# ABACUS: A Distributed Middleware For Privacy Preserving Data Sharing Across Private Data Warehouses * 

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#### Abstract

Recent trends in the global economy force competitive enterprises to collaborate with each other to analyze markets in a better way and make decisions based on that. Therefore, they might want to share their data with each other to run data mining algorithms over the union of their data to get more accurate and representative results. During this process they do not want to reveal their data to each other due to the legal issues and competition. However, current systems do not consider privacy preservation in data sharing across private data sources. To satisfy this requirement, we propose a distributed middleware, ABACUS, to perform intersection, join, and aggregation queries over multiple private data warehouses in a privacy preserving manner. Our analytical evaluations show that ABACUS is efficient and scalable.


## 1 Introduction

Recent trends in the global economy force competitive enterprises to collaborate with each other for the purpose of market analysis. One of the most important examples of such collaboration is data sharing to mine and understand the market trends to be used in decision making. However, although enterprises are willing to share information with each other, they do not want to reveal their data. Due to the legal issues and competition in the market, datasources want to preserve the privacy of their data while sharing them. For example, consider a scenario consisting of two hotels, $H_{1}$ and $H_{2}$, and two airlines, $A_{1}$ and $A_{2}$. Assume hotel $H_{1}$ wants to offer a new deal to each of its customers including hotel and flight expenses based on his/her flight history. Therefore, hotel $H_{1}$ needs to learn flight history of its customers from airlines $A_{1}$ and $A_{2}$. One method to learn flight history of customers is that airlines send all of their data to hotel $H_{1}$ so that hotel $H_{1}$ can extract desired information. However, these airlines also work with hotel $H_{2}$, which is a competitor of hotel $H_{1}$, and thus they may not want to send all of their data to hotel $H_{1}$. That is because hotel $H_{1}$ can discover the customers of hotel $H_{2}$ and try to attract them. Therefore, if airlines want to work with both hotels, they cannot send their data to any of these hotels. Similarly, hotel $H_{1}$ cannot send its data to airlines so that airlines can extract the information that hotel $H_{1}$ needs since airlines will discover each other's customers. In order to be able to collaborate, hotel $H_{1}$ should take its customers' information from airlines in a way that airlines $A_{1}$ and $A_{2}$ share their data with

[^0]hotel $H_{1}$ by only revealing common customers (i.e., revealing $H_{1} \cap A_{1}$ to $H_{1}$ and $A_{1}$ and $H_{1} \cap A_{2}$ to $H_{1}$ and $A_{2}$ ). By using such a method hotel $H_{1}$ cannot discover new customers which may be customers of hotel $H_{2}$ and also airlines cannot discover new customers which may be customers of the other airline. In addition to this, hotel $H_{1}$ may want to know the total amount of its customers' travel expenses or total expenditure of a customer for its future business decisions and offers. Other enterprises may be willing to collaborate with hotel $H_{1}$, if they can preserve their privacy. The essential operations to perform these collaborations are privacy preserving intersection, join and aggregation queries. Unfortunately, we cannot use traditional query processing techniques since they do not consider privacy issues. Therefore, there is a need for privacy preserving query processing and data sharing across multiple private data warehouses.

Data integration and sharing has emerged as an important practical problem from a data management point of view [3,4,7-9]. Techniques used for this purpose commonly assume that the data sources are willing to allow access to all their data without privacy concerns during query processing. This assumption, however, is unrealistic in real life since most of the time data sources are competing enterprises. There have been several techniques in the areas of database and cryptography for privacy preserving data sharing. One of them is to use trusted third parties such that data sources hand over their data and a third party performs the computation on their behalf [1,10]. The level of trust may not be acceptable in these methods. Another approach is using secure multi-party computation where given $m$ parties and their respective inputs $x_{1}, x_{2}, . ., x_{m}$, a function $f\left(x_{1}, x_{2}, \ldots, x_{m}\right)$ is computed such that all parties can only learn $f\left(x_{1}, x_{2}, \ldots, x_{m}\right)$ but nothing else $[6,7,11]$. The computation and the communication costs make this method impractical for database operations working over a large number of elements.

In this paper, we address the problem of privacy preserving data sharing over multiple private data warehouses. We propose a distributed middleware, ABACUS, to perform intersection, join, and aggregation queries over multiple private data warehouses in a privacy preserving manner. Privacy preservation means that parties involved in the query would only be able to learn the query result but nothing else. In addition, we introduce new types of aggregation queries needed in this context and propose efficient techniques to process them. ABACUS operates as a proxy among private data warehouses and allows users to pose queries over multiple private data warehouses. Our analytical evaluations demonstrate that ABACUS provides an efficient and scalable scheme to perform intersection, join, and aggregation queries.

The rest of the paper is organized as follows. Section 2 formulates the problem and presents the architecture overview. Section 3 describes intersection and join query processing. Aggregation query processing is discussed in Section 4 and the analysis is presented in Section 5. The last section concludes the paper.

## 2 Problem Definition and Architecture Overview

Enterprises gather data from their multiple operational databases into a data warehouse, which is one the most popular ways of storing data to support decision-making in organizations. Data warehouse systems or OLAP (Online Analytical Processing) systems are different than OLTP (On-Line Transaction Processing) systems which are designed for fast updates. Thus, large enterprises have both OLAP and OLTP systems to support both an on-line community who expect fast response time for executing transactions and
off-line users who expect to analyze the data in a reasonable amount of time. Most enterprises create a large data warehouse, and periodically extract data from OLTP systems into data warehouse to be able to analyze data without interfering with online users. Data Warehouses usually use star schema for fast execution of queries over aggregated data. Star schema has dimension tables and a fact table containing a foreign key for each of the dimension tables. Furthermore, it is usually not normalized for efficient query response time since fewer joins, a bottleneck in query processing, are performed. Figure 1 shows an instance of a star schema with the fact table, Sales, and the dimension tables, Customers, Times, and Locations. Current commercial data warehouses support efficient methods to examine data. However, they do not support privacy preserving data sharing across multiple private data warehouses, which is useful for analyzing the market instead of a single company's data.


Fig. 1. An Example of Star Schema

The problem of query processing across multiple private data warehouses is defined as follows:

Let $D_{1}, D_{2}, \ldots, D_{m}$ be the data warehouses (defined with a star schema) of a set of $m$ data sources $P=\left\{P_{1}, \ldots, P_{m}\right\}$ and $q$ be a query spanning $D_{1}$ through $D_{m}$. The problem is to compute the answer of $q$ without revealing any additional information to any of the data sources.
Agrawal et al. [2] solved the problem of privacy preserving query processing across private databases by restricting it to two data sources with some relaxation in an honest-but-curious environment [6] for intersection and equijoin operations. The honest-butcurious environment means that parties follow the protocols correctly but keep all messages sent and received during the course of the query processing. The relaxation reveals the sizes of the tables or lists in the database to the other party. However, the proposed technique has two shortcomings: 1) Encryption is a computation intensive operation which is not suitable for database operations where large numbers of items need to be processed. 2) It does not support aggregation queries, which are among the most important queries.

In this paper, we propose $A B A C U S$ for privacy preserving data sharing across multiple private data warehouses. ABACUS eliminates the need for third parties by taking advantage of the star schema and executes intersection, join and aggregation queries in


Fig. 2. The architecture of a ABACUS node
a privacy preserving manner. In addition, we introduce new types of aggregation operators which are useful in the context of data warehouse and solve them efficiently. ABACUS also operates in an honest-but-curios environment and it reveals the size of tables and lists similar to [2].

ABACUS is a distributed middleware operating on top of any commercial database as shown in Figure 2. It provides a user interface where users can pose queries over multiple private data warehouses. ABACUS executes queries by running ABACUS nodes operating on different data warehouses. Each ABACUS node interacts with its data warehouse via SQL interface supplied by the underlying commercial database. Then, it contacts other ABACUS nodes and shares its data with them to process queries in a privacy preserving manner using the protocols proposed in this paper.

