Self-organizing Spatio-temporal Pattern Formation in Two-dimensional Daisyworld

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Abstract. Watson and Lovelock's daisyworld model [1] was devised to demonstrate how the biota of a world could stabilise it, driving it to a temperature regime that favoured survival of the biota. The subsequent studies have focused on the behaviour of daisyworld in various fields. This study looks at the emergent patterns that arise in 2D daisyworlds at different parameter settings, demonstrating that a wide range of patterns can be observed. Selecting from an immense range of tested parameter settings, we present the emergence of complex patterns, Turing-like structures, cyclic patterns, random patterns and uniform dispersed patterns, corresponding to different kinds of possible worlds. The emergence of such complex behaviours from a simple, abstract model serve to illuminate the complex mosaic of patterns that we observe in real-world biosystems.

1 Introduction

Self-organization is a property of processes, in which patterns emerge at global scale due to the short-range local interactions of the system, with no external intervention [2]. A wide range of pattern formation processes have been observed in fields from chemistry (Turing's reaction-diffusion system [3], Belousov-Zhabotinsky reaction [4]) and fluid dynamics (Bénard convection cells [5]) to biology (patterns on animals' coats [6]).

In this paper, we investigate the emergence of self-organized, spatio-temporal vegetation patterns in two-dimensional (2D) Daisyworld [1]. First of all, we show that the surface temperature of planet is self-regulated around the mean 295.5K by the daisyworld dynamics of our model as in previous 2D daisyworld models [7, 8]. Then we observe the self-organizing properties of daisyworld, and qualitatively analyse pattern formation in daisyworld with Laplacian diffusion. We also quantify the emergence of vegetation patterns using a statistical measure, Moran's I. Pattern formation in daisyworld has been previously touched on in both 1D [9] and 2D [10] cellular automaton models. However our daisyworld model is extended to a coupled 2D chaotic system; we investigate daisyworld in diffusively coupled logistic systems in a 2D tordoidal regular lattice with chaotic local dynamics, a setting which has not been studied previously.

In our daisyworld model, depending on model parameter values we observe a variety of behaviours, such as 1) complex patterns (stationary/dynamic), 2) smoothly dynamic spatio-temporal patterns (Turing-like structures), 3) cyclic patterns (periodic spirals), 4) random patterns (stationary/dynamic) and 5) dispersed patterns (uniform).

2   Background

2.1   Daisyworld

Daisyworld was proposed by Watson and Lovelock [1] as a means to study planetary ecodynamics. This simple self adaptive system has one environmental variable (temperature) and two types of life (black and white daisies). The colour of the daisies influences the albedo (i.e. the reflectivity for sunlight) of the planet, and thereby influences the temperature of the planet. Though the daisies do not interact with each other directly, they do interact via the planetary temperature (environment); the behaviour of one type of daisies modifies the temperature, which affects the behaviour of the other, and vice versa. In general, increased growth of a particular type of daisy alters the temperature in a direction which is unfavourable for itself, but suitable to the other type. When black daisies start spreading more, the local temperature rises. Conversely, when white daisies spread, the temperature drops. Due to the contrary behaviours of the species, the temperature is self-regulated and daisies persist on the planet.

2.2   Turing instabilities

Turing instability of a homogeneous steady state in a two-species reaction-diffusion system provides theoretical mechanisms for pattern formation. Generally diffusion is perceived as an homogenising process. By contrast, Turing, in his remarkable paper(1952 [3]), demonstrated that a simple system of coupled reaction-diffusion equations could give rise to spatial patterns in chemical concentrations, through a process of chemical instability. The chemicals (morphogens or species), characterised as activator and inhibitor, react and diffuse throughout the tissue, and due to this physico-chemical process patterns such as spots and stripes can form in an animal's coat. These spatial patterns emerge from a homogeneous equilibrium state, due to symmetry-breaking via diffusion. This diffusion-driven instability forms the basis of pattern formation in physics, chemistry and biology. A wide spectrum of complex self organized patterns such as stationary, waves, spirals or turbulence emerge from reaction-diffusion systems.

