Energy-efficient Subcarrier Allocation in SC-FDMA Wireless Networks based on Multilateral Model of Bargaining

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Abstract— **In this paper the problem of subcarrier and power allocation in multiuser single-carrier frequency division multiple access (SC-FDMA) wireless networks is addressed. Given the inherent complexity of the corresponding joint optimization problem, in this paper we propose an approach where we treat the problem in three consecutive phases. First, the subcarrier allocation in terms of number of subcarriers allocated per user is obtained, by adopting a multilateral bargaining model, i.e. Rubinstein's bargaining model. Then, different realistic subcarrier mapping policies are proposed, towards determining the specific subcarrier assignment. Subsequently, an optimization problem with respect to user's uplink transmission power is formulated and solved, in order to determine the optimal power allocation per subcarrier assigned to each user in the previous phases. Emphasizing on energy efficiency, users' satisfaction is formulated through a properly defined utility-based framework that reflects the tradeoff between users' actual uplink throughput performance and their corresponding energy consumption. The performance evaluation of the proposed approach is achieved via modeling and simulation, and its superiority against other existing alternatives is demonstrated.**

Keywords-. Subcarrier and power allocation, SC-FDMA Wireless, Rubinstein bargaining model.

I. INTRODUCTION

Single Carrier Frequency Division Multiple Access (SC-FDMA) emerged as a promising radio access technology for 4th generation (4G) mobile communication systems. As such, it has been chosen as the uplink multiple access scheme in the Third Generation Partnership Project-Long Term Evolution (3GPP-LTE) standard [1]. In SC-FDMA, the total bandwidth is divided into orthogonal subcarriers in order to be allocated to multiple users. There are two types of subcarrier mapping: localized (L-SC-FDMA) and distributed (D-SC-FDMA), where the subcarriers are allocated to a user either consecutive or distributed, respectively [2]. L-SC-FDMA takes advantage of frequency selective fading in maximizing throughput, while D-SC-FDMA avoids allocating subcarriers in deep fade and provides resilience to frequency selective fading, thus resulting in lower peak-to-average power ratio (PAPR) [3]. In our framework, both distributed and localized SC-FDMA mapping are considered.

In this paper, we provide a robust and unified framework that allows the energy efficient utility based treatment of subcarriers and users' uplink transmission power allocation, via a bargaining model and a power optimization framework. As it is shown the proposed approach converges to a feasible and stable solution, while numerical results demonstrate its performance superiority when compared against existing schemes applied in SC-FDMA.

A. Related Work

In [4], a utility-based subcarrier and power allocation scheme is proposed, considering two different utility functions: aggregate user data rate for maximizing system capacity and aggregate logarithmic user data rate for maximizing proportional fairness. Furthermore, in [4], [5], the authors instead of directly treating and solving the optimization problem of resource allocation, they provide a greedy subcarrier allocation scheme, based on the improvement of users' marginal utility. The subcarriers are organized in chunks and the chunks are allocated to the users in order to minimize the convergence time of the greedy algorithm.

In [6], a weighted-sum rate maximization problem in SC-FDMA is considered, which approximates proportional fair scheduling using judiciously chosen weights and a reformulation of this resource allocation into a pure binary-integer problem is realized. An extension to [6] is proposed in [7], where two additional resource allocation problems are considered: rate constraint satisfaction with minimum number of subcarriers and sum-power minimization subject to rate constraints. Furthermore, an energy-efficient resource allocation in multiuser localized SC-FDMA with synchronous hybrid automatic repeat request is considered in [8], while a variable complexity sub-optimal approach towards minimizing the energy in resource allocation is proposed.

B. Paper Contribution & Approach

In this paper we propose an approach where we treat the corresponding problem in three consecutive phases. First we adopt a multilateral bargaining model, i.e. Rubinstein's bargaining model, to obtain a feasible and stable subcarrier allocation, in terms of the number of subcarriers allocated per user. Subsequently, different realistic subcarrier mapping policies can be followed to determine the specific subcarrier assignment. Finally, targeting energy efficiency, an optimization problem with respect to user's uplink transmission power is formulated and solved, in order to determine the optimal power allocation per subcarrier assigned to each user in the previous two phases.

The basic features, contributions and differences of our proposed approach and framework lie in the following:

- Users' satisfaction is defined as an enhanced utility function of their occupied system's resources, that represents the tradeoff between users' actual uplink throughput performance and their corresponding energy consumption, instead of simply expressing the achievable data rate of each user using the Shannon formula [4],[5]. Based on that, an Energy-efficient Utility-based Subcarrier and Power Allocation (EUSPA) non-cooperative game is formulated, towards maximizing the personal utility perceived by each user, rather than studying the centralized problem of maximizing the overall utility of all users as in [4], [5].

- A bargaining model based approach is followed – based on Rubinstein's *N*-person multilateral bargaining model - towards obtaining an initial subcarrier allocation to all users (in terms of the number of subcarriers allocated per user), instead of adopting greedy [4], [5] or heuristic approaches, which have the drawback of high convergence time to the stable outcome [6]-[8].

- Three mapping policies, to determine the assignment of specific subcarriers ID to each user are discussed. The mapping policies consider and exploit users' channel gain.

- Given the subcarrier allocation and assignment, users' uplink transmission powers are allocated in an optimal way, concluding to a unique stable point and achieving energy-efficiency, instead of simply adopting the non-energy-efficient equal-bit-equal-power (EBEP) allocation for each subcarrier ([4], [5]).

- An iterative, distributed and low-complexity algorithm is proposed, which converges to the stable power allocation and its convergence is proven.

- The performance of the proposed approach is evaluated in detail in order to gain insight about its operational characteristics, while its superiority compared to other state of the art approaches, i.e. [4], [5] is illustrated.

