A Risk-defined Trust Transitivity Model for Group Decisions in Social Networks

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***Abstract*—In a social network group decision making process, trust is a critical source of information. Trust transitivity is necessary in many cases to evaluate experts' trustworthiness through referring paths. However, existing trust transitivity models have hidden assumptions regarding the risk attitude of the buyer. Moreover, some users are more conservative than others regarding the risk bearing on referrals. To address this issue, this paper proposes a risk-defined trust transitivity model for group decision making in social networks with four tuple information, namely, trust, distrust, uncertainty and inconsistency. Firstly, a novel method of measuring knowledge degree is defined and applied to trust ranting. Afterwards, we develop a risk-defined trust propagation operator that can propagation trust and distrust information based on risk bearing level. To aggregate multiple possible paths, we propose a trust aggregation operator based on path centrality. Finally, we demonstrate the effectiveness of the trust model through an example of trusted service selection and verify the effectiveness of the proposed method through comparison with existing models.**

***Index Terms*—Group decision making; Social network; Risk attitude; Trust transitivity model; Closeness centrality analysis; Four tuple information.**

I. Introduction

Social network group decision-making (SNGDM) is a process of selecting the best alternative where two or more experts within the social network provide recommendations. The input from the experts are then aggregated into a collective decision [1, 2]. With the increasing influence of social networks and E-commerce [3], it is more and more common that someones decision is influenced by the opinions from his or her friends with high reputation [4, 5], many online service decision making problems can be regarded as SNGDM problems [6-8]. For example, social network platform CouchSurfing [9] (www.couchsurfing.com) is a home-share website where the recommendation scores of lodging places are determined by a group of users. Users can evaluate their experiences with other logging providing users through filling an online form that reflects trust and distrust scores on other users. Indeed, the opinions from others who have experience with the evaluatee are highly valuable information for individuals to make decisions through a social network. SNGDM problems have become a new key component of group decision-making (GDM) and have raised a great deal of attention from scientific researchers [1].

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Compared to the typical GDM [10, 11], a main feature of SNGDM is that the trust relationship between the members of a group can offer a reliable source to determine the expert weights of the members. Thus, it is necessary to research trust modeling in SNGDM [12-15]. The first problem needed to be solved is the preference values provided by group experts. Many existing trust models use a crisp value (i.e., $T \in [0,1]$) or a binary value (i.e., $T \in \{0,1\}$) to represent the trustworthiness of an evaluatee. However, the crisp values and the binary value are unable to address the subjectivity and uncertainty that characterize the social relationship occurring among experts. More recently, many studies [1, 13-19] used uncertainty mathematical methods to model trust such as fuzzy sets [13-15], interval theory [1, 16], intuitionistic fuzzy sets [17], linguistic terms [18, 19]. Although these methods can describe the inherent subjectivity of trust, they are limited to a single trust dimension. In our SNGDM model, experts may provide the other types of information besides the trust information, namely, the distrust information, the uncertainty information and the inconsistency information, which we call the four-tuple information.

The four-tuple information model has been used in the literature. For example, Wu et al. [20-21] and Liu et al. [22] model individual decision information with trust decision making space, which include four tuple components: trust, distrust, inconsistency and hesitancy. One of the foremost research topics in the field of GDM is the trust transitivity modeling [20-22]. However, existing works have overlooked the fact that the decision makers (DM) may have different risk tolerance levels. This is because in some practical trust transitivity environment, DMs with different risk tolerance may produce different results when evaluate the trust relationship between unknown experts. The lower the DMs risk tolerance, the faster trust attenuates through transitivity. Therefore, the risk tolerance of DMs should be regarded as an influence factor in trust transitivity process.

To address the aforementioned issue, we propose a risk-defined trust transitivity model (RDTTM) to provide a solution to SNGDM problem based on four tuple information. The main contributions of this paper are listed as follows: (1) we propose a novel distance-based knowledge degree (KD) of trust function (TF) model and apply it to compare TFs; (2) we develop a novel risk attitudinal trust transitivity model to infer indirect trust relationships between experts; (3) we adopt a trust based aggregation method to aggregate group preference information; (4) we present a new approach to solve SN-GDM problems with four tuple information.

The rest of the paper is organized as follows. In Section 2, we briefly review some relevant concepts of trust decision making space and trust network centrality analysis. In Section 3, a risk attitudinal trust transitivity model is developed. In Section 4, we propose our method for SN-GDM with four tuple information. In Section 5, we provide a trusted service selection example and comparision analyses to demonstrate the feasibility of the proposed method. Section 6 conducts some concluding remarks.

II. Preliminaries

In this section, we present some basic concepts of trust decision making space and trust network centrality analysis.