2.3   Moran's I

Moran's I [11] is a measure of spatial autocorrelation, and is often used to analyse spatial patterns. The specific values -1, 0, +1 indicate spatially dispersed, random, and clustered patterns respectively. It is defined by equation 1:

$$I = \frac{C}{\sum_i \sum_j w_{ij}} \frac{\sum_i \sum_j w_{ij}(P_i - \bar{P})(P_j - \bar{P})}{\sum_i (P_i - \bar{P})^2} \tag{1}$$

where C is the number of spatial units(cells) indexed by i and j, P is the size of the population, $\bar{P}$ is the mean population and w_{ij} is a spatial weight matrix. In our work, $w_{ij} = 1$ if i and j are horizontal or vertical neighbours, and $w_{ij} = 0$ otherwise.

3   Self-organization and pattern formation in Daisyworld

3.1   Positive-negative feedback

Positive and negative feedback are the two basic modes of interactions among components of self-organizing systems [2]. Self-enhancing positive feedback coupled with antagonistic negative feedback leads to the formation of striking patterns in nature (ripples in sand dunes, schooling of fish, flocking of birds, coat of animals). A stationary state becomes unstable due to amplification, and this provokes pattern in the system. Since amplification purely in one direction leads to destruction, negative feedback takes control of the system and plays a critical role in inhibiting amplification and shaping the process and pattern, for example in colonial nesting of male bluegill sunfish [2].

In daisyworld, an initial growth of black daisies, due to positive feedback, increases the temperature of system and thus reinforces the change in the same direction. Hence it has the potential to explode the system. But in daisyworld, at a higher temperature, white daisies bloom and keep the positive feedback under control. In the same manner, the growth of white daisies, due to positive feedback, decreases the temperature in the same direction. But at a colder temperature, black daisies bloom and inhibit the system behaviour. The initial growth of daisies promotes new changes in the system state but on the other hand, increased growth counteracts the changes. The combined effect of positive and negative feedback couplings result in a negative feedback loop, which stabilises the physiological processes and leads to homeostasis [12]. Due to this behaviour, we can see that daisyworld is a self-organizing system.

3.2   Activator-inhibitor principle

Turing pattern formation in activator-inhibitor systems forms the paradigm of self-organization [13], and provides a theoretical explanation of animal coat patterns [6]. The activator and inhibitor species destabilize the homogeneous state of the system due to diffusion, and lead to the spontaneous emergence of spatial patterns.

We can view daisyworld as a Turing reaction-diffusion system of two morphogens: black and white daisies. In cooler temperatures, black daisies absorb the available sun light and amplify their growth, functioning as activators (autocatalysis) because they activate their own production. But further increase in black daisies increases the surface temperature to the level at which cooler white daisies grow well, thereby acting as an inhibitor (self-suppression) because they inhibit their own production. This process is reversed for white daisies. Depending on the temperature, black and white daisies act both as inhibitor and as activator. Here, the diffusion of the two species, coupled by nonlinear local rules, result in the generation of patterns and spatial self-organization due to symmetry-breaking instability.

4   Model

The interacting components (cells) of the system are arranged on a toroidal 2D square ($N \times N$) regular lattice. Each cell is designated as a habitat with a maximum carrying capacity of 10,000 individuals. All sites in the lattice are randomly initialised with

a population size in $[0, 100]$ for both black and white daisies. The temperature is initialised randomly depending on the overlap between the optimal temperatures of black and white daisies (refer Table 2). The interaction rules among these system components are based only on local information (microscopic level) and not on the global emergence (macroscopic level). The black and white daisies, along with the temperature, diffuse around the world via their neighbourhood. The daisyworld model has two parts: the local component describing the effects on growth of daisies and change in temperature local to the particular cell; and the interaction component describing the migration of black and white daisies and diffusion of heat to their neighbours.