The rest of the paper is organized as follows: Section II initially describes the system model and the adopted utility function. Then the optimization problem of the energy-efficient utility-based subcarrier and users' uplink transmission power allocation is formulated. Section III presents a feasible and stable subcarrier allocation methodology based on a multilateral bargaining model, and section IV provides an optimization framework to treat the power allocation problem. Section V discusses different subcarrier assignment/mapping policies, while an iterative distributed algorithm to realize the overall proposed subcarrier and users' uplink transmission power allocation approach is described. The performance evaluation of the proposed framework is achieved via modeling and simulation in Section VI, and finally section VII concludes the paper.

II. SYSTEM MODEL AND BACKGROUND INFORMATION

The time synchronized uplink for a single cell of a multiple users wireless communications system employing SC-FDMA with system bandwidth *B Hz* is considered. The time axis is divided into timeslots (*0.5 ms* duration) as a basic unit of time scheduling. The system bandwidth *B Hz* is divided into a set $\mathbb{S}_{sub} = \{ s_i^j / i \in \mathcal{N} = \{1,2,...,i,...,N\}, j = 1,2,...,K_i \}$, where K_i denotes the number of subcarriers occupied by user i and $\mathbb{S}_i = \left\{ s_i^j / j = 1,2,...,K_i \right\}$ refers to the corresponding set. The total number of users residing in the cell is N and the corresponding set is denoted by $\mathcal{N} = \{1,2,...,i,...,N\}$ while the overall number of subcarriers in the system is $S = \sum_{i=1}^{N} K_i$. User $i \in \mathcal{N}$ at subcarrier $s_i^j \in \mathbb{S}_i$ has channel gain G_{i,s_i^j} and uplink transmission power P_{i,s_i^j}. User's total uplink transmission power is upper limited by his maximum power P_i^{Max}, due to his physical limitations, i.e. $\sum_{j=1}^{K_i} P_{i,s_i^j} \le P_i^{Max}$. Furthermore, the received bit energy to interference density ratio, γ_{i,s_i^j}, at the base station for user i at subcarrier s_i^j is given by:

$$\gamma_{i,s_i^j} = \frac{P_{i,s_i^j} G_{i,s_i^j}}{\sigma^2_{s_i^j}} \qquad (1)$$

where $\sigma^2_{s_i^j}$ denotes the noise power of subcarrier s.

In order to align user's various flow characteristics under a common optimization framework each mobile user is associated with a suitable utility function U_{i,s_i^j}, that refers to the level of satisfaction the user receives as a result of his actions and corresponding allocated resources.

Aiming at considering utilities from the users' point of view, the unique characteristics of the wireless transmission environment, in terms of transmission errors occurred at the receiver (which are strongly related with the achieved signal-to-interference ratio (SIR)), play a key role in the definition of the utility. On one hand, higher SIR level at the receiver will result in a lower bit-error rate and hence higher throughput, while on the other hand, achieving a high SIR level usually requires the user to transmit at a high power, which in turn, results in lower battery lifetime. In this work we consider that user's utility represents his degree of satisfaction at his allocated subcarrier $s_i^j \in \mathbb{S}_i \subseteq \mathbb{S}_{sub}$ in relation to the expected tradeoff between his actual uplink throughput performance and the corresponding energy consumption.

Therefore, the adopted utility function [9] is expressed as:

$$U_{i,s_i^j}(P_{i,s_i^j}) = \frac{R_{opt} f\left(\gamma_{i,s_i^j}\right)}{P_{i,s_i^j}} \tag{2}$$

where R_{opt} is user's fixed designed transmission rate, depending on user's requested service and $f\left(\gamma_{i,s_i^j}\right)$ is his efficiency function. The latter represents the probability of a successful packet transmission for user *i* at subcarrier s_i^j and is an increasing function of his bit energy to interference density ratio γ_{i,s_i^j}. User's efficiency function can be represented by a sigmoidal-like function of his power allocation for various modulation schemes [9]. Therefore, user's $i \in \mathcal{N}$ efficiency function has the following properties:

1. $f\left(\gamma_{i,s_i^j}\right)$ is an increasing, continuous, twice differentiable and sigmoidal function with respect to $\gamma_{i,s}$.
2. $f(0) = 0$ to ensure that $U_{i,s_i^j}(0) = 0$ when $P_{i,s_i^j} = 0$.
3. $f(\infty) = 1$.

The validity of the above properties has been demonstrated in several practical scenarios with reasonably large packet sizes *M*, i.e. $M \geq 100bits$ [10].

Therefore the overall user *i* satisfaction can be expressed as:

$$U_i(\boldsymbol{P}_{\boldsymbol{i},\boldsymbol{s}_i^j} = \left[P_{i,s_i^1}, \ldots, P_{i,s_i^{K_i}}\right], K_i) = \sum_{j=1}^{K_i} U_{i,s_i^j}(P_{i,s_i^j}) \tag{3}$$

where K_i denotes the number of subcarriers allocated to user *i*

A. Energy-efficient Utility-based Subcarrier and Power Allocation (EUSPA) Non-cooperative Game Formulation

Let $G_{S-P} = \left[\mathcal{N}, \left\{\mathbb{S}_i, \mathcal{P}_i\right\}, \left\{U_i\left(\boldsymbol{P}_{\boldsymbol{i},\boldsymbol{s}_i^j}, K_i\right)\right\}\right]$ denote the energy-efficiency utility-based subcarrier and power allocation (EUSPA) game in SC-FDMA wireless networks. Each user $i \in \mathcal{N}$ selects a number of subcarriers K_i and the corresponding transmission power P_{i,s_i^j} for each of his allocated subcarriers $s_i^j \in \mathbb{S}_i$, in order to maximize his overall expected utility U_i, via minimizing his power consumption and in parallel satisfying his Quality of Service (QoS) prerequisites. Therefore, the overall optimization problem is formulated as:

$$\begin{gathered} \max_{\substack{P_{i,s_i^j} \in \mathcal{P}_i \\ 0 \prec K_i \leq S}} U_i(\boldsymbol{P}_{\boldsymbol{i},\boldsymbol{s}_i^j} = \left[P_{i,s_i^1}, \ldots, P_{i,s_i^{K_i}}\right], K_i) = \sum_{j=1}^{K_i} U_{i,s_i^j}(P_{i,s_i^j}) \\ s.t. \ \sum_{j=1}^{K_i} P_{i,s_i^j} \leq P_i^{Max}, i \in \mathcal{N}, \ S = \sum_{i=1}^{N} K_i \end{gathered} \tag{4}$$

where $\mathcal{P}_i = [0, P_i^{Max}]$ denotes the set of user's $i \in \mathcal{N}$ feasible uplink transmission power, which is a compact and convex set with maximum and minimum constraints. As it is observed via the EUSPA optimization problem (4), each user may transmit with a different transmission power P_{i,s_i^j} at each of his allocated subcarrier to achieve energy-efficiency.