ABACUS does not aim to solve the problem of revealing additional information to a datasource which poses multiple queries and combines their results in order to obtain additional information about the data. In addition, it does not solve the problem of data discovery and schema mediation. Solutions to these problems are discussed briefly in [2] and they could be used in ABACUS.

## 3 Intersection and Join Query Processing

Intersection and join queries are the two important types of queries supported by current commercial databases without privacy concerns. However, in the context of data sharing across multiple private data warehouses we need privacy preserving intersection and join queries. Therefore, we will define the problems of privacy preserving intersec-
Costumers

| SSN | Name | Surname | Address | Phone |
| :---: | :---: | :---: | :---: | :---: |
| 6565 | Jack | Brial | 6616 K Rd. Xyz 93090 ZT | $890-908-4545$ |
| 7070 | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| 8080 | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

$\mathrm{T}_{1}$

| SSN | Name | Surname | Address | Phone |
| :---: | :--- | :--- | :--- | :--- |
| 6565 | Jack | Brial | 6616 K Rd. Xyz 93090 ZT | $890-908-4545$ |
|  |  |  |  |  |
| 8080 |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

$\mathrm{T}_{2}$


Fig. 3. Illustration of Aggregated Intersection
tion, and join queries and also show how to solve them efficiently in Section 3.1 and Section 3.2 respectively.

### 3.1 Aggregated Intersection

Intersection queries constitute the first step for collaboration over common data items. For example, a company may need to know other companies' opinions about its customers. For this kind of collaboration, two companies need to find the common customers as a first step, i.e., intersection. The intersection of two customer lists can be found easily unless they do not hesitate to reveal their customers to each other. However, most of the time companies may not want to reveal their customer lists but only common customers to each other due to legal issues or competition. To support such a type of collaboration, a method for privacy preserving intersection where parties can only learn items in the intersection but nothing else is needed. Therefore, we first define the problem of finding the intersection of lists in the context of data warehouse while preserving the privacy called aggregated intersection query processing as follows:

Let $L_{1}, L_{2}, \ldots, L_{m}$ be the lists containing secret data stored by a set of datasources $P=\left\{P_{1}, P_{2}, \ldots, P_{m}\right\}$ respectively. For each data source $P_{i}$, the problem is to find all other data sources, $P_{j}$, with $e \in L_{j}$ for each item $e \in L_{i}$ in a privacy preserving manner, i.e., if $P_{j}$ does not have $e$ in $L_{j}$, then $P_{j}$ will not know $e \in L_{i}$.

Example 1. We illustrate the aggregated intersection problem with an example. Consider three datasources $P_{1}, P_{2}$ and $P_{3}$ involved in executing aggregated intersection query with customer tables $T_{1}, T_{2}$ and $T_{3}$ respectively as shown in Figure 3. At the end of aggregated query processing, datasources will only learn the common customers they share with other data sources but nothing else. In this example, all data sources will know 6565 exists in tables $T_{1}, T_{2}$, and $T_{3} . P_{1}$ and $P_{2}$ will also know that 8080 is common in their tables. Similarly, $P_{1}$ and $P_{3}$ will know that 7070 is common in tables $T_{1}$ and $T_{3}$. However, $P_{2}$ should not be able to know that $P_{1}$ and $P_{3}$ have 7070 in their tables. Similarly, $P_{3}$ must not know that $P_{1}$ and $P_{2}$ have 8080 in their tables.

Our solution to the above problem is based on using one-way secure cryptographic hash functions. These hash functions are widely used in many real life applications such as password protection, message authentication, and digital signatures. The examples of such hash functions include SHA-1, MD4, and MD5 [12]. A simple solution to the aggregated intersection problem could use one-way hash functions and compare hashed values of items to determine whether they are the same or not. Basically, data source $P_{i}$ computes the hashed list of list $L_{i}$ by computing the hash value of each item in $L_{i}$. Then, it sends the hashed list to data source $P_{j}$ so that it can compare the incoming hashed list with its own hashed list to find the common items in $L_{i}$ and $L_{j}$. According to this scheme, in Example 1, data source $P_{1}$ uses a hash function $H$ and sends the list of hashed values, $\{H(6565), H(7070), H(8080)\}$, to $P_{2}$ and $P_{3}$. Then, $P_{2}$ compares the hashed list with its own hashed list, $\{H(6565), H(8080)\}$, and determines that 6565 and 8080 are common. Since the hash function is a one-way hash function, $P_{2}$ will not be able to know 7070 is in $L_{1}$. This basic solution, however, suffers from the following two problems: 1) If the domain size is small, then item $x$ whose hash value is $H(x)$ could be computed by exhaustively searching the whole domain. 2) Hash collisions might produce inaccurate results.

In the context of data warehouses, data sources usually have more information about the secret items. For example, all data sources in Example 1 keep name, last name, phone and address information as well as SSN (Social Security Number) of a customer in their customers tables. If all of these information is used in hashing, then the domain will become large. For instance, instead of hashing SSN, a concatenation of SSN, name, last name, phone and address could be used in hashing, i.e., H(6565|Jack|Brial|6616KRD $X_{y z Z T 93090 \mid 8909084545)}$ could be used instead of $H(6565)$ for a customer with an SSN 6565. This method allows us to enlarge the domain size and makes exhaustive search impossible. The aggregated intersection problem is to find the common secret items in the dimensions tables in the context of data warehouse. ABACUS uses the common attributes in all of the tables to hash secret items i.e., the values of common attributes are used instead of a value of a primary key. For example, the attributes SSN, Name, Surname, Phone, and Address are common in $T_{1}, T_{2}$ and $T_{3}$ in Example 1. If 5 attributes each of which is 10 characters long are used in hashing, the domain size would be $28^{50} \approx 2^{250}$ which makes exhaustive search impossible.

As mentioned before, hash collisions might result in sharing a secret item which is not in the intersection. $H$ maps values to $|\operatorname{DomH}|$ which is assumed to be arbitrarily large compare to the intersection size. Let $N=|D o m H|$; in the random oracle model, the probability that $n$ hash values have at least one collision equals [2]: $\operatorname{Pr}[$ collision $]=$ $1-\exp \left(\frac{-n(n-1)}{2 N}\right)$. For 1024 bit hash values and $n=1$ million, this probability is $10^{-295}$ [2]. Thus, the solution to expand the domain size minimizes the probability of data
vulnerability by exhaustive search and also helps in reducing the probability of hash collisions, and therefore, errors in the queries are significantly reduced.

### 3.2 Aggregated Join Queries

One of the most important query operators supported by current commercial database systems is the join operator. Privacy preserving join operations have not been previously considered in database research or in current database management systems. However, they might be needed in data sharing across private data sources. For example, a company (e.g. a hotel) might want to know the transaction details of its customers in other companies (e.g. airlines) in the market to classify them according to their transactions. For instance, a hotel can identify the customers that travel frequently and offer special promotions to them. To be able to do this, it needs to join its customers table with other companies' sales tables. Since other companies may benefit from this process, they might be willing to share transaction details. However, during this process companies are not willing to reveal any information about a customer who is not a customer of the other company as well as his/her existence. Traditional join query processing techniques cannot be used to process these queries since they do not consider privacy issues. In order to satisfy these requirements, we propose a new join operator, the aggregated join query operator, to be used for privacy preserving data sharing across private data warehouses. We first formally define the aggregated join query processing problem and then propose a solution.