A. Trust decision making space

Atanassov et.al.[23] introduced Intuitionistic fuzzy sets (IFs) in 1967. The model can be used to handle uncertainty and ambiguity of decision-making problems in various scenarios, including social networks. Let $A = \{< x, \mu_A(x), v_A(x) > |x \in X\}$ be an intuitionistic fuzzy set (IFS) in a universe of discourse X, where $\mu_A(x) : X \to [0,1]$ and $v_A(x) : X \to [0,1]$, with the condition $\mu_A(x) + \mu_A(x) \in [0,1]$. $\mu_A(x)$ and $v_A(x)$ are, respectively, the *membership degree* and *non-membership degree* of the element x in A. A derived third parameter $\pi_A(x) = 1 - \mu_A(x) - v_A(x)$, $x \in X$ is called the *hesitation degree* of x in A [24]. Obviously, $\pi_A(x) \in [0,1], x \in X$. For convenience, Beliakov et al. [24] named the tuple $\alpha = (\mu_\alpha, v_\alpha)$ an *intuitionistic fuzzy value*, where $\mu_\alpha \in [0,1]$, $v_\alpha \in [0,1]$, $\mu_\alpha + v_\alpha \in [0,1]$, $\mu_\alpha + v_\alpha + \pi_\alpha = 1$. However, In some real-world scenarios of IFs, $\mu_\alpha + v_\alpha > 1$ may appear in the context in which decision making information is inconsistent. For example, someone may rate an expert to be both highly trustable and highly untrustable. The primary reason is that their knowledge on a certain decision-making problem is limited or their assessments are irrational. To overcome the aforementioned limitation of Intuitionistic fuzzy value, we use the following definition of trust decision making space [20].

Definition 1 (Trust Function (TF)). Let tuple $\lambda = (t, d)$ be a trust function where the parameters t and d indicate, respectively, the trust degree and distrust degree of λ, with $0 \le t \le 1$, $0 \le d \le 1$. The set of trust functions will be denoted by

$$\Lambda = \{\lambda = (t, d)|t, d \in [0,1]\}$$

In order to better comprehend the concept of knowledge degree of TF, the distance of two TFs is given as follows.

Definition 2 For two TFs $\lambda_1 = (t_1, d_1)$ and $\lambda_2 = (t_2, d_2)$, we define the distance between the two TFs to be the Euclidean distance between two tuples as follows:

$$D(\lambda_1, \lambda_2) = \sqrt{\frac{(t_1 - t_2)^2 + (d_1 - d_2)^2}{2}} \qquad (1)$$

According to Definition 1 and the properties of the intuitionistic fuzzy value, four types decision making information can be derived to consist the trust decision making space as follows.

Definition 3 (Trust decision making space (TDMS)). The trust decision making space is made up of the following three elements: the set of TFs (Λ), a trust hesitancy space (THS) and a trust inconsistency space (TCS). It can be represented as $TDMS = (\Lambda, THS, TCS)$ with $THS = (\lambda \in \Lambda | t + d \le 1)$ and $TCS = (\lambda \in \Lambda | t + d > 1)$.

Notice that information is conflicting if trust degree plus distrust degree is greater than 1. For example, in an real online service evaluation, the trustworthiness of a service provider can be expressed by a TF (0.6, 0.7), which means that the trust degree is 0.6, the distrust degree is 0.7. Correspondingly, the hesitation degree of intuitionistic fuzzy value is a negative value, which reflects the scenario of inconsistency.

Nguyen et al. [25] pointed out that the distance of a fuzzy set (FS) from the most FS can be used to define knowledge degree. Inspired from their idea, we can use the level of knowledge of a TF to be the distance between this TF and the TF with least knowledge (i.e, when both trust and distrust are 0). Therefore, we define a new TS and KD as shown in Definition 4.

Definition 4. Let $\lambda = (t, d) \in \Lambda$ be a TF, then the TS and KD of this TF are defined as follows

$$TS(\lambda) = \frac{(t - d + 1)}{2} \qquad (2)$$

$$KD(\lambda) = \sqrt{\frac{(t^2 + d^2)}{2}} \qquad (3)$$

are TS and KD of λ, where $TS(\lambda) \in [0,1]$, $KD(\lambda) \in [0,1]$. The larger the $TS(\lambda)$, the greater the TF λ. If the trust scores of given two TFs are equal, the larger the $KD(\lambda)$, the greater the TF λ.

Definition 5. Let $\lambda_1 = (t_1, d_1) \in \Lambda$ and $\lambda_2 = (t_2, d_2) \in \Lambda$ be two TFs, then

1) If $TS(\lambda_1) > TS(\lambda_2)$, then $\lambda_1 > \lambda_2$.
2) If $TS(\lambda_1) = TS(\lambda_2)$, then:
   If $KD(\lambda_1) < KD(\lambda_2)$, then $\lambda_1 < \lambda_2$;
   If $KD(\lambda_1) > KD(\lambda_2)$, then $\lambda_1 > \lambda_2$.