More specifically, allocating subcarriers and users' uplink transmission powers by solving a standard form of the optimization problem (4) is extremely complex due to the following reasons: (i) The objective function in (4) is formulated as a complex form dependent on both subcarrier and power allocation, and (ii) there is an additional power constraint for each user, i.e. $\mathcal{P}_i = [0, P_i^{Max}]$.

In the following, at the first step of our approach we determine a feasible and stable subcarrier allocation of EUSPA game (i.e. a vector of users' subcarriers $\boldsymbol{K}^* = \left(K_1^*, K_2^*, \ldots, K_i^*, \ldots, K_N^*\right)$), based on Rubinstein's *N*-person multilateral bargaining model. As clarified before, the subcarrier allocation procedure based on the *N*-person multilateral bargaining model determines the total number of subcarriers that should be allocated to each user. However it does not directly identify the actual subcarriers to be assigned and used by each user (i.e. subcarrier IDs) within the available range and spectrum. Thus, at the second step of our approach different assignment (mapping) policies are followed depending on various prioritization factors and objectives, and as a result different mappings can be obtained and realized. For example, one such assignment could be based on sequential subcarrier selection following the localized (L-SC-FDMA) mapping approach, while alternative assignments favoring specific users (e.g. distant users vs. closer ones to base station and vice versa) would allow for assignment of subcarriers that are distributed over the entire available frequency band (D-SC-FDMA). In Section V several such alternative policies are presented, while their performance and impact is evaluated in detail in Section VI. After determining the specific subcarrier allocation, at the third step of our approach an optimization problem is formulated (section IV), where each user aims to maximize his utility $U_{i,s_i^j}\left(P_{i,s_i^j}\right)$ at each of his allocated subcarrier. Specifically, the optimization problem uses K_i^* to determine the optimal user's *i* uplink transmission power allocation. It should be noted that, we follow the aforementioned sequential multi-phase approach in order to deal with the high complexity of the combined problem of subcarrier and power allocation. As a consequence, our final solution though results to a feasible and stable allocation, does not guarantee optimality with respect to the initial joint resource allocation problem. Nevertheless, it guarantees optimal power assignment to the allocated subcarriers thus achieving energy-efficiency, while as demonstrated by numerical results the proposed approach prevails over greedy and heuristic alternatives.

III. SUBCARRIER ALLOCATION BASED ON MULTILATERAL MODEL OF BARGAINING

To allocate the total number of subcarriers *S* to the *N* users residing at the cell per time slot, various greedy or heuristic subcarrier allocation schemes have been proposed [3]-[8].

However, these allocations are neither optimal nor converge always to a stationary point. In the following the subcarrier allocation is obtained adopting a multilateral bargaining model.

Rubinstein [12] models a two-person bargaining in order to share a pie of size *1*, as an extensive game, by making use of a game structure in which two players take turns in making offers. The pie will be shared only after the players reach an agreement. Each player, in bargaining round *t* offers a partition of the pie $x_i(t)$ and his opponent may agree to the offer (Y) or reject (N) it. Acceptance of the offer ends the bargaining. After rejection, the rejecting player then has to make a counter offer to the next bargaining round and so on. Furthermore, there are no rules which bind the players to any previous offers they have made. Considering the infinite-horizon game, the existence of a unique equilibrium has been proven in [12]:

$$x^* = \left(x_1^*, x_2^*\right) = \left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right) \tag{5}$$

where δ is the same discount factor for each player *i*. The game can be summarized in the following graph:

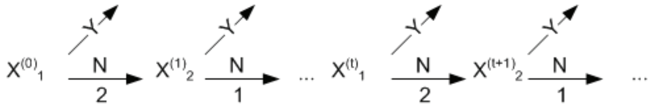

Figure 1. Bilateral Bargaining Model

In this paper in order to allocate the "pie" of the total number of subcarriers *S* to the *N* users, an extension of Rubinstein's alternating-offers bargaining model to the case of *N* players is adopted and is called *N*-person multilateral bargaining model [13]. The *N*-person bargaining game $G(t;1,2,...,i,...,N)$ over a pie of size *S* with identical players is defined as:

Player *1* makes the first offer at bargaining round *t=1*, offering $x = \left(x_1, x_2, ..., x_i, ..., x_N\right)$, where x_i is user/player *i*'s proposed share/partition and $\sum_{i=1}^{N} x_i = 1$. Player *2* to player *N* then respond sequentially, each saying either accept (Y) or reject (N) to the proposal. In the case of all responders accept player's *1* offer, then the game ends with each player *i* obtaining a partition x_i. A rejection takes the game to the next bargaining round, where player *2* now makes an offer and players *(3,...,i,...,N,1)* sequentially respond. In the case of the responders reject at bargaining round *t=2*, then the game goes to the next stage with player *3* making the offer and so on. If the offer *x* is accepted by all responders in bargaining round *t*, the payoff is $\delta^{t-1}x_i$ to player *i*, where $\delta<1$ is the common discount factor. Absence of agreement, that is bargaining forever, leads to a payoff of *0* for all players. To overcome the obstacle of bargaining forever, discount factor δ close to one is adopted, corresponding to a short time interval between bargaining rounds, as in this case the first players' advantage to the initial offer is small. In [13], it is proven that the bargaining game's Nash equilibrium is given as follows:

$$\boldsymbol{x} = \left(\frac{1-\delta}{1-\delta^N}, \delta\cdot\frac{1-\delta}{1-\delta^N}, ..., \delta^{t-1}\cdot\frac{1-\delta}{1-\delta^N}, ..., \delta^{N-1}\cdot\frac{1-\delta}{1-\delta^N}\right) \tag{6}$$

In our approach, we adopt the Rubinstein's *N*-person multilateral bargaining model and allocate the total number of subcarriers to the users residing in the cell according to its proven unique Nash equilibrium point.