The aggregated join query processing problem is formally defined as follows:
Assume data source $P_{1}$ has a dimension table $P_{1} . T_{d}$ and data sources $P_{2}, P_{3}, \ldots$, $P_{m}$ have fact tables $P_{2} . T_{f}, P_{3} . T_{f}, \ldots, P_{m} . T_{f}$ respectively with common attribute $A$. Then, the goal is to compute $P_{1} \cdot T_{d} \bowtie P_{2} \cdot T_{f} \cup P_{1} \cdot T_{d} \bowtie P_{3} \cdot T_{f} \cup \ldots \cup P_{1} \cdot T_{d} \bowtie$ $P_{m} . T_{f}$ such that none of the data sources learn any extra information other than the query result. Query poser $P_{1}$ will learn only the tuples $t$ such that $t \in P_{i} . T_{f}$ for which $t . A \in P_{1} . T_{d} . A$. In other words, $P_{i}$ shares a list, $L_{v}$, of tuples in $P_{i} . T_{f}$ for each value $v \in P_{i} . T_{f} . A$ with $P_{1}$ if $\exists t \in P_{1} . T_{d}$ such that $t . A=v$, and nothing else where $i=2,3 \ldots, m$.
We illustrate the problem with an example. Assume the three data warehouses in Example 1 want to execute an aggregated join query. And assume $P_{1}$ poses the aggregated join query to find the aggregated join of its dimension table, Customers Table, with the fact tables, Sales Table, of the other data sources as shown in Figure 4. The problem is to provide an answer to this query without revealing any additional information. For this example, $P_{2}$ will return the tuples with SSN 6565 and 8080 in its Sales table without knowing $P_{1}$ has 7070 in its customers table. Similarly, $P_{3}$ will return all tuples with SSN 6565 and 7070 without knowing $P_{1}$ has a customer with an SSN 8080. In addition, $P_{1}$ will not be revealed the transaction details of other customers which are not in its Customers table, e.g. a customer with an SSN 9090.

ABACUS executes the aggregated join query, $\bigcup_{i=2}^{m} P_{1} . T_{d} \bowtie P_{i} . T_{f}$, in two phases: Intersection Phase and Join Computation Phase. In the intersection phase, $P_{1}$ and $P_{j}$ compute the intersection of their dimension tables, $P_{1} \cdot T_{d} \cap P_{j} \cdot T_{d}$ with the method discussed in Section 3.1 (i.e., $P_{1}$ sends a hashed list of its customers so that $P_{j}$ can know common customers). Then, $P_{j}$ sends all tuples $t \in P_{j} . T_{f}$ where $t . A \in \Pi_{A}\left(P_{1} \cdot T_{d} \cap P_{j} . T_{d}\right)$ to $P_{1}$.


| SSN | Name | Surname | Address | Phone |
| :--- | :--- | :--- | :--- | :--- |
| 6565 | Jack | Brial | 6616 K Rd. Xyz 93090 ZT | $890-908-4545$ |
| 7070 |  |  |  |  |
| 8080 |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| Customers Table at $\mathrm{P}_{1}$ |  |  |  |  |

Fig. 4. Illustration of Aggregated Join

During the query processing, no extra useful information gets revealed. In the intersection phase, all data sources compute the intersection of the dimension tables and in the join computation phase, all data sources other than the query poser send the related tuples from their fact tables. As a result, no site gains extra useful information other than the intersection and the join results.

## 4 Aggregate Query Processing

The traditional aggregation operation is generally used to compute the aggregate of a list of values such as SUM, AVERAGE or MIN/MAX. One kind of privacy preserving aggregation can be thought of as computing the aggregation of values in the union of lists coming from different data sources such that each data source will only know the final aggregate but nothing else. To execute these queries, each data source can compute its local aggregate and the final aggregate can be computed in such a way that none of the data sources will know the local aggregate of other data sources (Secure multiparty computation or the technique described in this section can be used to compute the final aggregate value for SUM and AVERAGE). However, data sources may not be willing to execute aggregation operations over their whole data or may want to know more than the sum of the values in several lists. Therefore, there is a need for new types of aggregation queries. In this section, we will introduce Row-Based Aggregation and Column-Based Aggregation queries. We formally define Row-Based Aggregation queries and show how to process them in Section 4.1. Then, we will present Column-Based Aggregation queries and techniques to execute them efficiently in Section 4.2.

### 4.1 Row-Based Aggregation

Enterprises may want to know the total expenditure of a customer in the market. For example, hotels and airlines may want to classify their customers based on their travel
expenses. Without privacy concerns it is easy to perform this classification task. One of the enterprises may collect data from all enterprises and perform the computation. However, they may not be willing to reveal their value during this operation. For example, an airline company may not be willing to reveal an expenditure of a customer to other airlines since other airlines may try to attract this customer. For instance, if a customer's expenditure in company $C_{1}$ is 80 , and another company $C_{2}$ knows that his/her expenditure in $C_{1}$ is 80 , then $C_{2}$ can offer a new deal to this customer and try to attract him/her using this information. Although enterprises may not be willing to reveal their earnings from a customer, they may want to know the total expenditure of the customer without revealing their values. For example, these hotels and airlines may be willing to know the total expenditure of a customer in these hotels and airlines (i.e., total expenditure in the market) without revealing their earnings from this customer so that competing hotels and airlines protect their private information from each other. Since the traditional aggregation operation is not strong enough to support these needs, ABACUS proposes a new type of aggregation queries, Row-Based Aggregation queries, and a new technique to execute them in a privacy preserving manner in the context of data warehouses.

For the sake of this discussion, we will first define the row-based aggregation on a table with two attributes namely Key and Value. Then, we will discuss how this can be generalized to support queries in data warehouses. The Row-Based Aggregation query processing problem is defined as follows:

Let $T_{1}, T_{2}, \ldots, T_{m}$ be the tables stored by a set of source peers $P=\left\{P_{1}, P_{2}, \ldots, P_{m}\right\}$ ( $m \geq 3$ ) respectively containing a Key and a Value attributes. Each data source, $P_{i}$, would like to learn the aggregate for each $K e y \in T_{i}, \sum_{j=1}^{m}$ Value $\exists[$ Key, Value $] \in$ $T_{j}$. Then, the problem is to obtain the answer of the query without revealing any additional information.

The above problem formulation is for SUM queries. We solve the above problem with some relaxation. The relaxation is that a data source with Key in its database can learn who else has the same Key (Note that this information is the same as the result of aggregated intersection). However, it is impossible to learn the Value associated with that Key at the other data warehouses. Extending our solution to support AVERAGE queries is straightforward and discussed briefly at the end of this section.
Example 2 Let us illustrate the problem with an example. Consider four companies, $P_{1}, P_{2}, P_{3}$ and $P_{4}$, that want to classify their customers according to their total expenditures from these companies. They have tables $T_{1}, T_{2}, T_{3}$ and $T_{4}$ each of which with two attributes customer SSN and the amount of expenditure as [Key,Value] pairs. The contents of the tables are as follows:
$T_{1}=\{[6565,10],[7070,20],[8080,30]\}$
$T_{2}=\{[6565,50],[8080,30]\}$
$T_{3}=\{[6565,10],[7070,20],[8080,30]\}$
$T_{4}=\{[6565,10],[7070,20]\}$
To classify customers, one should know their total expenditures in the market. In other words, a row-based aggregation is needed for this process so that at the end of query processing $P_{1}, P_{2}, P_{3}$ and $P_{4}$ will get the following lists respectively as an answer without knowing any additional information: $\{[6565,80],[7070,60],[8080,90]\},\{[6565,80],[8080,90]\}$, $\{[6565,80],[7070,60],[8080,90]\},\{[6565,80],[7070,60]\}$. The first item in the above first list,
[6565, 80] means that the customer with SSN 6565 has a total expenditure of 80 in companies $P_{1}, P_{2}, P_{3}$ and $P_{4}$. ABACUS can process row-based aggregation queries in a privacy preserving manner while revealing some information which is typically acceptable in an honest-but-curious environment. For example, each company will learn who else has a customer with the same SSN as in its list. For instance, $P_{1}$ will know that 6565 exists in all data sources, while 7070 also exists in $P_{3}$ and $P_{4}$, and 8080 exists in $P_{2}$ and $P_{3}$. During this query processing, none of the data sources will be able to learn the value of a specific key of the other data sources. For example, $P_{1}$ will not learn that 6565 has an expenditure of 50 in $P_{2}$, but will learn that 80 is the total expenditure of 6565 in all of the companies. Note that, if only two data sources have the same key, they may not share their values with each other by rejecting aggregation on that key (because they can learn each other's value for that key). ABACUS allows users to configure their privacy policies for this kind of policy related issues and handle them efficiently. We will discuss these issues later in this section.

