

3D Aerial Base Station Position Planning based on Deep Q-Network for Capacity Enhancement

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Abstract—When the existing traditional terrestrial base station is insufficient to meet the sudden traffic demand or is not available, deploying the aerial base station (aerial-BS) is a fast and effective solution for achieving network capacity enhancement. How to plan the best 3D location of the aerial-BS according to the user's business needs and service scenarios is a key issue to be solved. At present, the conventional optimization algorithms that solve this problem have high time complexity and it is difficult to utilize experience. However, applying the deep reinforcement learning model can quickly get an optimal solution by historical experience feedback training. Therefore, it is suitable for solving the optimal 3D location planning problem of the aerial-BS. In this paper, firstly, aiming at the maximum spectral efficiency of the system, considering the effects of line-of-sight and non-line-of-sight path loss, a mathematical optimization model for the location planning of the aerial-BS is proposed. For this model, the model definition and training process of deep Q-Network are constructed, and through the large-scale pre-learning experience of different user layouts in the training process to gain experience, improve the timeliness of the training process. The simulation results show that the proposed method can achieve the spectral efficiency of more than 91% of the theoretical maximum spectral efficiency, which has lower time complexity than traditional genetic algorithms (such as hill climbing algorithm and simulated annealing algorithm).

Keywords—aerial base station, deep reinforcement learning, DQN, mobility management

I. INTRODUCTION

Due to the sharp increase in user demand, more and more data and services are constantly required, which has put significant strain on infrastructure-based macro cellular networks due to the inefficiency in handling these traffic demands, cost-effectively. The 5G communication network will be highly dependent on the architecture of large base stations but will use a large number of small base stations to increase network density and coverage to achieve higher transmission rates and network capacity. The aerial-BSs can be used as aerial access points or relays between disconnected networks and enhanced connectivity. Using the aerial-BS as the air support of the existing cellular network can handle such high traffic conditions more economically, and better enhance the network capacity.

The aerial-BS can overcome the physical limitation that the traditional terrestrial cellular base station can only be fixed at a specific location, without pre-installing any equipment, reducing the deployment cost of the base station, and improving the flexibility of the cellular network, so that the network can flexibly and dynamically provide enhanced mobile communication services for the burst high traffic area and remote weak coverage areas, to increase the number of users, the increase of the coverage area and the service quality, and the network utilization, etc. Such a system is practically advantageous for events where a large number of users in

small area access high-speed data simultaneously, for example, concerts, mass rallies, sport, and cultural events, etc. Such architecture has been previously proposed for rescue operations in case of disasters, where terrestrial cellular networks become non-functional

The use of aerial-BSs in wireless networks has gained attention in recent years, enabling rapid deployment solutions to provide the needs of wireless networks. In [1], a scheme for deploying an aerial-BS to enhance the capacity of a data traffic burst area is proposed. However, the location deployment of the aerial-BS has become one of the key challenges. Unlike traditional fixed-location terrestrial base stations, the location of the aerial-BS is flexible, can move spatially and intelligently, and finally determine the optimal height and angle. Therefore, the location planning of the aerial-BS is a 3D deployment problem. In [2], a heuristic algorithm is proposed for determining the 3D position of an aerial-BS and serving multiple users while using a minimum number of aerial-BSs. However, in a dynamic environment where the network topology changes, the heuristic algorithm needs to be re-initialized and run for the new topology, which brings a lot of computational complexity to the system. In [3], the vertical and horizontal dimensions of the aerial-BS are separated, and an aerial-BS deployment scheme that serves the maximum number of users with minimum transmit power is proposed. In [4], an algorithm is proposed which considers user base station association and wireless backhaul bandwidth allocation, and also performs 3D deployment of the aerial-BS with the goal of maximizing the user's total logarithm rate.

In the previous literature, the mobility of the users was not considered. In [5], the 3D deployment algorithm of the aerial-BS based on the reinforcement learning Q-learning considering user mobility is proposed. However, when the state space dimension increases, the Q table will consume a lot of memory and bring a lot of time overhead.

In view of the above problems, in this paper, we use Deep Q-Network (DQN) [6] algorithm to solve the 3D deployment problem of aerial-BS. DQN uses Q network to fit the Q table, which solves the dimensional disaster problem well. First, learn and save the model for different user-distributed environments, and then apply the model to quickly find the optimal 3D deployment location during testing.