Table 1. Daisyworld Parameter Settings (last two rows are variable parameters, others are fixed)

Parameter	Value(s)	Parameter	Value(s)
Fixed			
Number of cells	100×100	Bare ground Albedo (A_g)	0.5
Heat Capacity (C)	2500	Albedo of black daisies (A_b)	0.25
Diffusion constant (D_T)	500	Albedo of white daisies (A_w)	0.75
Stefan-Boltzmann constant (σ_B) $E^{-8}Wm^{-2}K^{-4}$	5.67	Mean growth temperature	295.5 K
Luminosity (L)	1	Solar Insolation (S) Wm^{-2}	864.65
Variable			
Diffusion rate of black daisies	$[0, 1]$	Diffusion rate of white daisies	$[0, 1]$
Noise Level	$[0, 10]$	Natural Rate of Increase	$[1, 4]$

Table 2. Optimal temperature overlap

Overlap(%)	Opt. temp. (K) black daisies	Opt. temp. (K) white daisies	Initial values (K)
0	278	313	$[270, 320]$
10	284.5	306.5	$[280, 310]$

Local Dynamics: The population size of each unit is computed based on the logistic equation with carrying capacity, refer equation 2:

$$P_{(t+1)} = P_t(1 + r[1 - \frac{P_t}{K}]) \tag{2}$$

where r is the bifurcation parameter (i.e. intrinsic capacity for increase). The parameter r provides positive feedback and the component $[1 - \frac{P_t}{K}]$ provides the negative feedback, with their combined effect regulating the population. Though the regulatory mechanism is built in, chaos emerges from the system [14]. The temperature change at each location

is based on the energy balance equation [15] (refer equation 3):

$$\sigma_B T^4 = SL(1 - A) \qquad (3)$$

where σ_B is the Stefan-Boltzmann constant, T is the temperature, S is the solar constant, L is the luminosity and A is the albedo.

Global Emergence: The behavioural rules in the system specify that black daisies grow better in colder temperatures, while white daisies grow better in hotter temperatures, both diffusing to their nearest neighbours. These simple components at the local level generate complicated dynamics at the global level due to the collective behaviour of the whole system, without the influence of any external forces. The interaction pattern is determined by the neighbourhood topology and the diffusion term. In our model, we use von Neumann neighbourhoods, consisting of a central cell and its four orthogonal neighbours (top, down, left and right, without diagonal interactions), and a Laplacian diffusion model for both species and temperature. The key ingredients leading to global emergence are the local nonlinear interactions among the components, physiological and behavioural rules, and physical constraints. The inhibition of self-growth can also arise from the physical constraints, due to the self-limiting factor of carrying capacity.

Albedo: The albedo of the planet can be computed by equation 4:

$$A = A_b \alpha_b + A_w \alpha_w + A_g \alpha_g \qquad (4)$$

where $\alpha_b, \alpha_w, \alpha_g (= 1 - \alpha_w - \alpha_b) \in [0, 1]$ are the relative areas occupied by black, white daisies and bare ground, A_b is the albedo of ground covered by black daisies, A_w that of ground covered by white daisies and A_g that of bare ground. We assume that $A_w > A_g > A_b$, with corresponding values of 0.75, 0.5, 0.25.

Temperature: The local temperature update is defined by diffusion, heat radiation, solar absorption and Gaussian noise, as in equation 5:

$$C \cdot \delta T_{(i,j,t)} = D_T \bigtriangledown^2 T_{(i,j,t)} - \sigma_B T_{(i,j,t)}^4 + SL(1 - A_{(i,j,t)}) + C\epsilon_{(i,j,t)} \qquad (5)$$

where $C = 2500$ is the heat capacity, $\delta T_{i,j,t} = T_{i,j,t+1} - T_{i,j,t}$ is the change in temperature at (discrete) time t, $1 \leq (i,j) \leq N$ indicates a 2D lattice point, N is the lattice length, $D_T = 500$ is the diffusion constant, $\bigtriangledown^2 T$ is the Laplacian operator, σ_B is the Stefan-Boltzmann constant, S is the solar constant, L is the luminosity, A is the albedo and ϵ represents Gaussian white noise (with mean zero and standard deviation 1.0) multiplied by the noise level.