A Nash equilibrium in subcarriers is formally defined as:

Definition 1: A subcarrier vector $\boldsymbol{K}^* = \left(K_1^*, ..., K_i^*, ..., K_N^*\right)$ is a Nash equilibrium of the EUSPA game $G_{S-P} = \left[\mathcal{N}, \left\{\mathbb{S}_i^*, \mathcal{P}_i\right\}, \left\{U_i\left(\boldsymbol{P}_{i,s_i^j}, K_i^*\right)\right\}\right]$, where $\mathbb{S}_i^* = \left\{s_i^j \,/\, j = 1, ..., K_i^*\right\}$, if $U_i\left(\boldsymbol{P}_{i,s_i^j}, K_i^*, \boldsymbol{K}_{-i}^*\right) \geq U_i\left(\boldsymbol{P}_{i,s_i^j}, K_i^{*\prime}, \boldsymbol{K}_{-i}^*\right)$ for every $i \in \mathcal{N}$ and for all $\mathbb{S}_i^{*\prime} = \left\{1, ..., K_i^{*\prime}\right\} \in \mathbb{S}_{sub}$,where $\boldsymbol{K}_{-i}^* = \left(K_1^*, ..., K_{i-1}^*, K_{i+1}^*, ..., K_N^*\right)$ is the *N-1* dimensional vector of user subcarriers that does not contain user *i's* subcarriers.

Theorem 1: A Nash equilibrium in subcarriers for game $G_{S-P} = \left[\mathcal{N}, \left\{\mathbb{S}_i^*, \mathcal{P}_i\right\}, \left\{U_i\left(\boldsymbol{P}_{i,s_i^j}, K_i^*\right)\right\}\right]$ exists, is unique and is given by:

$$K_i^* = \frac{\left(1-\delta\right)\delta^{i-1}}{1-\delta^N}\cdot S,\ 0 \prec \delta \prec 1,\ N \geq 2 \tag{7}$$

Proof: The proof is a straightforward application of the corresponding proof in [13] where it is shown that the Rubinstein's *N*-person ($N \geq 2$) multilateral bargaining model concludes to a unique Nash equilibrium. In our case the pie to be allocated is the total number of subcarriers and as a result the unique Nash equilibrium is given by (7). ■

Based on theorem 1 we obtain a closed form solution in allocating the total number of subcarriers to the users. However, due to the fact that the subcarriers to be allocated to the users is a discrete source, in the following we utilize the round of K_i^*, thus the number of subcarriers allocated to each user *i* is obtained as:

$$K_i^* = \left[\frac{\left(1-\delta\right)\delta^{i-1}}{1-\delta^N}\cdot S\right],\ 0 \prec \delta \prec 1,\ N \geq 2\,. \tag{8}$$

Based on extensive simulations we have verified that the difference between using the allocation given by relation (7) and the corresponding one resulting from the rounding process is less than one percent with respect to the total number of allocated subcarriers.

Furthermore, two important observations are noted concerning the subcarrier allocation:

A. The earlier a player has been admitted to the EUSPA game, the greater portion of resources he takes. Thus, aiming at achieving fairness and parity among the competitive users, as well as attaining a short time interval between bargaining rounds, adopting discount factors close to one is the most appealing strategy. However, if aiming at differentiating the users according to QoS prerequisites or different service classes, lower value of discount factor can be selected, giving priority to the first users inserted in the bargaining rounds.

B. As the number of users increase, the achievable resources allocated to each user are reduced, since the total resources are finite and divided among more players.

In the following we first analyze the optimal power allocation among the subcarriers assigned to each user, assuming that both subcarrier allocation and respective assignment/mapping policy have already been performed. Then in section V different subcarrier assignment policies are discussed and analyzed.

IV. Power Allocation Towards Energy-Efficiency

In this section for the sake of presentation and without loss of generality, we assume user *i* has been allocated a total number of K_i^* subcarriers, namely subcarriers $s_i^1, s_i^2, ..., s_i^j, ..., s_i^{K_i^*}$. In order to achieve users' power allocation, given their subcarrier allocation, we formulate an optimization problem considering each user's satisfaction (i.e. utility) per each of his allocated subcarriers. Specifically, we formulate a pure power control optimization problem, given subcarriers assignment to user *i*:

$$\begin{aligned} &\max_{P_{i,s_i^{K_i^*}} \in \mathcal{P}_i} U_{i,s_i^j}(P_{i,s_i^j}) \\ &s.t. \quad \sum_{j=1}^{K_i^*} P_{i,s_i^j} \le P_i^{Max} \end{aligned} \tag{9}$$

Considering the above optimization problem, at each of user's allocated subcarrier, each user computes his optimal transmission power vector $\boldsymbol{P}_{\boldsymbol{i,s_i^j}}^* = \left[P_{i,s_i^1}^*, ..., P_{i,s_i^{K_i^*}}^*\right]$ $\forall s_i^j \in \mathbb{S}_i^* \subseteq \mathbb{S}_{sub}$ and determines his resulting utility vector $U_i(P_i^*) = [U_{i,s_i^1}(P_{i,s_i^1}^*), U_{i,s_i^2}(P_{i,s_i^2}^*), ..., U_{i,s_i^j}(P_{i,s_i^j}^*), ..., U_{i,s_i^{K_i^*}}(P_{i,s_i^{K_i^*}}^*)]$

In the following, we study the existence and uniqueness of an optimal stable point in users' uplink transmission powers for the proposed optimization problem.

