Analytical Performance Evaluation of Distributed Multicast Algorithms for Directional Communications in WANETs

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Abstract. Two distributed algorithms DMMT-OA and DMMT-DA have been recently proposed to maximize the multicast lifetime for directional communications in wireless ad-hoc networks. The experimental results have shown their superior performance than other centralized algorithms; however, their theoretical performance in terms of approximation ratio is still unknown. In this paper, we use graph theoretic approach to derive the approximation ratio for both algorithms. Furthermore, we have discovered by the first time that both ratios are bounded by a constant number.

Key words: Wireless Ad Hoc Networks, Approximation Algorithm, Multicast, Distributed Algorithm, Directional Communications

1 Introduction

Over the last few years, energy efficient communication in Wireless Ad Hoc Networks (WANETs) with directional antennas has received more and more attention. This is because directional communications can save transmission power by concentrating RF energy where it is needed [1]. On the other hand, the broadcast / multicast communication is also an important issue as many routing protocols for WANETs need this mechanism to maintain the routes between nodes. Therefore, one would be interested in finding an algorithm that would provide the maximum lifetime to the multicast session. The optimization metric is typically defined as the duration of the network operation time until the battery depletion of the first node in the network [2][3].

Some work has considered maximizing the network lifetime in a WANET with omni-directional antennas for a broadcast session, *e.g.*, [2][3][4][5], or a multicast session, *e.g.*, [5][6][7][8]. The same problem with directional antennas has been studied in [1][9][10][11] and shown to be a NP-hard problem [11]. The only exact solution for such difficult problem is the MILP formulation presented in [10].

We note that all the solutions in [1][10][11] are centralized, meaning that at least one node needs global network information in order to construct an energy efficient multicast tree. Sometimes, this centralized approach is impractical for resource-constrained WANETs.

The most desirable work has been presented in [9], in which two distributed maximum-lifetime algorithms have been proposed for directional communications. Simulation results have also shown that they outperform other centralized multicast algorithms, *e.g.*, [1], significantly. However, their theoretical performance in terms of approximation ratio is still unknown so far. In this paper, we would like to explore the approximation ratio for these two heuristic algorithms from a graph theoretic approach.

2 Network Model

We model our wireless ad hoc network as a simple directed graph G with a finite node set N and an arc set A corresponding to the unidirectional wireless communication links. We assume a free-space propagation model and an adaptive antenna model, in which the antenna at each node v can switch its orientation to any desired direction with transmission power uniformly distributed across its adjustable beamwidth θ_v between constant numbers θ_{min} and 2π. The transmission power p_{vu} to support a link $(v\ ,\ u)$ separated by a distance r_{vu} $(r_{vu} > 1)$ is therefore

$$p_{vu} = \frac{\theta_v}{2\pi} \cdot r_{vu}^{\alpha} \tag{1}$$

We consider a source-initiated multicast with multicast members $M = \{s\} \cup D$, where s is the source node and D is the set of destination nodes. All the nodes involved in the multicast form a multicast tree rooted at the node s, *i.e.*, a rooted tree T_s, with a tree node set $N(T_s)$ and a tree arc set $A(T_s)$. Let the battery supply ε_v be the energy level associated with each node v. We assign the tree arc weight function w_{vu} as the reciprocal of the maximum lifetime of the arc (v, u) as defined as follows.

$$w_{vu}(\theta_v) = \frac{\theta_v \cdot r_{vu}^{\alpha}}{2\pi \cdot \varepsilon_v} \tag{2}$$

Note that the beamwidth θ_v applied by node v in the multicast tree T_s is a function of node v's child node set c_v, *i.e.*, $\theta_v = \varphi(c_v)$ and $c_v = \{u|(v,u) \in A(T_s)\}$. Such function $\varphi(c_v)$ is defined as the smallest possible beamwidth at node v in the range between θ_{min} and 2π to provide beam-coverage of c_v in the tree T_s.