Growth: The growth rate of daisies is defined as a parabolic function as in equation 6:

$$\beta(T) = 1 - \left[\frac{(T_{opt} - T)^2}{17.5^2}\right] \qquad (6)$$

where T is the local temperature, and T_{opt} is the optimal temperature of the species, which depends on their colour. The optimal temperature for black daisies is lower than for white. In general, growth rates of daisies depend on local temperature, which in turn depends on the albedo and thus on the proportions of the daisy species and bare ground.

Population Size: The local population update is computed based on Laplacian diffusion and local population growth, governed by the logistic equation with carrying capacity, as in equation 7:

$$\delta P_{(i,j,t)} = D \bigtriangledown^2 P_{(i,j,t)} + r P_{(i,j,t)}[\beta(T) - \frac{P_{(i,j,t)}}{K}] \tag{7}$$

where at location (i, j) and time t, $P_{i,j,t}$ is the population size, D is the fraction of the population being dispersed to its neighbours, r is the natural rate of increase, $\beta(T)$ is feedback coefficient, and K is the carrying capacity. Deterministic chaos can also be induced by diffusion and hence our system doesn't require high intrinsic growth rates.

4.1   Assumptions and Limitations

The model parameters are represented in Table 1. They reflect the general daisyworld literature and parameter values [1, 7]. The last two rows of Table 1 are variable parameters, while the rest are fixed parameters. The range of temperatures in which daisies can survive is the viability constraint of our model, and is based on the overlap between the optimal temperatures of black and white daisies as in Table 2. We define the optimal temperature of black daisies within the range $[278, 295.5]$K and of white in the range $[313, 295.5]$K so as to have an overlap in the range $[0, 100]$%. For this work, we use a 10% overlap for most of experiments, with a 0% overlap for one. As we have a huge parameter space, we have restricted our experiments to an increase rate of $r = 1$. We used the same diffusion constant for both black and white daisies in all our experiments. We limited our focus to studying the effects of species diffusion (which influences the species distribution) and noise (which influences the temperature) in detail.

5   Results

The results presented here are samples from over 15,000 experiments with different variable parameter settings, including species increase rates in the range $[1, 4]$, species dispersion rates in $\{0, 0.001, 0.01, 0.1, 0.2, 0.5, 1\}$, overlap in $\{0, 5, 10, 25, 50, 100\}$%, noise level in $\{0, 0.001, 0.005, 0.01, 0.05, 0.1, 0.2, 0.5, 1, 2.5, 5, 7.5, 10\}$, and temperature diffusion in $\{0, 0.05, 0.1, 0.2, 0.3, 0.5, 1\}$. We examined the results for the emergence of interesting patterns both qualitatively (by capturing snapshots of each experiment and inspecting visually) and quantitatively (using the value of Moran's I). Our preliminary investigation found interesting behaviours in the 0% and 10% overlap settings, and we present these here. Our future aim is to develop suitable metrics to automatically select interesting cases from the parameter space.

   We present the results by showing the state of the Daisyworld at different epochs. Since the behaviour is different in each case, the epochs chosen differ in each case. In these visualisations, a location is shown as black if the population of black daisies at a particular location is larger than that of white, and vice versa. The variable parameter values used in each case are detailed in the caption of the figure.

   As it is impossible to show all spatial snapshots over 5000 epochs, we have plotted the temporal dynamics of the global temperature, which characterises the nature of the

daisyworld. We note that in all cases, the temperature self-regulates around 295.5K, confirming the persistence of the daisies as an emergent property of daisyworld. We have also plotted the value of Moran's I for both black and white daisies.