B. Trust network centrality analysis

In real SNGDM environment, a trust network $\tilde{G} = (E, \tilde{R})$ can be seen as social network consisting of an expert set E and an edge set $\tilde{R}$ between experts. The *closeness centrality* is the easy degree to get to other nodes which can be used to assess nodes' influence in a social network. We define the closeness centrality on an expert as follows:

Definition 6. Given a trust network $\tilde{G} = (E, \tilde{R})$, let l_{ij} be the length of the shortest path from expert e_i to e_j, then the closeness centrality of expert e_i is defined as

$$C_e(e_i) = \frac{(n-1)}{\sum_{j=1}^{n} l_{ij}} \qquad (4)$$

where n is the total number of experts. We can see that the larger $C_e(e_i)$, the higher the influence of expert e_i in its social network [26], We can see that an expert can indirectly derive the TF value of a non-adjacent expert based on the influence of each expert along the path [1]. Therefore, the larger the closeness centrality of a recommending expert, the higher influence of the path in which the recommend experts are located. Hereby we define the concept of *path centrality* as follows.

Definition 7. Given a trust network $\tilde{G} = (E, \tilde{R})$, $C_e(e_i)$ is the closeness centrality of e_i. If a path p_k have t experts $e_1, e_2, \cdots, e_t$ from e_1 to e_t, then the *path centrality* of the path p_k is defined as

$$C_p(p_k) = \frac{\sum_{i=1}^{t} C_e(e_i)}{t} \tag{5}$$

For instance, consider a path $p_1 = (e_3, e_1, e_2)$ in Fig. 1, using Eq. (4), we get the closeness centrality of all experts $C_e(e_1) = 0.44$, $C_e(e_2) = 0.5$, $C_e(e_3) = 0.57$. From Eq. (5), we get the path centrality $C_p(p_1) = 0.5$.

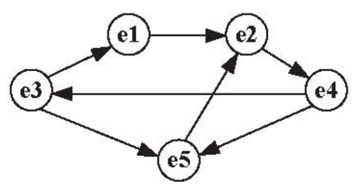

Fig. 1. An example of experts' trust network

III. A RISK DEFINED TRUST TRANSITIVITY MODEL

Trust transitivity is a process that a source user measures the trustworthiness of a target user along paths between them. To complete the transitivity process, it always consists of two parts trust propagation and trust aggregation. The first part performs trust propagation to derive each trust value between two users along each path. In the second part, the final estimated trust value between the source user and the target user derived from multiple paths by aggregation operator.

A. Trust propagation

In most practical SNGDM situations, if there is no direct interactive relationship between two experts, and one of them needs to evaluate the trust of the other, trust of the evaluatee can be propagated by recommending information from third parties. A more recent work from Wu et al. [20] proposed a uninorm propagation operator to propagate both trust and distrust simultaneously. Although it can avoid trust information loss effectively, both the attenuation of trust information and the growth of distrust information are overly fast, reflecting that the risk attitude of the decision maker (DM) is conservative. This situation may limit the flexibility of propagation operators applications. To overcome these problem, we propose a novel risk defined propagation operator and the details follow.

Definition 8. Given two TFs $\lambda_1 = (t_1, d_1)$ and $\lambda_2 = (t_2, d_2)$ on a propagation path, the risk defined trust propagation operator P_θ on Δ is a mapping function $P : \Delta \times \Delta \to \Delta$ which can be expressed using:

$$\begin{aligned} P_\theta(\lambda_1, \lambda_2) =& (\log_\theta(1 + \frac{(\theta^{t_1} - 1)(\theta^{t_2} - 1)}{\theta - 1}), \\ & 1 - \log_\theta(1 + \frac{(\theta^{1-d_1} - 1)(\theta^{1-d_2} - 1)}{\theta - 1})) \end{aligned} \tag{6}$$

where $\theta > 1$ denotes the DM's risk attitude. The larger θ is, the more conservative the DM's risk attitude. When the DM's risk attitude is more conservative, both the decrease of trust degree and the increase of distrust degree are faster for P_θ. According to definition 5, for two given λ_1 and λ_2, the propagated trust becomes weaker when the DM's risk attitude becomes more conservative.

Let $\lambda_1 = (t_1, d_1)$ be TF of expert e_1 to expert e_2, and $\lambda_2 = (t_2, d_2)$ be TF of expert e_2 to expert e_3, then the TF of expert e_1 to expert e_3 can be derived by P_θ. Next we are going to demonstrate that our proposed trust propagation operator P_θ follows some desirable properties in trust propagation.