Theorem 2: A stable point in user's *i* uplink transmission power for the proposed optimization problem (9) exists, is unique and is given by $\boldsymbol{P}_{\boldsymbol{i,s_i^j}}^* = \left[P_{i,s_i^1}^*, ..., P_{i,s_i^{K_i^*}}^*\right]$, where $P_{i,s_i^j}^*$ is the unique global maximization point of user's *i* utility function.

$$P_{i,s_i^j}^* = \min\left\{\frac{\gamma_{i,s_i^j}^* \sigma_{s_i^j}^2}{G_{i,s_i^j}}, \left(P_i^{Max} - \sum_{\substack{u \neq j \\ u=1,...,K_i^*}} P_{i,s_i^u}\right)\right\} \tag{10}$$

Proof: In Section II, it was described that the efficiency function $f\left(\gamma_{i,s_i^j}\right)$ is a sigmoidal function of γ_{i,s_i^j}. Moreover, the received bit energy to interference density ratio, γ_{i,s_i^j}, at the base station for user *i* at subcarrier s_i^j, has a strict linear relationship to the corresponding uplink transmission power P_{i,s_i^j}, in accordance to (1), thus it follows that the reliably transmitted bits to the base station, i.e. $R_{opt} f\left(\gamma_{i,s_i^j}\right)$, formulate also a sigmoidal function with respect to user's uplink transmission power P_{i,s_i^j}. Due to the fact that $R_{opt} f\left(\gamma_{i,s_i^j}\right)$ is a sigmoidal function with respect to P_{i,s_i^j}, the ratio $\frac{R_{opt} f\left(\gamma_{i,s_i^j}\right)}{P_{i,s_i^j}} = U_{i,s_i^j}(P_{i,s_i^j})$ is a quasi-concave function of $P_{i,s_i^j} \in \left[0, P_i^{Max}\right]$, following the approach in [15]. Considering the first order derivative optimization condition, we conclude that $\partial U_{i,s_i^j}(P_{i,s_i^j}) \big/ \partial P_{i,s_i^j} = 0$. Thus, $U_{i,s_i^j}(P_{i,s_i^j})$ is maximized when

$$\gamma_{i,s_i^j} \frac{\partial f_{i,s_i^j}(\gamma_{i,s_i^j})}{\partial \gamma_{i,s_i^j}} - f_{i,s_i^j}(\gamma_{i,s_i^j}) = 0 \tag{11}$$

due to the fact that $\frac{\partial \gamma_{i,s_i^j}}{\partial P_{i,s_i^j}} = \frac{\gamma_{i,s_i^j}}{P_{i,s_i^j}}$, by (1). Considering the sigmoidal type of $f_{i,s_i^j}(\gamma_{i,s_i^j})$, relation (11) has a unique solution $\gamma_{i,s_i^j}^*$. Thus, the unique uplink transmission power P_{i,s_i^j}' that maximizes $U_{i,s_i^j}(P_{i,s_i^j})$ is: $P_{i,s_i^j}' = \frac{\gamma_{i,s_i^j}^* \sigma_{s_i^j}^2}{G_{i,s_i^j}}$.

Concluding our proof, we examine the case of the maximum remaining uplink transmission power is less than P_{i,s_i^j}'. Then, $U_{i,s_i^j}\left(P_i^{Max} - \sum_{\substack{u \neq j \\ u=1,...,K_i^*}} P_{i,s_i^u}\right)$ is the highest value of $U_{i,s_i^j}(P_{i,s_i^j})$, since the utility function is increasing over $\left[0, P_{i,s_i^j}'\right]$. Thus the global maximum of $U_{i,s_i^j}(P_{i,s_i^j})$ is given by relation (10). ■

V. EUSPA Algorithm & Assignment/Mapping Policies

In this section, based on our previous analysis, we introduce an iterative and distributed algorithm that determines users' subcarrier and uplink transmission power allocation, while its convergence is studied. Furthermore, different subcarrier assignment/mapping policies are discussed, and therefore variations of EUSPA algorithm are introduced accordingly.

The first algorithm variation - in the following we refer to as Pure-EUSPA - allocates the subcarriers to the users based on Rubinstein's *N*-person multilateral bargaining model and the users are assigned only sequential subcarriers to transmit (i.e. local mapping policy), That is, *user 1* is sequentially assigned the first K_1^* subcarriers, *user 2* is assigned sequentially the next set of K_2^* subcarriers, etc. Alternatively, in the second variation - we refer to as Maximum Gain Selection EUSPA (MGS-EUSPA) - a distributed subcarrier mapping policy is followed which prioritizes users' transmission to their allocated subcarriers. More specifically, two alternatives of MGS-EUSPA are studied: (A) Max Gain Selection-Far-EUSPA (MGS-F-EUSPA), which allows the more distant users from the base station to choose first and transmit to those subcarriers that have the best resulting channel gain, and (B) Max Gain Selection-Near-EUSPA (MGS-N-EUSPA), which gives priority to the less distant users in a similar way. All variations of EUSPA algorithm are divided into two basic parts. The first part allocates and assigns the subcarriers to all users, and the second part, given the subcarrier allocation and mapping, determines the optimal users' power allocation.

EUSPA Algorithm

Step 1: (*Subcarrier Allocation*) At the beginning of time slot *t*, the optimal subcarrier allocation $\boldsymbol{K}^* = \left(K_1^*, K_2^*, \dots, K_i^*, \dots, K_N^*\right)$ is determined via equation (8), based on Rubinstein's multilateral bargaining model and announced to all users.

(Subcarrier Mapping)

Step 2a (Only for Pure-EUSPA Algorithm): The user with number ID *1* occupies and transmits to the first K_1^* subcarriers, the user with number ID *2* occupies the following K_2^* subcarriers and so on till all users are exhausted.

Step 2b (Only for MGS-F-EUSPA Algorithm): Starting from the more distant user *i*, the user selects and transmits to the K_i^* subcarriers with best channel gain.

Step 2c (Only for MGS-N-EUSPA Algorithm): Starting from the less distant user *i*, the user selects and transmits to the K_i^* subcarriers with best channel gain.

Step 3: (*Utility and Power Optimization*) Each user *i*, $i \in \mathcal{N}$ computes his uplink transmission power based on equation (10) for his assigned subcarrier $s_i^j \in \mathbb{S}_i^*$. Set *k=0*.