A wide spectrum of patterns emerge from our model, depending on the model parameter values. We have classified them into 1) complex patterns (stationary/dynamic), 2) smoothly dynamic spatio-temporal patterns (Turing-like structures), 3) cyclic patterns (periodic spirals), 4) random patterns (stationary/dynamic) and 5) dispersed patterns (uniform). We were able to find examples of most behaviours with an overlap of 10%; however for Turing-like structures, we used an overlap of 0%.

5.1   Self-organized complex pattern

| (a) epoch 50 | (b) epoch 375 | (c) epoch 425 | (d) epoch 800 | (e) epoch 850 |
| (f) epoch 1425 | (g) epoch 1500 | (h) epoch 2950 | (i) epoch 3000 | (j) epoch 5000 |
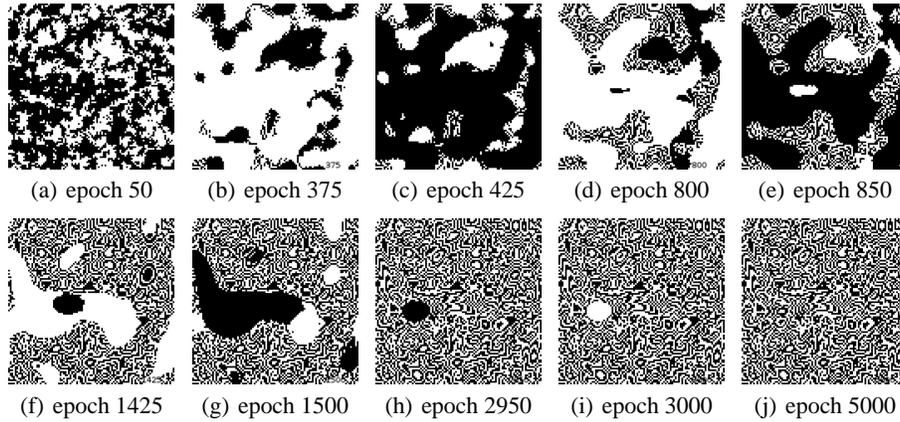
Fig. 1. Maze like organization for $D = 0.001$ and noise level$= 0.001$ in 2D 100×100 Lattice

(a) Surface Temperature   (b) MI(Black Daisy)   (c) MI(White Daisy)
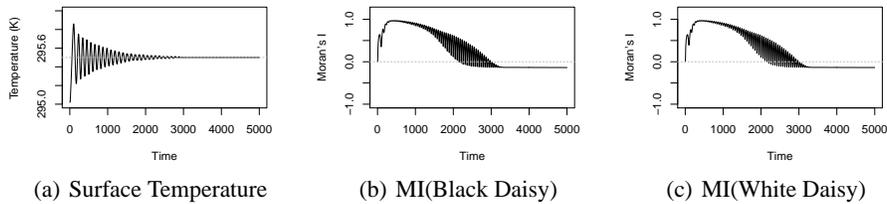
Fig. 2. Global dynamics of maze like self-organized pattern formation

A maze-like self-organizing pattern (refer Fig. 1) emerge from our model with a diffusion constant 0.001 for both species and a noise level of 0.001. Though the ini-

tial states look chaotic, the system finally converged to a stationary maze-like structure. The global spatial behaviour is plotted in Figure 2. The temperature initially fluctuates, but then converges, exhibiting a stationary pattern formation. Initially Moran's I is very high, indicating the formation of very large clusters, but it finally drops below 0, indicating a uniformly dispersed pattern. In between, the fluctuations show complex dynamics.