A simple technique to compute the sum of values (i.e., $V_{1}+V_{2}+V_{3}+V_{4}$ ) for a specific key Key in four key-value pairs [Key, $\left.V_{1}\right]$, $\left[K e y, V_{2}\right]$, $\left[K e y, V_{3}\right]$, and $\left[K e y, V_{4}\right]$ residing at four different parties $P_{1}, P_{2}, P_{3}$ and $P_{4}$ respectively without revealing $V_{1}$, $V_{2}, V_{3}$ and $V_{4}$ could be circulating a token with a label $H($ Key $)$. Using secure one-way hash function can prevent others from learning Key if they do not have Key. The process consists of two circulations. During the first circulation, every party, $P_{i}$, would add its value, $V_{i}$, and a random number, $r_{i}$, and pass the token to the next party. Therefore, $P_{1}$ creates a token with a label $H($ Key $)$ and adds $V_{1}+r_{1}$, then passes it to $P_{2}$. The other parties follow the same protocol and pass the token to the next one. At the end of first circulation, $P_{1}$ will get $V_{1}+r_{1}+V_{2}+r_{2}+V_{3}+r_{3}+V_{4}+r_{4}$ for a token with a label $H($ Key $)$. There is no way to determine the value of a specific party during the course of the first circulation because of the random numbers added. In the second circulation, all parties subtract the random numbers they added during the first circulation. Therefore, at the end of second circulation, $P_{1}$ would have a token with a label $H($ Key $)$ and the sum of the values for that Key, $V_{1}+V_{2}+V_{3}+V_{4}$. Although it seems secure, this basic technique has two problems. Since this process is needed for every item in the list, using the same random number for every item in the list may result in information leakage such as the difference between two values. To prevent this information leakage, parties should use a different random number for each item in their lists. Therefore, every data source should maintain a list of random numbers it used during this process which is not scalable for large lists. Another problem is that any two of the data sources could collude and learn the value of another data source. For example, $P_{2}$ and $P_{4}$ could learn the value of $P_{3}, V_{3}$. In the first circulation, $P_{4}$ would pass the token with a label $H(\mathrm{Key})$ and $V_{1}+r_{1}+V_{2}+r_{2}+V_{3}+r_{3}+V_{4}+r_{4}$ to $P_{1}$ and in the second circulation, $P_{1}$ would pass $V_{1}+V_{2}+r_{2}+V_{3}+r_{3}+V_{4}+r_{4}$ to $P_{2}$. Since $P_{2}$ and $P_{4}$ know $V_{1}+r_{1}+V_{2}+r_{2}+$ $V_{3}+r_{3}+V_{4}+r_{4}$ and $V_{1}+V_{2}+r_{2}+V_{3}+r_{3}+V_{4}+r_{4}$ they could figure out $r_{1}$, and thus $V_{1}$ (Note that $P_{1}$ passed $V_{1}+r_{1}$ to $P_{2}$ in the first circulation). Therefore, $P_{2}$ and $P_{4}$ could collude and reveal the value of $P_{1}$ without revealing their values to each other or to the other parties.

In order to compute aggregation securely, ABACUS uses Shamir's secret sharing technique, which allows one to compute any linear combination of secret values. ABACUS uses this property to perform SUM and AVERAGE queries thus computing aggregation without revealing individual values.

```
Algorithmus 1 Shamir's Secret Sharing Algorithm
    Input:
    \(v_{s}\) : Secret value
    \(D\) : Dealer of secret \(v_{s}\)
    \(P\) : set of peers \(P_{1}, \ldots, P_{n}\) to distribute secret;
    Output:
    share \(_{1}, \ldots\), share \(_{n}:\) Shares of secret, \(v_{s}\), for each peer \(P_{i}\)
    Procedure:
    \(D\) creates a random polynomial \(q(x)=a_{k-1} x^{k-1}+\ldots+a_{1} x^{1}+a_{0}\) with degree \(k-1\) and a constant term
    \(a_{0}=v_{s}\)
    \(D\) chooses publicly known \(n\) random points, \(x_{1}, \ldots x_{n}\), such that \(x_{i} \neq 0\)
    \(\dot{\mathrm{O}}: D\) computes share, share \(_{i}\), of each peer, \(P_{i}\), where share \({ }_{i}=q\left(x_{i}\right)\) and sends it to \(P_{i}\).
```


## Shamir's Secret Sharing

Shamir's secret sharing method [13] allows a dealer $D$ to distribute a secret value $v_{s}$ among $n$ peers $\left\{P_{1}, P_{2}, \ldots, P_{n}\right\}$, such that knowledge of any $k(k \leq n)$ peers is required to reconstruct the secret. Since, even complete knowledge of $k-1$ peers cannot reveal any information about the secret, Shamir's method is information theoretically secure. Dealer $D$ chooses a random polynomial $q(x)$ of degree $k-1$ where the constant term is the secret value, $v_{s}$, and a publicly known set of $n$ random points. The dealer computes the share of each peer as $q\left(x_{i}\right)$ and sends it to peer $P_{i}$. The method is summarized in Algorithm 1.

In order to construct the secret value $v_{s}$, any set of $k$ peers will need to share the information they have received. After finding the polynomial $q(x)$, the secret value $v_{s}=$ $q(0)$ can be reconstructed. $q(x)$ can be found using Lagrange interpolation such that $p\left(x_{i}\right)=$ share $_{i}$ where $i=1, \ldots, k$. The key observation is that at least $k$ points and the respective shares are required to determine a unique polynomial $q(x)$ of degree $k-1$.

## Row-Based Aggregation in ABACUS

ABACUS executes row-based aggregation queries in three phases: Distribution phase, Intermediate-Computation phase, and Final-Computation phase.

