \times p_1^2 + 0.3532 \times p_1 p_2 + 0.5000 \times p_2^2. \quad (12)$$

where, $p_1 + p_2 = 1$.

By minimizing Eq. (12), we obtain the minimum intersecting probability $p_{min} = 30.26\%$ and the optimal routing scheme: $p_1 = 0.6105$ and $p_2 = 0.3895$, *i.e.*, $(0.6105, 0.3895)$.

2) The NSFNET with Weighted Traffic Distribution:

We assume that the nodes 2 and 14 in NSFNET are the two data-center nodes. Then, we have its CM

$$CM_{NSF}^{Con} = \begin{bmatrix} 0.3554 & 0.2119 \\ 0.2119 & 0.3982 \end{bmatrix}$$

and its intersecting probability

$$p = 0.3554 \times p_1^2 + 0.4238 \times p_1 p_2 + 0.3982 \times p_2^2. \quad (13)$$

where, $p_1 + p_2 = 1$.

By minimizing Eq. (13), we get the minimum intersecting probability $p_{min} = 29.30\%$. The optimal routing scheme is $p_1 = 0.5648$ and $p_2 = 0.4352$, *i.e.*, $(0.5648, 0.4352)$.

3) The NJ-LATA under the Weighted Traffic Distribution:

Nodes 5 and 8 are assumed to the two data-center nodes in NJ-LATA. In this case, the CM becomes

$$CM_{NJ}^{Con} = \begin{bmatrix} 0.2758 & 0.0616 \\ 0.0616 & 0.3306 \end{bmatrix}$$

and the intersecting probability is given by

$$p = 0.2758 \times p_1^2 + 0.1232 \times p_1 p_2 + 0.3306 \times p_2^2. \quad (14)$$

where, $p_1 + p_2 = 1$.

Thus, the minimum intersecting probability is $p_{min} = 18.08\%$ and the optimal routing scheme is: $p_1 = 0.5568$ and $p_2 = 0.4432$, *i.e.*, $(0.5568, 0.4432)$.

VII. Numerical Results

In this section, we verify the effectivenesses of the two theoretic chains by simulations:

- The effectiveness of Theoretic Chain 2 in Fig. 3, *i.e.*, the conflict coefficients and the computing method for intersecting probability. In other words, whether the theoretical intersecting probability computed by the GOF can fit in with the realistic one.
- The effectiveness of Theoretic Chain 1 in Fig. 2, *i.e.*, the intersecting probability itself. In other words, whether the intersecting probability is positively correlated to the final MUFI of the RSA.

We conduct simulations in Ring (**R**), NSFNET (**NSF**) and NJ-LATA (NJ) with both the uniform (**U**) and the weighted (**W**) traffic distributions. In our simulations, after the lightpath routing, we utilize a same spectrum assignment algorithm MRSA in [8] to assign FS sets to requests. We consider six scenarios: **R-U**, **NSF-U**, **NJ-U**, **R-W**, **NSF-W**, **NJ-W**.

In each scenario, we first pre-compute two candidate paths for each source-destination pair, *i.e.*, the first and second shortest paths as mentioned in Section VI. Eleven routing schemes will be conducted and compared in each scenario by increasing p_1 (the percentage of requests routed on the first shortest path) from 0 to 1 with a step of 0.1. Hence, there are 66 cases in total for the six scenarios. Meanwhile, for each case, we compare the realistic intersecting probability p with the one theoretically estimated by the GOF through substituting the value of p_1 into the corresponding formulation obtained in Section VI. Here the realistic intersecting probability is $\frac{\#\{e\}}{\binom{|\mathcal{R}|}{2}}$, where $\#\{e\}$ is the number of realistic edges in the conflict graph.

For the bandwidth range and guard band size, we set $\alpha = 1$, $\beta = 4$, and $GB = 1$ respectively. The number of requests is set as $|\mathcal{R}| = 1000$ in each simulation. We repeat each simulation

100 times under the same circumstance to ensure sufficient statistical accuracy, and a 95% confidence interval is given to each numerical result. All the simulations have been run by MATLAB 2015a on a computer with 3.2 GHz Intel(R) Core(TM) i5-4690S CPU and 8 GBytes RAM.

A. Uniform Traffic Distribution

We first verify the three scenarios under the uniform traffic distribution: R-U, NSF-U and NJ-U. The corresponding results are demonstrated in Figs. 5, 6 and 7 respectively.