Step 4: Set *k:=k+1*, delete the subcarrier *s* in the set of user's *i* available subcarriers, i.e. $K_i^{*(k+1)} = K_i^{*(k)} - \left\{s_i^j\right\}$, renew user's *i* maximum transmission power, i.e. $P_i^{Max(k+1)} = P_i^{Max(k)} - P^*_{i,s_i^j}$, and if $P_i^{Max(k+1)} \neq 0$ or $\mathbb{S}_i^* \neq \varnothing$ go to step 3, otherwise stop.

A. Convergence

Both variations of EUSPA algorithm (i.e. Pure-EUSPA and MGS-EUSPA) can be characterized as single-valued best response algorithm. The complexity of the proposed EUSPA algorithms is low, due to the simplicity of the calculations in order to determine users' subcarrier and power allocation. Specifically, in our study we have considered the infinite-horizon game [13], where essentially convergence of EUSPA algorithm in subcarrier allocation is achieved based on single calculation process via equation (8). In this case, the overall convergence of the proposed approach is dominated by the last phase of our algorithm (i.e. optimal power allocation), and is discussed in more detail below and in the numerical evaluation section. It is noted also that if finite-horizon game were considered, then the convergence speed and the concluding allocation is influenced by the appropriate selection of the discount factor, i.e. selection of discount factors close to one, attains a short time interval between bargaining rounds.

The convergence of EUSPA algorithm in users' uplink transmission powers can be proven via the best response approach. Based on theorem 2, user's $i \in \mathcal{N}$ best response BR_i is denoted as follows.

$$BR_i(P_{i,s_i^j}) = \min\left\{\frac{\gamma^*_{i,s_i^j}\,\sigma^2_{s_i^j}}{G_{i,s_i^j}}, \left(P_i^{Max} - \sum_{\substack{u\neq j\\ u=1,\dots,K_i^*}} P_{i,s_i^u}\right)\right\} \qquad (12)$$

Theorem 3: EUSPA algorithm converges to the unique stable point with respect to users' uplink transmission powers, starting from any initial point.

Proof: In theorem 2, we have shown the uniqueness of a stable point in users' uplink transmission powers, which is denoted by $\boldsymbol{P}^* = \left(P^*_{1,s_1^j}, P^*_{2,s_2^j}, \dots, P^*_{i,s_i^j}, \dots, P^*_{N,s_N^j}\right)$. By definition, this stable point has to satisfy:

$$\boldsymbol{P}^* = \boldsymbol{BR(P^*)} \qquad (13)$$

where $\boldsymbol{BR(P)} = \left(BR_1(P^*_{1,s_1^j}), BR_2(P^*_{2,s_2^j}), \dots, BR_N(P^*_{N,s_N^j})\right)$ is the best response strategy of all users. The fundamental point to show the convergence of EUSPA algorithms to the unique stable transmission power is to show that the best response function $\boldsymbol{BR(P)}$ is standard [14]. A function is characterized as standard if it satisfies the following properties for all $\boldsymbol{x} \geq 0$, where $\boldsymbol{x} = (x_1, x_2, \dots, x_N)$ is a stable point of the optimization problem,

- Positivity: $f(\boldsymbol{x}) \succ 0$;
- Monotonicity: if $\boldsymbol{x} \geq \boldsymbol{x}'$ then $f(\boldsymbol{x}) \geq f(\boldsymbol{x}')$;
- Scalability: for all *α>1*, $\alpha f(\boldsymbol{x}) \geq f(\alpha \boldsymbol{x})$.

These properties can be easily verified for $\boldsymbol{BR(P)}$.

- $\boldsymbol{P} \succ 0$, thus $\boldsymbol{BR(P)} \succ 0$;
- if $\boldsymbol{P}' \geq \boldsymbol{P}$ then via equation (13) we conclude that $\boldsymbol{BR(P')} \geq \boldsymbol{BR(P)}$;
- for all *α>1*, then via equation (13) we conclude that $\alpha\boldsymbol{BR(P)} \geq \boldsymbol{BR(\alpha P)}$.

VI. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we provide numerical results to evaluate the performance of the proposed framework. Initially, we present some results in order to gain some insight about the operation and features of the proposed EUSPA framework, and demonstrate its effectiveness. Afterwards, a comparative study is provided aiming at showing the benefit and superiority of adopting Rubinstein's bargaining model in realizing the subcarriers allocation, rather than a greedy one [4], [5].

Specifically, the uplink of a single-cell time slotted SC-FDMA system is considered, supporting *N=20* continuously backlogged users that are allocated at distances

$d_{i+1} = d_i + 50$ *[m]*, with *d_1=300m* from the base station. This topology has been specifically chosen in order to reveal the operational characteristics of EUSPA algorithm, as it allows for better understanding and interpretation of the corresponding results and observations. Similar overall results are achieved for other random topologies as well. Therefore, in our study we emulate the scenario where users' channel conditions become worse as their ID value (*i.e. i=1,2,...,20*) increases. Each simulation lasts *10.000* time slots, while path-loss and shadowing effect are generated randomly and are assumed to be constant during each time-slot. Thus, each user is assumed to be stationary or slowly moving. Moreover, we assume that the multi-path fading component is time invariant over the time slot, but changes independently at each time slot. Therefore, unless otherwise explicitly indicated, we model users' path gains as $G_{i,s_i^j} = \Lambda_{i,s_i^j} \big/ d_i^a$, where d_i is the distance of user *i* from the base station, *a* is the distance loss exponent, and Λ_{i,s_i^j} is a log-normal distributed random variable with standard deviation *8dB*, which represents the multi-path fading effect. Moreover, we set users' maximum uplink transmission power to $P_i^{Max} = 2$ *Watts* and $\sigma^2_{s_i^j} = 5 \cdot 10^{-15}$.

A. Operation and Performance of Proposed Approach

For simplicity in the presentation, only the Pure-EUSPA algorithm is considered in this subsection. In addition we assume that the closest to the base station users (i.e. those with smaller IDs) are the first ones to enter the bargaining rounds (i.e. in step 1 of the algorithm), while the most distant users (those with larger IDs) are the last ones to be imported.