Property 1. (Trust non-accumulation) For any $\theta > 1$, the trust degree of P_θ does not increase, i.e., $\log_\theta(1 + \frac{(\theta^{t_1}-1)(\theta^{t_2}-1)}{\theta-1}) \leq t_1$ and $\log_\theta(1 + \frac{(\theta^{t_1}-1)(\theta^{t_2}-1)}{\theta-1}) \leq t_2$.

Property 2. (Distrust non-reduction) For any $\theta \in [0, 1]$, the distrust degree of P_θ does not decrease, i.e., $1 - \log_\theta(1 + \frac{(\theta^{1-d_1}-1)(\theta^{1-d_2}-1)}{\theta-1}) \geq d_1$ and $1 - \log_\theta(1 + \frac{(\theta^{1-d_1}-1)(\theta^{1-d_2}-1)}{\theta-1}) \geq d_2$.

Property 3. (Commutativity) $P_\theta(\lambda_1, \lambda_2) = P_\theta(\lambda_2, \lambda_1)$.

Property 4. (Associativity) $P_\theta(P_\theta(\lambda_1, \lambda_2), \lambda_3) = P_\theta(\lambda_1, P_\theta(\lambda_2, \lambda_3))$.

Property 5. (Fully distrust) If $\lambda_1 \vee \lambda_2 = (0, 1)$, then $P_\theta = (0, 1)$.

Obviously, the above Properties are easy to prove. According to Property 1 and Property 2, after propagation the trust will not increase and the distrust will not decrease. Property 5 implies that if one expert fully distrusts another expert, then e_1 will fully distrusts e_3 no matter what the DM's risk attitude is.

In a real social network GDM problem, there are often more than three experts in a propagation path. For instance, a path $\rho_1 = (e_4, e_3, e_1, e_2)$ between e_2 and e_4 in Figure 1 contains two other experts e_3 and e_1. Under this situation, using the associativity property, a generalized risk defined trust propagation operator can be derived as follows.

$$\begin{aligned} & P_\theta(\lambda_1, \lambda_2, \cdots, \lambda_n) \\ =& P_\theta((t_1, d_1), (t_2, d_2), \cdots, (t_n, d_n)) \\ =& (\log_\theta(1 + \frac{\prod_{i=1}^{n}(\theta^{t_i} - 1)}{(\theta - 1)^{n-1}}), 1 - \log_\theta(1 + \frac{\prod_{i=1}^{n}(\theta^{1-d_i} - 1)}{(\theta - 1)^{n-1}})) \end{aligned} \tag{7}$$

This generalized risk defined trust propagation operator (7) can be proved by mathematical induction on n as follows:

Proof.

(1) For $n = 2$, Eq. (7) deduces to Eq. (6).

Hence, Eq. (7) is correct.

(2) Suppose Eq. (7) holds for $n = k$, that is

$$P_\theta((t_1,d_1),(t_2,d_2),\cdots,(t_k,d_k))$$
$$=(\log_\theta(1+\frac{\prod_{i=1}^{k}(\theta^{t_i}-1)}{(\theta-1)^{k-1}}),1-\log_\theta(1+\frac{\prod_{i=1}^{k}(\theta^{1-d_i}-1)}{(\theta-1)^{k-1}}))$$

then, when $n=k+1$, we have

$$\begin{aligned}&P_\theta((t_1,d_1),(t_2,d_2),\cdots,(t_{k+1},d_{k+1}))\\&=(\log_\theta(1+\frac{\prod_{i=1}^{k}(\theta^{t_i}-1)}{(\theta-1)^{k-1}}),\\&\quad(1-\log_\theta(1+\frac{\prod_{i=1}^{k}(\theta^{1-d_i}-1)}{(\theta-1)^{k-1}}),(t_{k+1},d_{k+1}))\\&=(\log_\theta(1+\frac{\prod_{i=1}^{k}(\theta^{t_i}-1)}{(\theta-1)^{k-1}}\frac{(\theta^{t_{k+1}}-1)}{(\theta-1)}),\\&\quad 1-\log_\theta(1+\frac{\prod_{i=1}^{k}(\theta^{1-d_i}-1)}{(\theta-1)^{k-1}}\frac{(\theta^{1-d_{k+1}}-1)}{(\theta-1)})\\&=(\log_\theta(1+\frac{\prod_{i=1}^{k+1}(\theta^{t_i}-1)}{(\theta-1)^{k}}),1-\log_\theta(1+\frac{\prod_{i=1}^{k+1}(\theta^{1-d_i}-1)}{(\theta-1)^{k}}))\end{aligned}$$

i.e. Eq. (7) holds for $n=k+1$. Thus, Eq. (7) is correct for all n.

Apparently, the propagated value satisfies the conditions of Definition 1. Thus it is also an TF.

It is easy to prove that the general risk defined trust propagation operator $P_\theta(\lambda_1,\lambda_2,\cdots,\lambda_n)$ also satisfies the above 5 properties.