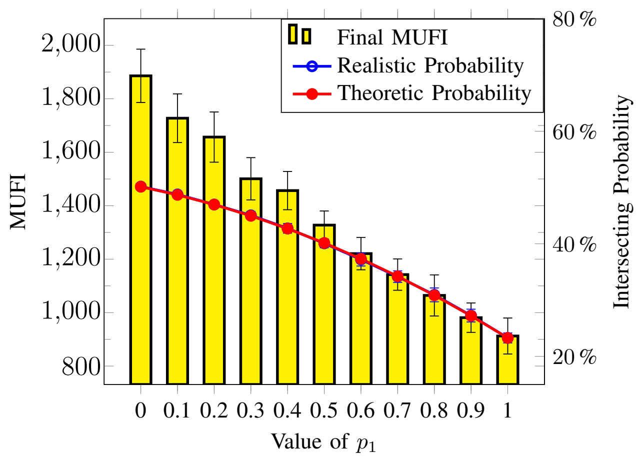


Fig. 5. Numerical results for R-U Scenario.

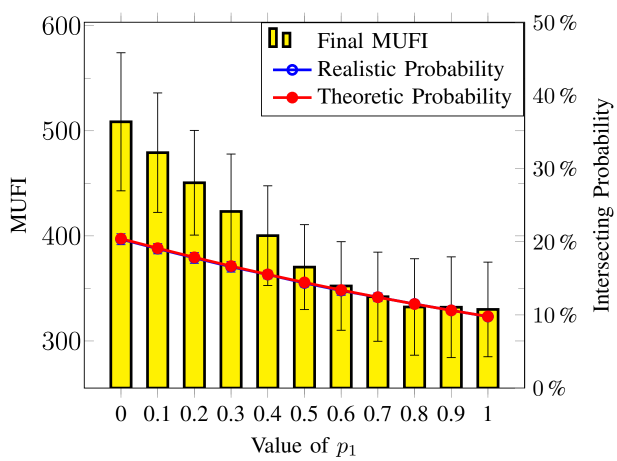


Fig. 6. Numerical results for NSF-U Scenario.

From these results, we can observe that in all the three scenarios, the realistic intersecting probabilities (marked by blue lines) perfectly fit in with the theoretic ones (marked by red lines) that we can barely see the blue lines in the three figures.

The results of intersecting probabilities prove the effectiveness of our Theoretic Chain 2 in Fig. 3, *i.e.*, the conflict coefficients and the computing method of GOF.

For R-U and NSF-U, with the intersecting probabilities gradually decreasing, the corresponding MUFIs reduce and reach the minimum when $p_1 = 1$ in Figs. 5 and 6, *i.e.*, their own optimal routing schemes. For NJ-U in Fig 7, as we can observe that the intersecting probability p is within a narrow range of $[8.94\%, 11.57\%]$ which means the difference of intersecting probabilities is less than 3%. Thus, the difference of the corresponding conflict graphs is so subtle that only a small volatility of MUFIs ([357.38, 379.06]) is observed.

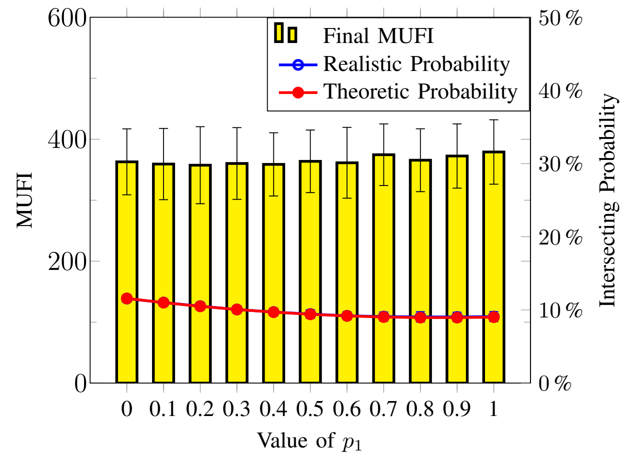


Fig. 7. Numerical results for NJ-U Scenario.

This results of MUFIs prove the effectiveness of our Theoretic Chain 1 in Fig. 2, *i.e.*, the intersecting probability is positively correlated to the final MUFI.

B. Weighted Traffic Distribution

Here, we evaluate the other three scenarios with the weighted traffic distribution: R-W, NSF-W and NJ-W. The corresponding results are demonstrated in Figs. 8, 9 and 10 respectively.