## Distribution Phase

After the query is posed, $m$ data sources decide on the degree of the polynomial that is going to be used in Shamir's secret sharing (the degree of the polynomial should be greater than or equal to $m-1$ ). They also choose $n \geq m$ random values $X=$ $\left\{x_{1}, \ldots, x_{n}\right\}$. Without loss of generality, we will use a polynomial of degree of $m-1$ and $n=m$ in our setting. Each data source $P_{i}$ has a list of Key-Value pairs, $L_{i}=$ $\left\{\left[K_{1}, V_{1}\right], \ldots,\left[K_{\left|L_{i}\right|}, V_{\left|L_{i}\right|}\right]\right\} ; P_{i}$ creates $m$ shares from $L_{i}, \operatorname{share}\left(L_{i}, P_{1}\right), \ldots, \operatorname{share}\left(L_{i}, P_{m}\right)$, one for each of the data sources $P_{1}$ through $P_{m}$ respectively (including itself).
$P_{i}$ creates the shares by applying a one-way hash function and Shamir's secret sharing algorithm to each of the elements in $L_{i}$. For every element [Key,Value] in $L_{i}, P_{i}$ computes the share of data source $P_{j}, \operatorname{sh}\left([K e y, V a l u e], P_{j}\right)=\left[H(K e y), q\left(x_{j}\right)\right]$, using a hash function $H$ and Algorithm 1 with $q(x)$ and $X$ (the constant term in $q(x)$ will be replaced by the secret value, Value, to compute $q(x)$ in Shamir's secret sharing). Therefore, the list of shares of data source $P_{j}$ from $L_{i}$ is $\operatorname{share}\left(L_{i}, P_{j}\right)=\left\{\operatorname{sh}\left(\left[K_{1}, V_{1}\right], P_{j}\right), \ldots\right.$, $\left.\operatorname{sh}\left(\left[K_{\left|L_{i}\right|}, V_{\left|L_{i}\right|}\right], P_{j}\right)\right\}$. Then, $P_{i}$ sends $\operatorname{share}\left(L_{i}, P_{j}\right)$ to the data source $P_{j}$. Note that $P_{i}$ keeps its share, $\operatorname{share}\left(L_{i}, P_{i}\right)$, for itself and since using the same $q(x)$ would results in information leakage, a random polynomial is used for each of the item in the list.

```
Algorithmus 2 Distribution Phase
    Input:
    \(X\) : Random Values \(X=\left\{x_{1}, . ., x_{m}\right\} ;\)
    \(H\) : Secure one-way hash function
    \(L_{i}:\) Secret list of Key-Value pairs at data source \(P_{i} ;\)
    Output:
    \(\operatorname{share}\left(L_{i}, P_{1}\right), \ldots, \operatorname{share}\left(L_{i}, P_{m}\right)\) : Shares of secret list, \(L_{i}\), for each data source \(P_{j}\);
    Procedure:
    for Each secret Key-Value pair \(\left[K e y, V_{s}\right] \in L_{i}\) do
        Find share \(s h\left(\left[\right.\right.\) Key,\(\left.\left.V_{s}\right], P_{j}\right)\) of each data source \(P_{j}\) for \(\left[K e y, V_{s}\right]\) with Algorithm 1 using a random polyno-
        mial \(q(x)\) where \(q(x)=a_{k-1} x^{k-1}+\ldots+V_{s}\) and the hash function \(H\) such that \(\operatorname{sh}\left(\left[K e y, V_{s}\right], P_{j}\right)=\)
        [H(Key), \(\left.q\left(x_{j}\right)\right]\).
            Add \(\operatorname{sh}\left(\left[\right.\right.\) Key, \(\left.\left.V_{s}\right], P_{j}\right)\) into \(\operatorname{share}\left(L_{i}, P_{j}\right)\).
        end for
    .
    for For each data source \(P_{j}\) do
        Send \(\operatorname{share}\left(L_{i}, P_{j}\right)\) to data source \(P_{j}\)
        end for
```

Therefore, random polynomials $q_{1}$ through $q_{\left|L_{i}\right|}$ are used for the items 1 through $\left|L_{i}\right|$ in $L_{i}$.

In Example 2, assume $P_{1}$ with a list, $L_{1}=\{[6565,10],[7070,20],[8080,30]\}$ and four data sources decided on four random points $X=\{27,65,90,123\}$. Since there are four data sources, a polynomial $q(x)$ of a degree three would be used with a hash function $H$ while calculating the share of each data source. As a first step, $P_{1}$ chooses three random polynomials for each item in its list: $q_{1}(x)=2 x^{3}-2 x^{2}+10, q_{2}(x)=x^{3}-5 x^{2}+20$, $q_{3}(x)=x^{3}-13 x^{2}+30$. Observe that the constant term of polynomial $q_{i}$ is value of the $i$ th item in $L_{1}$ and $q_{i}$ is used for the $i$ th item in $L_{1}$. Then, the shares of key-value pairs in $L_{1}$ for data source $P_{2}$ are calculated as follows:
$s h\left([6565,10], P_{2}\right)=\left[H(6565), q_{1}\left(x_{2}\right)\right]=\left[H(6565), q_{1}(65)\right]$
$\operatorname{sh}\left([7070,20], P_{2}\right)=\left[H(7070), q_{2}\left(x_{2}\right)\right]=\left[H(7070), q_{2}(65)\right]$
$s h\left([8080,30], P_{2}\right)=\left[H(8080), q_{3}\left(x_{2}\right)\right]=\left[H(6565), q_{3}(65)\right]$.
Therefore, the share list for $P_{2}, \operatorname{share}\left(L_{1}, P_{2}\right)$, is: $\operatorname{share}\left(L_{1}, P_{2}\right)=\left\{\left[H(6565), q_{1}(65)\right],[H(7070)\right.$, $\left.q_{2}(65)\right],\left[H(8080), q_{3}(65)\right]$. Similarly, other data sources' share lists are computed and are sent to them. $P_{1}$ would keep $\operatorname{share}\left(L_{1}, P_{1}\right)$ for itself and sends $\operatorname{share}\left(L_{1}, P_{2}\right)$, $\operatorname{share}\left(L_{1}, P_{3}\right)$, and $\operatorname{share}\left(L_{1}, P_{4}\right)$ to $P_{2}, P_{3}$, and $P_{4}$ respectively.

Distribution phase at data source $P_{i}$ is summarized in Algorithm 2.

## Intermediate-Computation Phase

After receiving their shares from the data sources, $P_{1}, \ldots, P_{m}$, each data source, $P_{i}$, calculates intermediate result lists, $I R\left(L_{1}, P_{i}\right), \ldots, I R\left(L_{m}, P_{i}\right)$, corresponding to the lists $\operatorname{share}\left(L_{1}, P_{i}\right), \ldots, \operatorname{share}\left(L_{m}, P_{i}\right)$ respectively. The $k$ th element of $\operatorname{share}\left(L_{j}, P_{i}\right)$ is a key-value pair i.e., share $\left(L_{j}, P_{i}\right)[k]=\left[H(\right.$ Key $)$, Value $\left.{ }^{*}\right]$ which is the share of $P_{i}$ from the $[$ Key, Value $]$ pair in $L_{j}\left(\operatorname{share}\left(L_{j}, P_{i}\right)[k][1]=H(\right.$ Key $)$ and $\operatorname{share}\left(L_{j}, P_{i}\right)[k][2]=$ Value ${ }^{*}$ ). $P_{i}$ computes the intermediate result lists as follows:
$\operatorname{IR}\left(L_{j}, P_{i}\right)[k][1]=\operatorname{share}\left(L_{j}, P_{i}\right)[k][1]$
$\operatorname{IR}\left(L_{j}, P_{i}\right)[k][2]=\sum_{h=1}^{m}\left(\operatorname{share}\left(L_{h}, P_{i}\right)[g][2]\right.$ s.t. $\exists g$ where $\left.\operatorname{share}\left(L_{h}, P_{i}\right)[g][1]=\operatorname{IR}\left(L_{j}, P_{i}\right)[k][1]\right)$, i.e., $I N T E R-R E S_{i}$.