Fig. 1 illustrates the number of subcarriers allocated to each of the *N=20* users in the cell, under various discount factors. It is observed that as the discount factor decreases, the first users inserted in the bargaining rounds are favored compared to the rest and a larger portion of the subcarriers is allocated to them. However, aiming at a fair allocation among the users, a discount factor *δ* close to one is a more appropriate choice.

Fig.2 and Fig. 3 illustrate total user's uplink transmission powers and total achieved user's uplink transmission rates at the stable point of the optimization problem (9) for each user (differentiated by users' ID) under two different discount factors, considering the Pure-EUSPA algorithm. The corresponding results reveal that users' uplink transmission powers are inversely proportional to their instantaneous channel conditions, while based on the proposed power optimization model users do not exhaust their maximum uplink transmission power, thus extending their battery life.

Furthermore, due to the fact that as the discount factor increases (i.e. *δ=0.95*) more subcarriers are allocated to the distant users, which are the last that are imported in the bargaining rounds, the total distant users' uplink transmission power increases when the discount factor increases. For the same reason, as observed by the results in Fig. 3 when considering lower discount factor, the first users inserted in the bargaining rounds and simultaneously the ones with better channel achieve higher uplink transmission rate. This difference is reversed, considering the distant users, which occupy more subcarriers for a high discount factor than for a low value of discount factor, thus achieving higher uplink transmission rate.

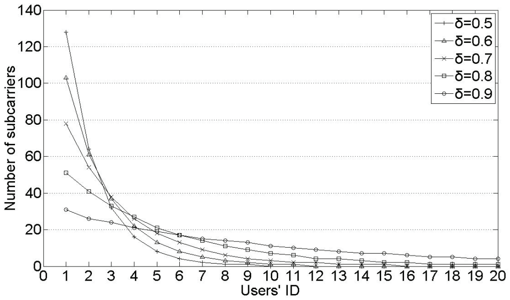

Figure 1. Number of subcarriers allocated to N=20 users, considering various discount factors.

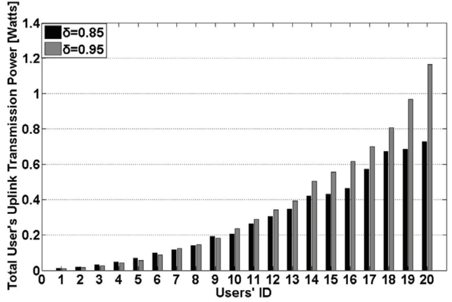

Figure 2. Total users' uplink transmission powers with various discount factors.

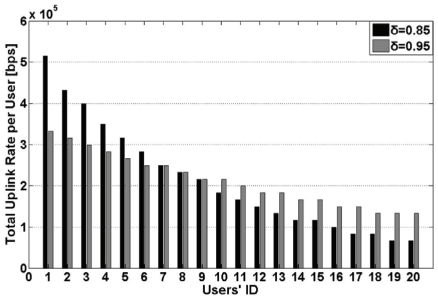

Figure 3. Total users' uplink transmission rates with various discount factors.

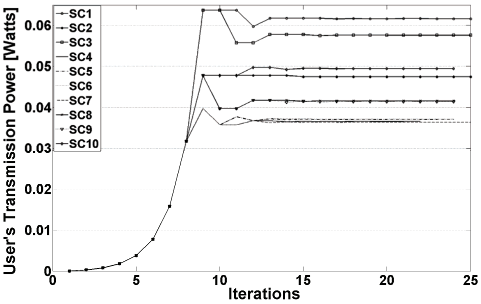

Figure 4. User's Transmission Power Convergence.

Fig. 4 illustrates user's uplink transmission power evolution per each of his allocated subcarrier, as a function of the iterations required for Pure-EUSPA algorithm to converge at its equilibrium point. We observe that the convergence of the proposed algorithm is very fast since in less than twenty iterations all subcarriers have reached the equilibrium point.

B. Comparative Study of Pure-EUSPA and MGS-EUSPA

In this subsection we provide a comparative study of the introduced EUSPA variations, i.e. Pure-EUSPA and the two versions of Max Gain Selection EUSPA (that is MGS-F-EUSPA which gives priority in subcarrier selection to the more distant users and MGS-N-EUSPA which gives priority to the less distant users).

Specifically, in Fig. 5 and Fig. 6 we compare the users' average uplink transmission power and the corresponding total utility-based performance of all the three algorithms. The corresponding results reveal the benefits of MGS-F-EUSPA algorithm vs. Pure-EUSPA and MGS-N-EUSPA algorithms, in terms of lower users' average uplink transmission powers (Fig.5) and higher users' total utility-based performance (Fig.6). This behavior is observed due to the fact that giving the opportunity to the distant users (which are already unfavored due to their position in the cell) to select and transmit to those subcarriers where they experience better channel gains, they achieve high utility and simultaneously lower uplink transmission powers. This is significant especially taking into account the scarcity of the radio resources in wireless networks, the mobile nodes' physical limitations and users' time varying channel conditions. Furthermore, we note that similar trend - in terms of increasing power consumption and decreasing utility - is observed for all algorithms as we move towards users with higher ID (i.e. more distant from the base station).

C. Tradeoff between Energy Consumption and Rate

In this subsection the critical tradeoff between user's energy consumption and the corresponding actual uplink transmission rate is evaluated in more detail. The ratio of user's uplink transmission power to the corresponding actual uplink transmission rate is used as a single-value metric, to illustrate each user's "cost" (in terms of power) per bit/sec for his achieved transmission rate. Three different scenarios are comnsidered: (a) Pure-EUSPA algorithm considering a logarithmic, concave utility function of user's signal-to-interference density ratio, which approximates the capacity via the classical Gaussian formula [11] while adopting the equal-bit-equal-power (EBEP) allocation, (b) MGS-N-EUSPA and (c) MGS-F-EUSPA algorithm, both considering the utility function, as expressed in (2) and adopting the optimal power allocation (described in Section IV).