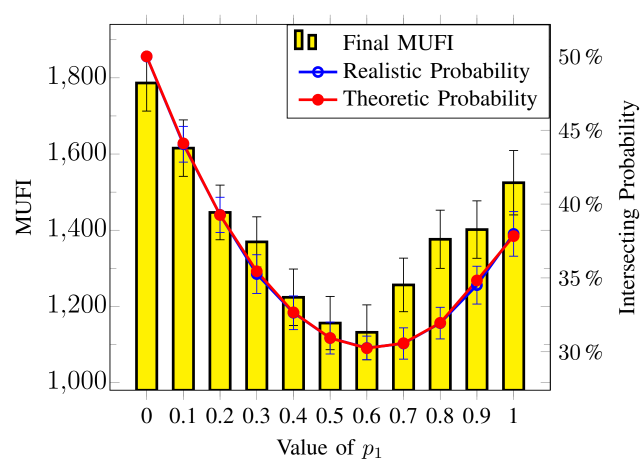


Fig. 8. Numerical results for R-W Scenario.

Similarly, the realistic intersecting probabilities match very well with the theoretical ones in the three scenarios, which again prove the effectiveness of Theoretic Chain 2 of the conflict coefficients and GOF.

From the aspect of final MUFIs, the three scenarios of R-W, NSF-W and NJ-W represent a common characteristic: First, with the decreasing of the intersecting probabilities,

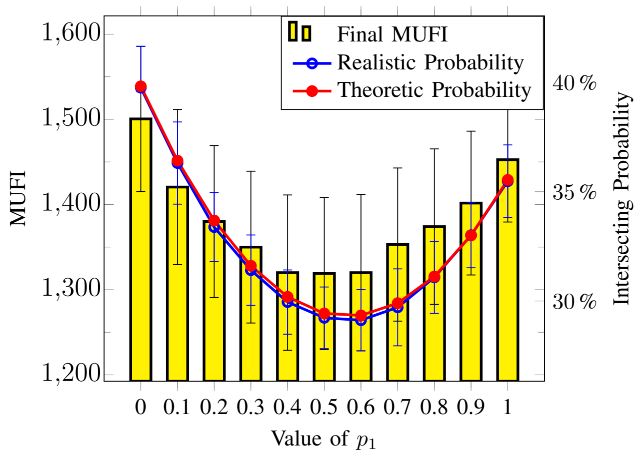


Fig. 9. Numerical results for NSF-W Scenario.

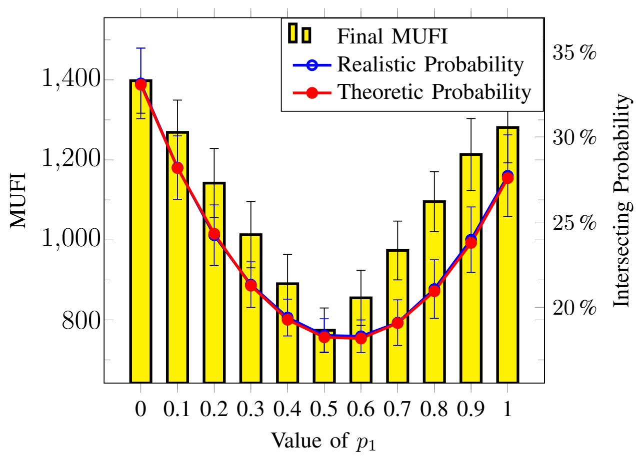


Fig. 10. Numerical results for NJ-W Scenario.

the corresponding MUFIs reduce. After passing their optimal routing schemes, the corresponding MUFIs keep increasing as the intersecting probabilities grow. These results further verify the effectiveness of Theoretic Chain 1 of the intersecting probability.

Meanwhile, the results also exhibited the importance of designing a better routing scheme to decrease the intersecting probability. When the information of the network topology and traffic distribution are obtained, how to optimally assign the requests on the K candidate paths to decrease the intersecting probability is crucial to the final performance of the RSA. Taking the three scenarios of R-W, NSF-W and NJ-W with $K = 2$ for examples, the (worst, best) MUFI pairs are (1786.51, 1132.32), (1500.23, 1319.02) and (1398.41, 774.08) respectively. **Thus, decreasing the intersecting probability can obtain a huge gain in the final spectrum usage.**

VIII. Conclusions

In this work, we developed two theoretical chains to reveal the implicit impacts of network topology, traffic distribution and routing scheme on the spectrum management in EONs. We first investigated the property of the conflict graph built upon the computed lightpaths. We proved that the optimal MUFI of the conflict graph is directly determined by its chromatic number, and the chromatic number has a strongly positive correlation to the edge existence probability in the conflict graph, *i.e.*, the intersecting probability. In other words, the smaller the intersection probability, the smaller the optimal MUFI, which constitutes our first theoretical chain. We then proposed the concept of conflict coefficients, which are important parameters decided by the network topology and traffic distribution. We further developed the quadratic programming GOF with the routing scheme to determine the intersecting probability. This constitutes our second theoretical chain. Finally, the proposed theoretical chains have been validated by extensive simulations in several well-known EONs.

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