In Example 2, $P_{1}$ would have lists $\operatorname{share}\left(L_{1}, P_{1}\right)$, $\operatorname{share}\left(L_{2}, P_{1}\right)$, $\operatorname{share}\left(L_{3}, P_{1}\right)$ and $\operatorname{share}\left(L_{4}, P_{1}\right)$ where
$\operatorname{share}\left(L_{1}, P_{1}\right)=\{[H(6565), 120],[H(7070), 320],[H(8080), 400]\}$

```
Algorithmus 3 Intermediate Computation Phase
    Input
    Share \(_{L}:\) Set of share lists, Share \(_{L}=\left\{\operatorname{share}\left(L_{1}, P_{i}\right), .\right.\). , share \(\left.\left(L_{m}, P_{i}\right)\right\}\)
    Output:
    Set of intermediate result lists \(\left\{I R\left(L_{1}, P_{i}\right), \ldots, I R\left(L_{m}, P_{i}\right)\right\}\) to send back to the data sources \(P=\left\{P_{1}, \ldots, P_{m}\right\}\)
    respectively;
    Procedure:
    for each list \(\operatorname{share}\left(L_{k}, P_{i}\right) \in S h a r e_{L}\) do
        for \(j=1 ; j \leq\left|\operatorname{share}\left(L_{k}, P_{i}\right)\right|\) do
        \(\operatorname{IR}\left(L_{k}, P_{i}\right)[j][1]=\operatorname{share}\left(L_{k}, P_{i}\right)[j][1]\)
        if \(\operatorname{share}\left(L_{k}, P_{i}\right)[j][1]=\operatorname{share}\left(L_{l}, P_{i}\right)[o][1]\) such that \(\exists l\) and \(o\) where \(l \leq m\) and \(1 \leq o \leq\)
        share \(\left(L_{l}, P_{i}\right) \mid\) then
            \(\operatorname{IR}\left(L_{k}, P_{i}\right)[j][2]=\operatorname{IR}\left(L_{k}, P_{i}\right)[j][2]+\operatorname{share}\left(L_{l}, P_{i}\right)[o][2]\)
        end if
        end for
    end for
    Send \(I R\left(L_{1}, P_{i}\right), \ldots, I R\left(L_{m}, P_{i}\right)\) to \(P_{1}, \ldots, P_{m}\) respectively
```

```
share( }\mp@subsup{L}{2}{},\mp@subsup{P}{1}{})={[H(6565),100],[H(8080),600]
share(L}\mp@subsup{L}{3}{},\mp@subsup{P}{1}{})={[H(6565),3500],[H(7070),900],[H(8080), 90]
share(L}\mp@subsup{L}{4}{},\mp@subsup{P}{1}{})={[H(6565),110],[H(7070),80]
```

Then, in the intermediate computation phase, $P_{1}$ will compute $\operatorname{IR}\left(L_{1}, P_{1}\right) \operatorname{IR}\left(L_{2}, P_{1}\right)$,
$I R\left(L_{3}, P_{1}\right)$ and $I R\left(L_{4}, P_{1}\right)$ and send them to data sources $P_{1}, P_{2}, P_{3}$ and $P_{4}$ respec-
tively. For example, $\operatorname{IR}\left(L_{3}, P_{1}\right)$ is computed as follows: Since $H(6565)$ exists in all
lists, the values associated with it, 120, 100, 3500 and 110 in $\operatorname{share}\left(L_{1}, P_{1}\right)$ through
$\operatorname{share}\left(L_{4}, P_{1}\right)$ respectively, are added. Therefore, $\operatorname{IR}\left(L_{3}, P_{1}\right)[1]=H(6565)$ and $\operatorname{IR}\left(L_{3}, P_{1}\right)[2]=$
$120+100+3500+110=3830$. The same calculation is performed for all items in the
list resulting in $\operatorname{IR}\left(L_{3}, P_{1}\right)=\{[H(6565), 3830],[H(7070), 4900],[H(8080), 1090]\}$.

The intermediate computation process at data source $P_{i}$ is summarized in Algorithm 3.

## Final-Computation Phase

In the final computation phase, data source $P_{i}$ retrieves its intermediate result lists, $I R\left(L_{i}, P_{1}\right), \ldots, I R\left(L_{i}, P_{m}\right)$ from all $m$ data sources. Since all data sources compute the sum of their shares for a specific Key, the $k$ th entry of an intermediate list contains $H(K e y)$ and the sum of shares for Key. Therefore, for a Key-Value pair in $L_{i}$, the corresponding entry $k$ in the intermediate result lists are: $\operatorname{IR}\left(L_{i}, P_{1}\right)[k]=[H($ Key $), I N T E R-$ $\left.R E S_{1}\right], I R\left(L_{i}, P_{2}\right)[k]=\left[H(\right.$ Key $\left.), I N T E R-R E S_{2}\right], \ldots, I R\left(L_{i}, P_{m}\right)[k]=[H($ Key $), I N T E R-$ $\left.R E S_{m}\right]$.

In the final computation phase, data sources calculate the sum for each Key from the $m$ intermediate results. Since all data sources use a random polynomial degree of $m-1$ and compute the shares of all data sources using $m$ points, $X=\left\{x_{1}, x_{2}, . ., x_{m}\right\}$, these result in a polynomial $P(x)=a_{m-1} x^{m-1}+\ldots+a_{1} x^{1}+a_{0}$ where constant term, $a_{0}$, is the sum of the values for Key and $P\left(x_{i}\right)=I N T E R-R E S_{i}$. The coefficients of $P(x)$ and thus the sum of the values could be computed because the values of $P(x)$ are known at $m$ different points $\left(P\left(x_{i}\right)=I N T E R-R E S_{i}\right)$.

## Proof of Correctness

A data source $P_{j}$ constructs a random polynomial $a_{j} x_{i}^{m-1}+b_{j} x_{i}^{m-2}+\ldots+$ Value to hide the secret values for each [Key,Value] pair. After generating this random polynomial, it computes the share of $P_{i}$ as $\left(H(K e y),\left[a_{P_{j}} x_{i}^{m-1}+b_{P_{j}} x_{i}^{m-2}+\ldots+v_{P_{j}}\right]\right)$ for each secret
key-value pair, where $v_{P_{j}}=$ Value and sends the shares of the other data sources. After $P_{i}$ receives the shares from all $m$ data sources, it sends the sum of values which have the same key. Without loss of generality, assume $l$ of the $m$ data sources have the same Key with the secret values $v_{1}$ through $v_{l}$ respectively. Then the sum for that Key is in the following form:

$$
\begin{gathered}
a_{1} x_{i}^{m-1}+b_{1} x_{i}^{m-2} \ldots+v_{1}+ \\
a_{2} x_{i}^{m-1}+b_{2} x_{i}^{m-2} \ldots+v_{2}+ \\
\vdots \\
a_{l} x_{i}^{m-1}+b_{l} x_{i}^{m-2} \ldots+v_{l}
\end{gathered}
$$

Therefore, $P_{i}$ sends its results $I N T E R-R E S_{i}=\left(a_{1}+a_{2}+\ldots+a_{l}\right) x_{i}^{m-1}+\ldots+S U M$ to the parties having Key in their lists, where $S U M$ is the sum of the secret values ( $S U M=v_{1}+v_{2}+\ldots+v_{l}$ ) for the values that have the same key, Key.

Each data source receives $m$ results from each of the data sources (including itself) for each key in its [Key-Value] list:

$$
\begin{aligned}
I N T E R-R E S_{1} & =\left(a_{1}+a_{2}+\ldots+a_{l}\right) x_{1}^{m-1}+\ldots+S U M \\
I N T E R-R E S_{2} & =\left(a_{1}+a_{2}+\ldots+a_{l}\right) x_{2}^{m-1}+\ldots+S U M \\
& \vdots \\
I N T E R-R E S_{m} & =\left(a_{1}+a_{2}+\ldots+a_{l}\right) x_{m}^{m-1}+\ldots+S U M
\end{aligned}
$$

Since $X=\left\{x_{1}, x_{2}, . ., x_{m}\right\}$ is known by all data sources, there are a total of $m$ unknown coefficients including $S U M$ and $m$ equations in the above system of equations. Therefore, $S U M$ can be derived by using the above equations. The data source, $P_{j}$, cannot know the value of the other data sources, since the coefficients of the polynomials used by other data sources are not known by $P_{j}$.