B. Trust aggregation

Generally, within a trust network there could be multiple paths between two non-adjacent nodes. To aggregate multiple paths, many classical methods have been applied to deal with trust and distrust information, such as Matrix multiplications [25], Min-Max criterion [9], and method based on shortest path [1, 20]. In our work we use a weighted average aggregation (WAA) operator.

Definition 9. For a set of TFs $\lambda_i=(t_i,d_i)\in\Lambda$ $(i=1,2,\cdots,n)$, $\mathbf{w}=\{w_1,w_2,\cdots,w_n\}^{\mathrm{T}}$ is associated weight vector, where $w_i\in[0,1]$ and $\sum_{i=1}^{n}w_i=1$. The WAA operator is defined as:

$$WAA(\lambda_1,\lambda_2,\cdots,\lambda_n)=(\sum\nolimits_{i=1}^{n}w_it_i,\sum\nolimits_{i=1}^{n}w_id_i) \quad (8)$$

Given the fact that large path centrality means greater influence from the path, path centrality is used to measure the weight of an expert. For k path $p_i(i=1,2,\cdots,k)$ between e_i and e_j, if $w_i=\delta_i=(1/C_p(p_i))\Big/\sum_{i=1}^{k}1/C_p(p_i)$, then the path centrality based WAA (PCWAA) is defined as follows:

$$PCWAA(\lambda_1,\lambda_2,\cdots,\lambda_n)=(\sum\nolimits_{i=1}^{n}\delta_it_i,\sum\nolimits_{i=1}^{n}\delta_id_i) \quad (9)$$

IV. Risk defined trust transitivity model for GDM

The trust transitivity model and trust aggregation can be used to derive the trust of expert pairs who do not have direct interaction with each other but are connected by social paths. In this section, we propose a group decision making method utilizing our proposed risk defined trust propagation and aggregation methods.

A. Determine expert weights

After the complete social trust matrix is derived by our proposed risk defined trust transitivity model, the TS of each individual expert can be computed as follows:

Definition 10. In a group trust network $\tilde{G}=(E,\tilde{R})$, $S=(S_{lh})_{n\times n}$ is trust relationship matrix, in which S_{lh} is the TF from e_l to e_k. The TS of expert e_k is computed as

$$TS(e_h)=\frac{1}{n-1}\sum\nolimits_{1}^{n}TS(\lambda_{lh}) \quad (10)$$

In a GDM process, People tend to follow the opinions from highly trusted experts. In other words, the greater the TS value of an individual expert, the larger the expert's influence. Consequently, the expert weight can be computed as follows:

$$w_i=\frac{TS(e_h)}{\sum_{i=1}^{n}TS(e_h)} \quad (11)$$

B. Proposed algorithm for SN-GDM

Based on the aforementioned model and analysis, we designed an algorithm for SN-GDM with four tuple information and the process contains the following steps:

Step 1. Form an initial trust social matrix by known expert trust relations.

Step 2. Input the DM's risk attitude and compute the TSs of indirectly connected experts using the risk defined trust transitivity model including propagation operator Eq. (7) and aggregation operator Eq. (9). This step completes when a complete trust sociomatrix is obtained.

Step 3. Compute the TS of each expert using Eqs. (2) and (10). Then we can obtain the expert weights using Eq. (11).

Step 4. Aggregate the evaluation matrices into a collective overall weight for each expert according to Eq. (9).

Step 5. Choose the best one by Definition 5 and conduct a sensitivity analysis with the DM's risk attitude θ.

V. Numerical example

In this section we first use a numerical example of trusted service selection to illustrate the process of our proposed trust model and SNGDM method. Suppose John intends to buy a computer and there are four possible suppliers $\{o_1,o_2,o_3,o_4\}$. To evaluate the trustworthiness of these suppliers, four attributes (whose weighted vector is $\Omega=(0.32,0.26,0.18,0.24)^T$) are considered: service quality (a_1), product quality (a_2), shipping speed (a_3) and product value (a_4).

Due to the lack of knowledge on technology, John may want to consult his friends within his social network and see them as experts. Therefore, the trusted service selection becomes a SNGDM problem, in which the trust network of experts is described in Figure 1 and their evaluation matrices $X^k = (x_{ij}^k)_{4\times 4}$ are shown in Table I, where each tuple in the table represents the trust function provided by each expert regarding certain supplier and attribute.

TABLE I
THE EVALUATION MATRICES OF FOUR SUPPLIERS.