Specifically, Fig. 7 presents the corresponding tradeoff between the total consumed energy and the corresponding achieved actual transmission rate by all users as a function of the discount factor *δ*. It is observed that MGS-F-EUSPA algorithm, considering the utility function as expressed in relation (2) and adopting the proposed optimal power allocation achieves the lower cost, i.e. with lower power consumption achieves higher actual transmission rate. This outcome is significant due to the fact that a more efficient resource allocation is achieved from the system's point of view.

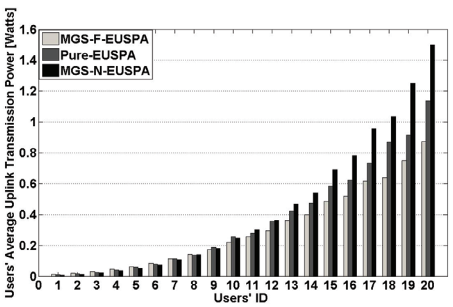

Figure 5. Users' average uplink transmisison powers of MGS-N-EUSPA, Pure-EUSPA and MGS-F-EUSPA with δ=0.95.

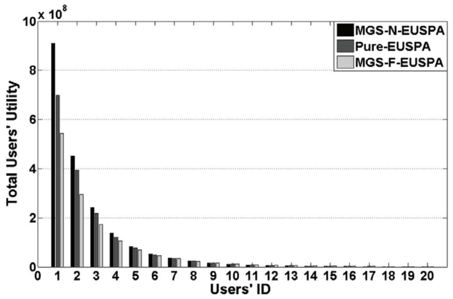

Figure 6. Total users' utilities of MGS-N-EUSPA, Pure-EUSPA and MGS-F-EUSPA with δ=0.95.

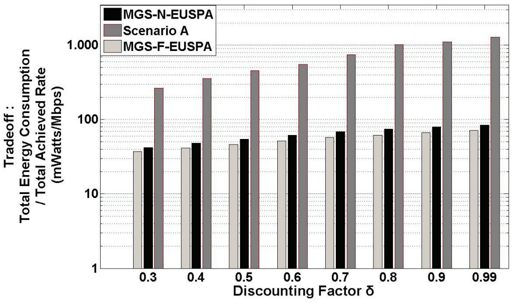

Figure 7. Tradeoff among energy consumption and achievable rate

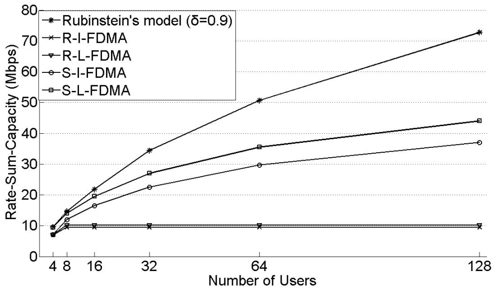

Figure 8. Rate sum capacity vs increasing number of users

D. Comparative Study to Greedy Approaches

In this subsection, a comparative study of our introduced subcarrier allocation approach based on the multilateral bargaining model against greedy frameworks adopted in recent literature [4], [5] where the subcarrier allocation is based on the improvement of users' marginal utility, is provided. The main objective of this comparison is to support and justify the choice of Rubinstein's bargaining model in terms of effectiveness in subcarrier allocation. For fairness in the comparison and in order to have a common basis and metric for all scenarios considered here, the non-energy-efficient equal-bit-equal-power (EBEP) allocation is used while user's maximum uplink transmission power is assumed to be: $P_i^{Max} = 0.25\ Watts$. Furthermore, towards maximizing spectral efficiency, a logarithmic, concave utility function of user's signal-to-interference density ratio, is adopted [11], which approximates the capacity via the classical Gaussian channel formula.

Fig. 8 presents the rate sum capacity where the sum of users' data rates is the considered utility function, as a function of increasing number of users. Five scenarios have been assumed: *(i)* R-L-FDMA: L-FDMA with *N* users selected by round-robin [4], *(ii)* S-L-FDMA: L-FDMA with the channel dependent scheduling (CDS) proposed in [4], *(iii)* R-I-FDMA: Interleaved-FDMA, i.e. users' selected subcarriers are equidistant from each other, with *N* users selected by round-robin, *(iv)* S-I-FDMA: I-FDMA with the CDS method proposed in [4] and *(v)* Rubinstein's multilateral bargaining model in order to allocate the total subcarriers to the users. The corresponding results clearly reveal the superiority of the proposed approach resulting in significantly higher rate sum capacity.

VII. CONCLUSIONS

In this paper we introduced an energy-efficient utility-based subcarrier and power allocation framework for application in the next generation SC-FDMA wireless networks. The proposed approach initially determines the subcarrier allocation via Rubinstein's *N*-person multilateral bargaining model thus decomposing the original joint resource allocation, and then realistic mapping policies are used, towards specifying the subcarriers ID assignment, via considering users' channel gain conditions. Afterwards, an optimization problem is formulated and solved, aiming at maximizing users' utilities per each of their allocated subcarrier and achieving energy-efficiency. The obtained numerical results, demonstrate that such an approach significantly outperforms existing resource allocation methods. Its superior performance stems from the appropriate choice of the order that the users enter the bargaining game, as well as the applied subcarrier mapping policy (i.e. favouring the users closer to the base station for achieving higher rate-sum-capacity (MGS-N-EUSPA), or favouring the ones with worse channel gains (MGS-F-EUSPA)).

It should be noted that among the key features of our approach, is the introduction of a holistic and flexible common framework, that allows for: a) the support of heterogeneous services and users with different QoS, b) the implementation of different users' priorities in accessing the available resources (i.e. through the use of different discount factors δ), c) the realization of different performance objective functions and metrics (i.e. maximize rate-sum-capacity, fairness, etc.). Towards this direction, the performance evaluation, study and analysis of the impact of the use of different subcarrier mapping policies along with the use of different discount factors δ under various realistic network topologies and user distributions, is of high research and practical importance and part of our current research.

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