For the average query, $P_{i}$ sends $I N T E R-R E S_{i}=\left[\left(a_{1}+a_{2}+\ldots+a_{l}\right) x_{i}^{m-1}+\right.$ $\ldots .+S U M] / l$ where $\left.I N T E R-R E S_{i}=\frac{\left(a_{1}+a_{2}+\ldots+a_{l}\right)}{l} x_{i}^{m-1}+\ldots .+A V G\right)$ and $A V G=$ $\frac{v_{1}+v_{2}+\ldots+v_{l}}{l}$. Therefore, each data source receives $m$ results:

$$
\begin{gathered}
I N T E R-R E S_{1}=\frac{\left(a_{1}+a_{2}+\ldots+a_{l}\right)}{l} x_{1}^{m-1}+\ldots+A V G \\
I N T E R-R E S_{2}=\frac{\left(a_{1}+a_{2}+\ldots+a_{l}\right)}{l} x_{2}^{m-1}+\ldots+A V G \\
\vdots \\
I N T E R-R E S_{m}=\frac{\left(a_{1}+a_{2}+\ldots+a_{l}\right)}{l} x_{m}^{m-1}+\ldots+A V G
\end{gathered}
$$

Again, since $X=\left\{x_{1}, x_{2}, . ., x_{n}\right\}$ is known by the data sources, there are $m$ unknown coefficients including $A V G$ and $m$ equations and thus, $A V G$ can be derived from the above equations.

## Row-Based Aggregation in Data Warehouses

After the query is posed, data sources create lists of [Key,Value] pairs using their fact and dimension tables so that row-based aggregation can be performed over them with the above technique. All information in the dimension table about a tuple in the fact table is used to form a Key for that tuple. The tuple from a fact table is added into the list as [Key,Value] pairs where Value is the value associated with that tuple. For example, data source $P_{2}$ in Figure 4 creates [Key-Value] pairs as follows: for a tuple with SSN 6565, it retrieves other information about 6565 from the customers table such as name, surname and address. Then, it combines those information to create the Key for this tuple and the amount is used as the Value.

## Properties of the Algorithm

Data sources use a one-way hash function to hide Key, and thus all of the data sources will learn $H(K e y)$. Only those data sources which have Key would be able to know Key and its existence at data source $P_{i}$. In addition, $P_{i}$ uses Shamir's secret sharing to hide the value associated with Key from other data sources. It uses a polynomial degree of $m-1$ and $m$ random points to compute shares of the $m$ data sources. Then, it keeps one of these shares for itself and sends the remaining $m-1$ shares to the other parties. Since all of the $m$ shares are needed to reveal the secret value in Shamir's secret sharing method, the other data sources would not be able to compute the value, even if they combine their shares coming from $P_{i}$.

In general, for any Key at any data source $P_{j}$, any data source $P_{i}$ can prevent execution of aggregation for that Key. Since one of the $m$ shares is sent to $P_{i}, P_{i}$ can prevent aggregation on Key by not sending the intermediate result to the other data sources. Therefore, other data sources would not be able to learn SUM for Key. Using this property, ABACUS allows data sources to control sharing the value of Key with other data sources. This might be needed since if only two data sources have Key, performing row-based aggregation will result in revealing the values to these two data sources (the result is the sum of the two values, and since these data source know their values, they can figure out the other value from the result). Note that, if Key exists in only one data source, then the owner can protect it from other data sources This can easily be done by preventing aggregation on Key. In addition to these, data sources cannot figure out something from their shares using the distribution of values since they are random values (i.e., a random polynomial is used for each item in the list to compute the shares).

### 4.2 Column-Based Aggregation

Enterprises might want to know the size of the market and some statistical information about the market where they compete. In addition, they might be interested in expenditures of their customers such as the ratio of their expenditures in their companies to their total expenditures in the market. In other words, a company might want to know how much it satisfies the needs of customers. Therefore, companies might be willing to collaborate to perform these kinds of operations however, they might not want to reveal extra information, for example a company might not want to reveal how much it satisfies the needs of its customers. One way to compute the market size in a privacy preserving manner is to aggregate the expenditures of all customers in that market. Formally, data sources $P_{1}, P_{2}, . ., P_{m}$ might want to know sum of their local sums $L S_{1}, L S_{2}, \ldots, L S_{m}$ respectively, and the global sum $G S=L S_{1}+\ldots+L S_{m}$, without revealing their local sums. This problem could be solved with the technique discussed in Section 4.1. However, in a competitive environment it is unrealistic to expect enterprises to share their local sums. For example, a big company with 1000 customers might not be willing to share its local sum which is the sum of its 1000 customers with a small company with 10 customers. Instead, it might want to collaborate for the common customers to compute their total expenditures, so that both companies could learn how much they satisfy the needs of their customers. However, during this process they do not want to reveal any additional information. In order to satisfy these needs, we introduce column-based aggregation.

Formally, the column-based aggregation query processing problem is defined as follows:

Let $T_{1}, T_{2}, \ldots, T_{m}$ be the tables stored by a set of data warehouses $P=\left\{P_{1}, P_{2}, \ldots, P_{m}\right\}$ ( $m \geq 3$ ) respectively containing a key and a value field. The data source $P_{i}$ would like to learn the aggregation of values for all Keys in $T_{i}$, i.e., $\sum_{\forall K e y \in T_{i}} \sum_{k=1}^{m}$ (Value s.t. $\exists[$ Key,Value $] \in T_{k} \wedge \exists$ Key $\in T_{i}$ ). Then the problem is to obtain the answer by only providing the aggregation result to $P_{i}$ while revealing only the common Keys to other data sources.

The goal of the query processing is to compute column-based aggregation such that the data source posing the query, $P_{i}$, would only know the result of the query, $\sum_{\forall K e y \in T_{i}} \sum_{k=1}^{m}\left(\right.$ Value s.t. $\exists[$ Key,Value $\left.] \in T_{k} \wedge \exists K e y \in T_{i}\right)$, while other data sources would only know the Keys in $T_{i}$ if they have those Keys. The query processing consists of three steps:

- Intersection Phase: Data source $P_{i}$ sends the list of hash values of Keys in $T_{i}$. On $P_{j}$ receiving this list, $P_{j}$ computes the common keys in tables $T_{i}$ and $T_{j}$ (by hashing its keys in $T_{j}$ and comparing them with the list coming from $P_{i}$ ).
- Local Aggregation for Intersection Phase: Data source $P_{j}$, computes the local sum of values, local sum, for the common keys between $P_{i}$ and $P_{j}$. Formally, the local sum, $L S_{j}$, at data source $P_{j}$ is: $L S_{j}=\sum_{\forall K e y \in T_{i}}\left(\right.$ Value s.t. $\exists[$ Key, Value $] \in T_{j} \wedge$ $\left.\exists K e y \in T_{i}\right)$.
- Global Aggregation Phase: Data sources compute the global sum, GS, which is the sum of local sum of $m$ data sources. They compute $G S=\sum_{i=1}^{m} L S_{i}$ without revealing the local sums with the technique discussed in Section 4.1 (One could think of the data sources, $P_{1}, \ldots, P_{m}$, have the following [Key,Value] pairs $\left[P_{i}, L S_{1}\right], \ldots,\left[P_{i}, L S_{m}\right]$ respectively and they want to compute the row-based aggregation for the key $P_{i}$, which is the global sum).

The proposed query processing method computes column-based aggregation queries correctly. The answer to the column-based aggregation query for data source $P_{i}$ is $\sum_{\forall K e y \in T_{i}} \sum_{k=1}^{m}\left(\right.$ Value s.t. $\exists[$ Key, Value $\left.] \in T_{k} \wedge \exists K e y \in T_{i}\right)$. The proposed technique computes the local sum at each data source in local aggregation for intersection phase where $L S_{j}=\sum_{\forall K e y \in T_{i}}\left(\right.$ Value s.t. $\exists[$ Key, Value $\left.] \in T_{j} \wedge \exists K e y \in T_{i}\right)$. Then, in global aggregation phase the sum of all the local aggregations are computed as answer which is $\sum_{k=1}^{m} L S_{k}$, i.e., $\sum_{\forall K e y \in T_{i}} \sum_{k=1}^{m}$ (Value s.t. $\exists[$ Key, Value $\left.] \in T_{k} \wedge \exists K e y \in T_{i}\right)$.