Experts	Suppliers	a_1	a_2	a_3	a_4
e_1	o_1	(0.6,0.5)	(0.7,0.1)	(0.8,0.1)	(0.8,0.3)
	o_2	(0.8,0.2)	(0.8,0.1)	(0.5,0.2)	(0.7,0.2)
	o_3	(0.8,0.3)	(0.8,0.1)	(0.6,0.5)	(0.5,0.3)
	o_4	(0.5,0.5)	(0.5,0.6)	(0.6,0.1)	(0.6,0.2)
e_2	o_1	(0.6,0.4)	(0.7,0.1)	(0.8,0.2)	(0.8,0.4)
	o_2	(0.7.0.4)	(0.8,0.3)	(0.5,0.4)	(0.6,0.5)
	o_3	(0.8,0.3)	(0.6,0.4)	(0.7,0.4)	(0.7,0.3)
	o_4	(0.5,0.2)	(0.8,0.1)	(0.6,0.3)	(0.8,0.1)
e_3	o_1	(0.7,0.2)	(0.7,0.1)	(0.8,0.3)	(0.8,0.2)
	o_2	(0.8,0.2)	(0.4,0.4)	(0.8,0.1)	(0.6,0.3)
	o_3	(0.7,0.4)	(0.6,0.2)	(0.8,0.1)	(0.7,0.2)
	o_4	(0.7,0.3)	(0.5,0.3)	(0.4,0.6)	(0.7,0.3)
e_4	o_1	(0.6,0.4)	(0.5,0.4)	(0.7,0.4)	(0.7,0.2)
	o_2	(0.6,0.1)	(0.6,0.3)	(0.6,0.2)	(0.7,0.2)
	o_3	(0.7,0.2)	(0.6,0.3)	(0.7,0.3)	(0.8,0.2)
	o_4	(0.8,0.3)	(0.7,0.3)	(0.7,0.1)	(0.6,0.3)
e_5	o_1	(0.6,0.4)	(0.9,0.2)	(0.7,0.5)	(0.7,0.3)
	o_2	(0.7,0.4)	(0.7,0.3)	(0.6,0.2)	(0.4,0.3)
	o_3	(0.7,0.2)	(0.7,0.4)	(0.7,0.3)	(0.5,0.5)
	o_4	(0.9,0.1)	(0.4,0.2)	(0.5,0.2)	(0.6,0.1)

In order to use the collected information to make a decision, the following steps are used to process information:

Step 1. The experts trust sociomatrix R is constructed according to Figure 1 as follows. Each tuple in the matrix represent the trust function between experts.

$$R = \begin{bmatrix} - & (0.6,0.2) & & & \\ & - & & (0.4,0.5) & \\ (0.6,0.1) & & - & & (0.6,0.3) \\ & & (0.5,0.4) & - & (0.6,0.2) \\ & (0.8,0.1) & & & - \end{bmatrix}$$

Step 2. Note that a typical sociomatrix is sparse since most people may not have direct experience with most others in the network. The missing TF of the sparse matrix can be filled by using our proposed trust transitivity model. For example, assuming John's risk bearing level is $\theta = 3$, there are three possible paths $\rho_1 = (e_4, e_3, e_1, e_2)$, $\rho_2 = (e_4, e_5, e_2)$ and $\rho_3 = (e_4, e_3, e_5, e_2)$ from e_4 to e_2. By Eq. (7), we have $r_{\rho 1} = (0.13, 0.60)$, $r_{\rho 2} = (0.46, 0.29)$ and $r_{\rho 3} = (0.20, 0.66)$. Using Eq. (4), we get the closeness centrality of all experts $C_e(e_1) = 0.44$, $C_e(e_2) = 0.5$, $C_e(e_3) = 0.57$, $C_e(e_4) = 0.67$ and $C_e(e_5) = 0.4$. From Eq. (5), we get the path centrality of three paths $C_p(p_1) = 0.55$, $C_p(p_2) = 0.53$, $C_p(p_3) = 0.52$. Then, their weights are $\delta_1 = 0.34$, $\delta_2 = 0.33$, $\delta_3 = 0.33$. Using the PCWAA operator, we obtain $r_{42} = (0.26, 0.52)$. Likewise, the complete trust sociomatrix R can be filled as follows:

$$R = \begin{bmatrix} - & (0.60,0.20) & (0.08,0.80) & (0.21,0.62) & (0.07,0.80) \\ (0.08,0.77) & - & (0.17,0.73) & (0.40,0.50) & (0.16,0.72) \\ (0.60,0.10) & (0.40,0.34) & - & (0.10,0.67) & (0.60,0.30) \\ (0.27,0.47) & (0.26,0.52) & (0.50,0.40) & - & (0.60,0.20) \\ (0.06,0.80) & (0.80,0.10) & (0.12,0.77) & (0.30,0.56) & - \end{bmatrix}$$

Step 3. Based on the above sociomatrix, the TSs of each expert are found to be $TS(e_1) = 0.358$, $TS(e_2) = 0.617$, $TS(e_3) = 0.271$, $TS(e_4) = 0.332$, $TS(e_5) = 0.426$. By Eq. (11), we can derive the weights of the five experts as $w_1 = 0.179$, $w_2 = 0.308$, $w_3 = 0.135$, $w_4 = 0.165$, $w_5 = 0.213$.