(a) epoch 1000   (b) epoch 2018   (c) epoch 2073   (d) epoch 4000   (e) epoch 5000
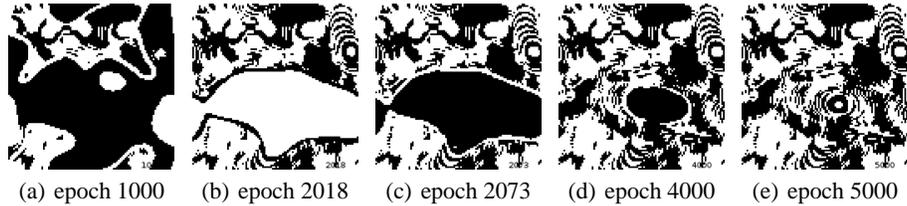
Fig. 3. Modern art complex pattern for $D = 0.1$ and noise level$= 0.001$

(a) Surface Temperature   (b) MI(Black Daisy)   (c) MI(White Daisy)
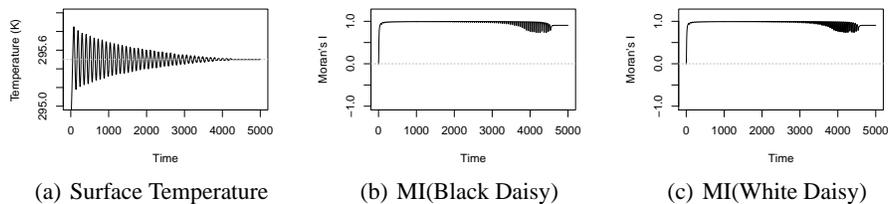
Fig. 4. Global dynamics of modern art complex pattern

Raising the diffusion constant to a still relatively low 0.1 for both species, and with a noise level of 0.001, we see the emergence of a 'modern art' complex pattern (Figure 3); Figure 4 shows its global dynamics. Moran's I stays very high throughout the run till the final epoch – and visually we can see clusters in some locations in that final epoch.

5.2 Turing-like structures

Turing-like patterns emerge with a diffusion constant of 0.5 for both species, a noise level of 0.2 and an overlap of 0% – refer to Figure 5. The corresponding global dynamics are plotted in Figure 6. The patterns which emerge are smooth and slowly changing, looking like an animal's coat (e.g. Holstein cow). The fluctuations in temperature in sub-figure (a) of Fig. 6 show the dynamicity of pattern formation. The high Moran's I throughout the run (except the initial transitions) corresponds to the emergence of large clusters in the patterns (see subfigures (b) and (c) of Figure 6).

(a) epoch 200   (b) epoch 2000   (c) epoch 3000   (d) epoch 4000   (e) epoch 5000
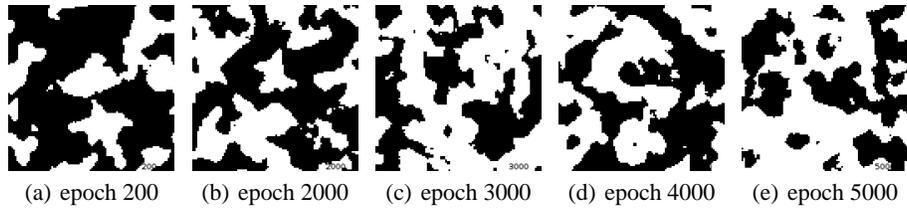
Fig. 5. Turing-like structures for $D = 0.5$, noise level$= 0.2$ and overlap0%

(a) Surface Temperature   (b) MI(Black Daisy)   (c) MI(White Daisy)
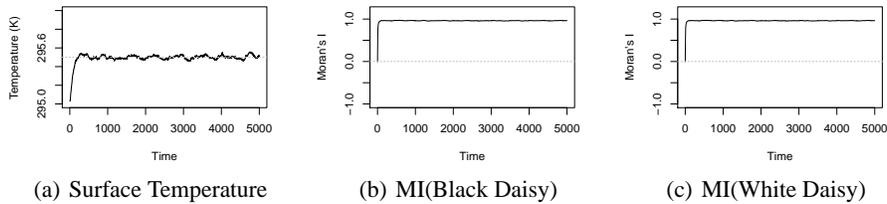
Fig. 6. Global dynamics of Turing-like structures