At the end of the query processing other data sources will only know their common Keys with the query poser $P_{i}$ and $P_{i}$ will only know the result of the column based aggregation query result but nothing else. After intersection phase, the other data sources will know the common elements between $P_{i}$ and them but nothing else, since one-way hash function is used to hide Keys. During local aggregation for intersection phase, the data sources would compute their local aggregates. Then, in the global aggregation phase, they compute the sum of the local aggregations without revealing their local aggregations to anybody with the row-based aggregation. Therefore, $P_{i}$ would only know the global aggregation result, which is column based aggregation result but not the local aggregations. And the other data sources would not know any other local aggregation and the global aggregation results unless $P_{i}$ wants them to know (Note that if $P_{i}$ does not send its intermediate result to other data sources, they cannot compute the global sum in the row-based aggregation in Section 4.1).

## 5 Analytical Evaluation

In this section, we compute the query response times for the proposed query processing techniques. The query responses time for intersection and join queries are studied in Section 5.1. Then, the query execution costs of row-based aggregation and columnbased aggregation queries are calculated in Section 5.2. Finally, we show the query response times of the queries over a sample scenario to demonstrate the scalability of our technique in Section 5.3.

### 5.1 Cost of Intersection and Join Query Processing

Data source $P_{i}$ hashes its list and sends to $m$ data sources. Then, it compares its list with other datasources to find the intersection. Therefore, the computation cost is the cost of hashing the list and the cost of comparisons. Let $C_{h}$ be the cost of hashing a single item and every hashed word is $b$ bits long. The computation time for hashing is: $C_{h} \times\left|L_{i}\right|$. The number of comparisons to compare the hashed list, $L_{i}$, with the other lists coming from other data sources is (assuming lists are sorted) less than $(m-1) \times\left|L_{i}\right|$ without loss of generality assume $L_{i}$ is the longest list. The time needed for this comparison is: $\frac{(m-1) \times\left|L_{i}\right|}{\text { CPU Speed }}$ seconds. Therefore, total computation time is: $C_{h} \times\left|L_{i}\right|+\frac{(m-1) \times\left|L_{1}\right|}{C P U S p e e d}$ The communication time is the sum of the time needed to send its own hashed list and the time to receive the $m-1$ hashed lists from other data sources. Therefore, total communication time is: $\frac{b \times\left(\left|L_{1}\right|+\ldots+\left|L_{m}\right|\right)}{\operatorname{Bandwwidth}}$. The query response time, the sum of the computation and communication cost, is:

$$
C_{h} \times\left|L_{i}\right|+\frac{(m-1) \times\left|L_{i}\right|}{\text { CPU Speed }}+\frac{b \times\left(\left|L_{1}\right|+\ldots+\left|L_{m}\right|\right)}{\text { Bandwidth }}
$$

In the aggregated join, the first step is aggregated intersection. After this first step, data source $P_{j}$ sends the related tuples to $P_{i}$. The query response time is sum of the cost of aggregated intersection and the cost of sending related tuples. Therefore, the query response time for aggregated join is:

$$
\text { The cost of intersection }+\frac{m \times|L| \times v}{\text { Bandwidth }}
$$

where $v$ is the size of a tuple $t \in T_{R}$.

### 5.2 Cost of Aggregation Query Processing

## The Cost of Row-Based Aggregation Query Processing

In the distribution phase, data sources compute the hash value of keys and the shares of $m$ data sources. Therefore, the computation cost is $\frac{m \times|L|}{C P U \text { speed }}+C_{h} \times|L|$. The communication cost is sending these shares to other data sources and receiving shares from other data sources, which is $\frac{2 \times m \times|L| \times b}{\text { Bandwidth }}$, where $b$ is the size of a Key-Value pair.

In the local aggregation phase, the computation cost is scanning all lists and adding the values for a specific key (computation of intermediate result lists). The amount of addition is less than $m \times|L|$. Thus the cost of computation in the local aggregation phase is $\frac{m \times L \mid}{C P U \text { speed }}$ (assuming that lists are sorted and are of the same size). After this
computation, $P_{i}$ sends intermediate results lists to $m$ data sources and receive its intermediate result lists from $m$ data sources. The communication cost for this operation is $\frac{2 \times m \times|L| \times b}{\text { Bandwidth }}$ (note that the size of intermediate lists is equal to the size of lists).

In the final aggregation phase, $P_{i}$ solves an equation system for each element in the list. Thus, the computation time is $\left|L_{i}\right| \times C_{e q}$, where $C_{e q}$ is the cost of solving an equation with $m$ unknowns.

The query response time for row-based aggregation query is (without loss of generality, assume all lists are size of $|L|$ ):

$$
\approx|L| \times C_{h}+\frac{4 \times m \times b \times|L|}{\text { Bandwidth }}+\frac{2 \times m \times|L|}{\text { CPUspeed }}+|L| \times C_{e q}
$$

## The Cost of Column-Based Aggregation Query Processing

The column-based aggregation query processing consists of three phases: 1) intersection phase 2) local aggregation for intersection 3) global aggregation. In the intersection phase, data source $P_{i}$ sends its hashed lists to $m$ data sources. The communication and computation cost for this phase is:

$$
C_{h} \times\left|L_{i}\right|+\frac{m \times b \times\left|L_{i}\right|}{\text { Bandwidth }}
$$

The cost of computation in local aggregation for intersection phase is $\frac{\left|L_{i}\right|}{\text { CPUspeed }}$ (data sources calculates the sum of values in the intersection). Remember that there is no communication in this phase. The cost of global aggregation phase is negligible since the cost of the summation of $m$ values using $m$ parties is negligible in this context. Therefore, the cost of column-based aggregation query processing is :

$$
\approx C_{h} \times\left|L_{i}\right|+\frac{m \times b \times\left|L_{i}\right|}{\text { Bandwidth }}+\frac{\left|L_{i}\right|}{C P U \text { speed }}
$$

### 5.3 Query Response Times over a Sample Scenario

We demonstrate the query response time of ABACUS for intersection and row-based and column based aggregation queries over a sample scenario to show that it is scalable and efficient. We compute the response times for queries in an environment where $m$ data warehouses each of which with a dimension table and a fact table size of 1 million. We execute the queries over these data warehouses by varying the bandwidth and the number of data warehouses involved, $m$. Figures 5,6 , and 7 show the query response time for intersection, row-based aggregation and column-based aggregation queries. During these calculations we take the size of key-value pair, $b$, as 1024 bits, the cost of hashing, $C_{h}$, as $10^{-4}$ [5] seconds and the cost of solving an equation, $C_{e q}$, as $10^{-5}$ seconds (the time needed to solve an equation system with 20 unknowns in Matlab). The analytical evaluations and the results over the sample scenario demonstrate that ABACUS is scalable in terms of the number of parties participating in queries and the cost is increasing linearly with the number of parties involved. In addition as results show that the query processing is communication intensive operation since ABACUS uses light-weight computations.


Fig. 5. The Query Response Time for Intersection Queries


Fig. 6. The Query Response Time for Row-based Aggregation Queries

## 6 Conclusion

In this paper, we propose a distributed middleware, ABACUS, to perform intersection, join and aggregation queries over multiple private data warehouses in a privacy preserving manner. In addition to this, we present new types of aggregation queries which are needed for privacy preserving data sharing. Analytical evaluations demonstrate that the proposed scheme is efficient and scalable.

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Fig. 7. The Query Response Time for Column-based Aggregation Queries
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