Step 4. According to Eq. (8), when $\theta = 3$, the collective overall evaluation of each supplier is derived as $o_1 = (0.699, 0.290)$, $o_2 = (0.658, 0.265)$, $o_3 = (0.691, 0.282)$, $o_4 = (0.653, 0.245)$.

Step 5. By Definition 5, the order relation of the four suppliers and the best supplier for different risk attitude are also listed in Table II. From Table II, the orders of the suppliers

TABLE II
THE ORDERS OF THE SUPPLIERS FOR DIFFERENT METHODS

	Method	Order	Best
Our method	$\theta = 1.25$	$o_1 \succ o_3 \succ o_4 \succ o_2$	o_1
	$\theta = 2$	$o_3 \succ o_1 \succ o_4 \succ o_2$	o_3
	$\theta = 2.5$	$o_3 \succ o_1 \succ o_4 \succ o_2$	o_3
	$\theta = 10$	$o_4 \succ o_3 \succ o_1 \succ o_2$	o_4
Wu et al 's method [20]		$o_1 \succ o_3 \succ o_4 \succ o_2$	o_1
Wu et al 's method [21]		$o_3 \succ o_1 \succ o_4 \succ o_2$	o_3

are remarkably different in various risk tolerance level. When $\theta = 1.25$, the order is $o_1 \succ o_3 \succ o_4 \succ o_2$ and the best supplier is o_1; when $\theta = 2$, the order is $o_3 \succ o_1 \succ o_4 \succ o_2$ and the best supplier is o_3; when $\theta = 10$, the order is $o_4 \succ o_3 \succ o_1 \succ o_2$ and the best supplier is o_4. We can see that if consumer is more conservative, the supplier o_4 should be chosen; if consumer is more risk taking, the supplier o_1 should be chosen. The change of ranking for different suppliers under different risk tolerance degree is shown in Fig.2. We can see that when the DM is increasingly risk taking (a.k.a. $1/\theta \rightarrow 1$), the ranking of o_1 also increases. DM with different risk attitude may choose different suppliers, while in the results of the method [20, 21], only a fixed supplier is chosen. This is because our proposed method takes the DM's risk tolerance into consideration, while existing methods neglect this factor. Thus, our proposed method is more flexible and practical.

VI. Conclusions

In this paper, we propose a risk-defined trust transitivity model for group decision making in social network with four tuple information. The significant advantages of the proposed SNGDM method are summarized as follows:

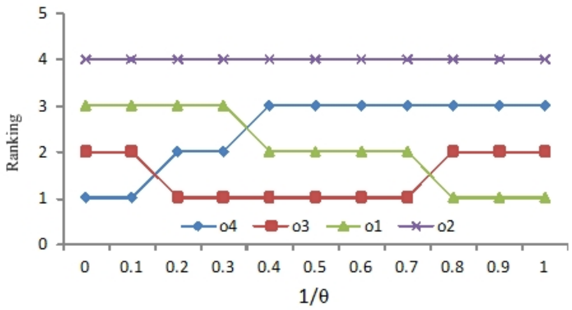

Fig. 2. Ranking of suppliers with different risk attitude

(1) We define a new distance-based KD and apply it to compare TFs that enrich the theories and approaches of trust-based decisions.

(2) We develop a risk defined trust transitivity model, which consists of two parts: trust propagation and trust aggregation. We present a new considering risk attitudinal propagation operator. Moreover, we study some desirable properties for the propagation operator. Based on social network analysis, we use an aggregation operator to merge multiple paths between two unknown individuals. Thus, the model can effectively avoid path information loss.

(3) Combining the proposed risk defined trust transitivity model with the ranking method of TFs, we develop a new SN-GDM method with four tuple information. Finally, we demonstrate the effectiveness of the trust model through an example of trusted service selection and verify the effectiveness of the proposed method through comparison with existing models.

As our future work, we will propose a method for large scale SNGDM problems with multiple tuple information.

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References

[1] J. Wu and F. Chiclana, "A social network analysis trust-consensus based approach to group decision-making problems with interval-valued fuzzy reciprocal preference relations," Knowledge-Based Systems, vol. 59 pp. 97–107, 2014.

[2] D. J. Packer, N. D. Ungson, "Group decision-making," Social Psychology: Revisiting the Classic Studies, vol. 182 pp. 182–200, 2017.

[3] R. Chen, J. Guo and F. Bao, "Trust management for SOA-based IoT and its application to service composition," IEEE Transactions on Services Computing, vol. 9(3) pp. 482–495, 2016.