5.3 Periodic Spirals

(a) epoch 50   (b) epoch 2440   (c) epoch 2600   (d) epoch 3800   (e) epoch 3850
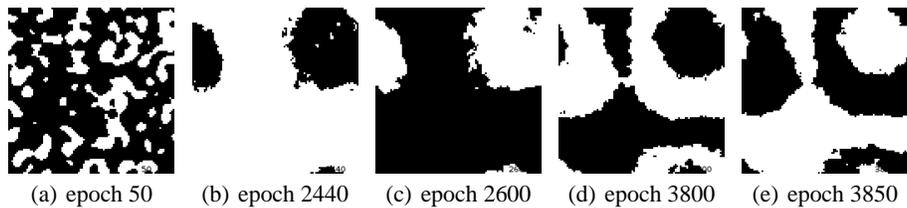
Fig. 7. Periodic spirals for $D = 0.2$ and noise level$= 0.05$

Periodic spirals can be seen in the behaviour when we use a diffusion constant of 0.2 for both species and a noise level of 0.05 (Figure 7). The corresponding global dynamics are plotted in Figure 8. The cycles in temperature seen in sub-figure (a) confirm the periodicity in pattern formation. Except for the initial states, Moran's I is high throughout the run, confirming the formation of large clusters (sub-figures (b) and (c)).

5.4 Random pattern

Random patterns evolve from the system for a noise level of 0.01 and with no species dispersion – see Figure 9 and the corresponding global dynamics in Figure 10. The near-zero value of Moran's I in sub-figures (b) and (c) of Fig. 10 confirms the formation of random patterns.

(a) Surface Temperature   (b) MI(Black Daisy)   (c) MI(White Daisy)
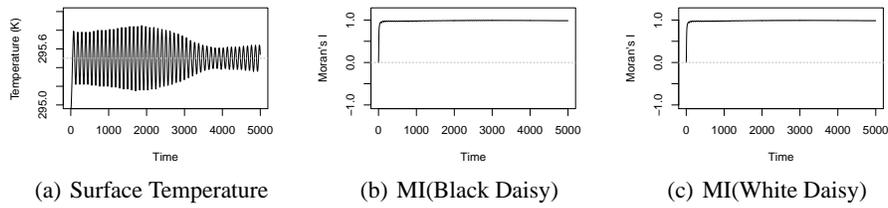
Fig. 8. Global dynamics of periodic spirals

(a) epoch 50   (b) epoch 100   (c) epoch 150   (d) epoch 2500   (e) epoch 5000
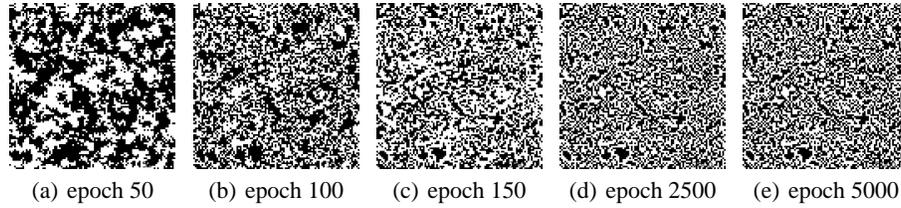
Fig. 9. Stationary random for $D = 0$ and noise level 0.01

5.5 Dispersed pattern

Uniformly dispersed patterns emerge at a of noise level 0.5 with no species diffusion (Figure 11); the corresponding global dynamics are depicted in Figure 12. Moran's I in sub-figures (b) and (c) of Figure 12 lie below zero, indicating dispersed patterns. Interestingly, we do see the emergence of a hint of a large scale grid in this setting, presumably resulting from the effects of temperature diffusion.