Let Ω_M be the family of all rooted multicast trees spanning nodes in M. It has been shown in [9] that the maximum-lifetime multicast problem is equivalent to the *min-max tree problem*, which is to determine a directed tree T_s spanning all the multicast members (*i.e.*, $M \subseteq N(T_s)$ such that the maximum arc weight is minimized, *i.e.*,

$$\min_{T_s \in \Omega_M} \max_{(v,u) \in A(T_s)} w_{vu}(\theta_v). \tag{3}$$

3 Two Approximation Algorithms

Two distributed maximum-lifetime algorithms DMMT-OA (Distributed Min-Max Tree algorithm for Omnidirectional Antennas) and DMMT-DA (Distributed Min-Max Tree algorithm for Directional Antennas) [9] have been proposed for directional communications. It has been proved that the degenerate versions of both distributed algorithms for omni-directional antennas are globally optimal. For both algorithms, the multicast tree is constructed in a distributed and incremental manner. Initially, the multicast tree T_s only contains the source node. It then iteratively performs the following *Search-and-Grow* procedure until the tree contains all the nodes in M.

Search-and-Grow: Find the link (v, u) connecting tree node set and non-tree node set with minimum weight w_{vu}, and then include it into the multicast tree. Consequently, the tree T_s would grow by including as many non-tree links (x, y) as possible into the multicast tree if $w_{xy} \leq w_{vu}$ until no more such links can be found.

The DMMT-OA algorithm disregards the beamwidth in the tree construction process, assuming using omnidirectional antennas, *i.e.*, $w_{vu}(\theta_v) = w_{vu}(2\pi)$ for each arc (v, u) in the graph. After the tree T_s is constructed, each internal node v set its antennas beamwidth to $\varphi(c_v)$. Unlike the arc weights $w_{vu}(2\pi)$ in the DMMT-OA, which remain unchanged throughout the execution of the algorithm, the D-DPMT algorithm dynamically updates the weights $w_{vu}(\theta_v)$ at each step to reflect the changes of the smallest beamwidth θ_v. In the following theorems, we shall show that both algorithms have bounded approximation ratios ρ_1 and ρ_2, respectively. The technical detail of the proofs can be found in [12].

Theorem 1. *The DMMT-OA algorithm has a bounded approximation ratio* ρ_1*,*

$$\rho_1 \leq \frac{\varphi_1}{\theta_{min}}, \tag{4}$$

where φ_1 *is the minimum beamwidth applied by the transmitting node* v *of the arc* (v, u) *in the final multicast tree* T_s *obtained by DMMT-OA, i.e.,* $\varphi_1 \equiv \varphi(c_v)$*, in which* (v, u) *satisfies*

$$w_{vu}(\varphi(c_v)) = \max_{(x,y)\in A(T_s)} w_{xy}(\varphi(c_x)). \tag{5}$$

Let T_s be the final multicast tree obtained from the DMMT-DA algorithm. We use T_s^k and c_v^k to denote the partially constructed tree rooted at s and the child-node set of node v, respectively, after the k-th node is added into the tree, where $k = 0, 1, \ldots, n-1$. We assume that the ending node u of the bottleneck arc (v, u) is the i-th node added into the tree T_s and the node chosen at the beginning of the same *search-and-grow* cycle as arc (v, u) is the j-th ($j \leq i$) one that was added into the tree. Note that c_v excludes all pruned nodes from c_v^{n-1}.

Theorem 2. *The DMMT-DA algorithm has a bounded approximation ratio* ρ_2*,*

$$\rho_2 \leq \frac{\varphi_2 \cdot \varphi(c_v)}{\theta_{min} \cdot \varphi(c_v^i)}, \tag{6}$$

where φ_2 *is the smallest beamwidth applied by the transmitting node* v' *before the j-th node added into the tree such that there exists an arc* (v', u') *with minimum weight* $w_{v'u'}(2\pi)$ *connecting node sets* X *and* $N - X$*, where* $X = N(T_s^{j-1})$*, i.e.,* $\varphi_2 \equiv \varphi(c_{v'}^{j-1} \cup \{u'\})$*, in which* (v', u') *satisfies*

$$w_{v'u'}(2\pi) = \min_{x \in X, y \in N-X} w_{xy}(2\pi). \tag{7}$$

4 Conclusion

The main contribution of this paper is to provide the fact that both DMMT-OA and DMMT-DA algorithms have bounded approximation ratios. These results would help us understand why they have superior performance than other proposals in simulation experiments.

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