[4] P. Meo, E. Ferrara, D. Rosaci and G. M. Sarn, "Trust and compactness in social network groups," IEEE transactions on cybernetics, vol. 45(2) pp. 205–216, 2015.

[5] G. Beigi, J. Tang, S. Wang, et al, "Exploiting emotional information for trust/distrust prediction," Proceedings of the 2016 SIAM International Conference on Data Mining, Society for Industrial and Applied Mathematics, pp. 81–89, 2016.

[6] Y. M. Li, C. Y. Lai, "A social appraisal mechanism for online purchase decision support in the micro-blogosphere," Decision Support Systems, vol. 59(1) pp. 190–205, 2014.

[7] Z. Huang, M. Benyoucef, "The effects of social commerce design on consumer purchase decision-making: An empirical study," Electronic Commerce Research and Applications, vol. 25 pp. 40–58, 2017.

[8] H. Alabool, A. Kamil, N. Arshad and D. Alarabiat, "Cloud service evaluation method-based multi-criteria decision-making: A systematic literature review," Journal of Systems and Software, vol. 139 pp. 161–188, 2018.

[9] P. Victor, C. Cornelis, M.D. Cock, and E. Herrera-Viedma, "Practical aggregation operators for gradual trust and distrust," Fuzzy Sets & Systems, vol. 184(1) pp. 126–147, 2011.

[10] H. Brandstatter, J.H. Davis, G. Stocker-Kreichgauer (Eds.), Group decision making, Academic Press, London, 1982.

[11] J. Lu, J. Han, Y. Hu and G. Zhang, "Multilevel decision-making: A survey," Information Sciences, vol. 346 pp. 463–487, 2016.

[12] V. Sadovykh, D. Sundaram and S. Piramuthu, "Do online social networks support decision-making?," Decision Support Systems, vol. 70 pp. 15–30, 2015.

[13] M. Brunelli, M. Fedrizzi and M. Fedrizzi, "Fuzzy m-ary adjacency relations in social network analysis: optimization and consensus evaluation," Information Fusion, vol. 17(1) pp. 36–45, 2014.

[14] L. G. Perez, F. Mata, F. Chiclana, G. Kou and E. Herrera-Viedma, "Modelling influence in group decision making," Soft Computing, vol. 20(4) pp. 1653–1665, 2016.

[15] N. Capuano, F. Chiclana, H. Fujita, E. Herrera-Viedma, V. Loia, "Fuzzy group decision making with incomplete information guided by social influence," IEEE Transactions on Fuzzy Systems, vol. 26(3) pp. 1704-1718, 2018.

[16] H. Shakeri and A.G. Bafghi, "Propagation of trust and confidence using intervals," Internet Technology and Secured Transactions, pp. 436–441, 2012.

[17] Y. F. Zheng, J. Xu, "A trust transitivity model for group decision making in social network with intuitionistic fuzzy information," FILOMAT, vol. 32(5) pp:1937-1945, 2018.

[18] L.G. Perez, F. Mata, F. Chiclana, "Social network decision making with linguistic trustworthiness-based induced owa operators," International Journal of Intelligent Systems, vol. 29(12) pp. 1117–1137, 2015.

[19] J. Wu, L. Dai, F. Chiclana, H. Fujita and E. Herrera-Viedma, "A minimum adjustment cost feedback mechanism based consensus model for group decision making under social network with distributed linguistic trust," Information Fusion, vol. 41 pp. 232–242, 2018.

[20] J. Wu, R. Xiong, F. Chiclana, "Uninorm trust propagation and aggregation methods for group decision making in social network with four tuple information," Knowledge-Based Systems, vol. 96(2) pp. 29–39, 2016.

[21] J. Wu, F. Chiclana, H. Fujita and E. Herrera-Viedma, "A visual interaction consensus model for social network group decision making with trust propagation," Knowledge-Based Systems, vol. 112(C) pp. 39–50, 2017.

[22] Y. Liu, C. Liang, F. Chiclana and J. Wu, "A trust induced recommendation mechanism for reaching consensus in group decision making," Knowledge-Based Systems, vol. 119 pp. 221–231, 2017.

[23] K. T. Atanassov, "Intuitionistic fuzzy sets," Fuzzy sets and Systems, vol. 20(1) pp. 87-96, 1986.

[24] G. Beliakov, H. Bustince, D. P. Goswami, U. K. Mukherjee, N. R. Pal, "On averaging operators for Atanassov's intuitionistic fuzzy sets," Information Sciences, vol. 181(6) pp. 1116-1124, 2011.

[25] Nguyen, H, "A new knowledge-based measure for intuitionistic fuzzy sets and its application in multiple attribute group decision making," Expert Systems with Applications, vol. 42(22) pp. 8766–8774, 2015.

[26] Minor and J, "Applied network analysis," Sage Publications, vol. 14(1) pp. 101–116, 1983.