6 Conclusions

6.1 Summary

The results illustrate that the underlying dynamics of daisyworld are not restricted to stationary patterns but extend to periodic and chaotic behaviour. We see stability as


(a) Surface Temperature   (b) MI(Black Daisy)   (c) MI(White Daisy)
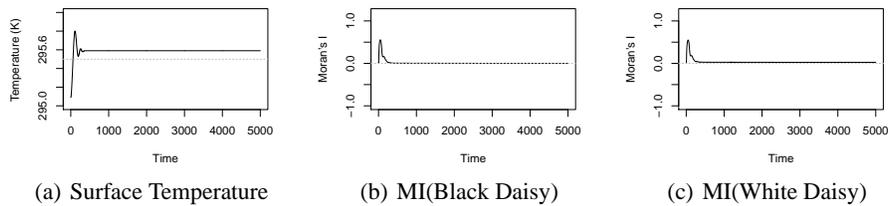
Fig. 10. Global dynamics of random pattern

(a) epoch 50   (b) epoch 300   (c) epoch 500   (d) epoch 2500   (e) epoch 5000
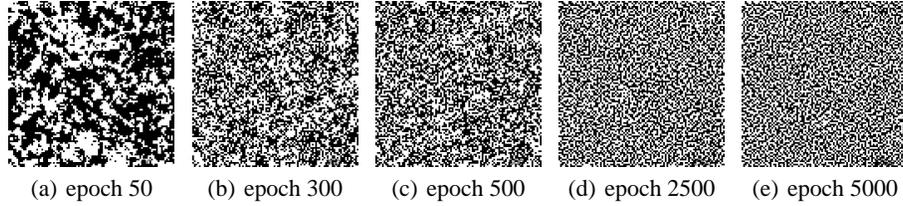
Fig. 11. Uniformly dispersed pattern for $D = 0$ and noise level 0.5

(a) Surface Temperature   (b) MI(Black Daisy)   (c) MI(White Daisy)
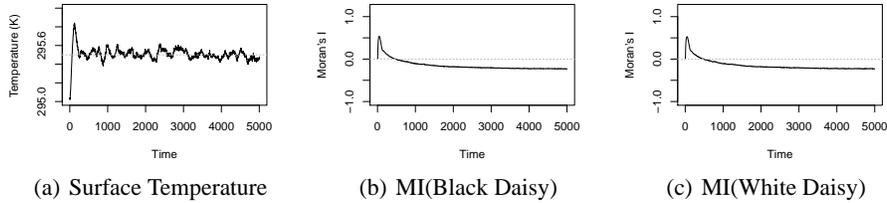
Fig. 12. Global dynamics of uniformly dispersed pattern

well as complexity emerge. The figures clearly show the spatial patterns and temporal behaviour of daisyworld. Moran's I quantifies the pattern formation accurately, demonstrating its reliability in spatially quantifying a wide spectrum of patterns.

The localised interaction of species (black and white daisies) in our daisyworld model generate an array of fascinating spatio-temporal patterns, due to endogenous self-organization resulting from "symmetry-breaking instability". In our nonlinear reaction-diffusion system, symmetry-breaking is induced by species dispersion, temperature diffusion and noise. A low species dispersion rate combined with noise leads to complex self-organized patterns (Figures 1 and 3). As species dispersion and noise increase, periodic (Figure 7) and dynamic chaotically evolving patterns (Figure 5) emerge. In the absence of species dispersion, random (Figure 9) and dispersed (Figure 11) patterns emerge. We thus see the constructive roles of spatial diffusion, nonlinear local rules and noisy fluctuations in influencing the physiological, behavioural rules and physical constraints and thereby self-organizing the daisyworld.

6.2   Future Work

Since our parameter space is huge, it is practically infeasible to categorise all domains of pattern formation in daisyworld by qualitative visualisation – some automated criterion is required. We used Moran's I, but it assists only with spatial analysis, omitting the temporal analysis. In order to to fully analyse both parameter and pattern formation space, we need objective statistical measures which can uniquely classify the patterns in both space and time. We hope that further work will help us to understand the intrinsic connection between spatio-temporal dynamics and pattern formation in daisyworld.

Acknowledgements

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