The contribution of this paper is that we apply the DQN algorithm to the 3D deployment of aerial-BSs and solve the problem of dimensional disasters. Considering the dynamic changes of the network topology environment, the user distribution is trained as part of the state, so that the model can be well adapted to different user distributions.

The rest of this paper is organized as follows. The system model is given in Section II. Section III gives DQN for 3D position planning of an aerial-BS. Section IV presents the

simulation results and finally the study is concluded in Section V.

II. SYSTEM MODEL

As shown in Fig. 1, a macro base station is deployed in the ground center, and the green shaded portion represents the coverage of the macro base station. When the stadium holds concerts and other activities, it will cause sudden traffic hotspots, which puts higher demands on the capacity of mobile data access. We can increase capacity for local networks by deploying aerial-BSs. The aerial-BS can also provide services to users outside the coverage of the macro base station. The deployment location of the aerial-BS not only affects the number of users in its coverage area, but also the quality of the air-to-ground link. Due to the characteristics of the aerial-BS, the air-to-ground channel is different from the terrestrial channel, and it has a higher line of sight connection opportunity.

In order to find the optimal location of the aerial-BS, we set the optimization goal to the average spectral efficiency of the system, that is, to simplify the problem to find the location of the maximum system spectrum efficiency. In this section, the air-to-ground path loss model is introduced in II.A, and II.B gives the bandwidth allocation method of the aerial-BS. Finally, the calculation formula of the average spectral efficiency of the system is obtained at II.C, thereby modeling the optimization problem of the deployment position of the aerial-BS.

A. Air-to-ground Path Loss Model

The aerial-BS is connected to the cellular base station on the ground through the wireless downlink, and provides wireless communication services for users in the area, as shown in Fig. 2. If there is no interference between the uplink and the downlink, it is further assumed that the aerial-BS uses the fixed transmission power p_{tx} (watts), the bandwidth B (in Hertz), and transmits data to the user at the center carrier frequency f (hertz) [7].

The ground distance between the user i and the aerial-BS j is defined as the distance between the user position of the ground and the position of the AERIAL-BS projected on the

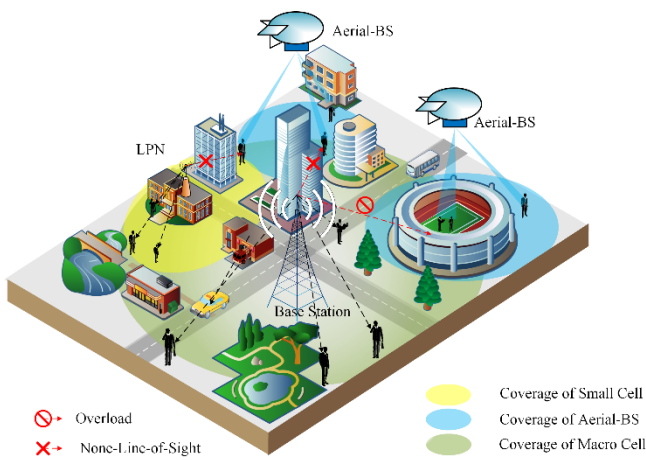


Fig. 1. Aerial-BS can assist network through hard to predict situations.

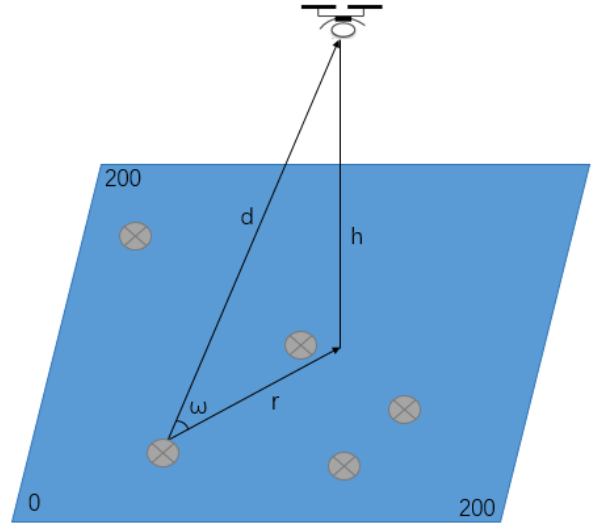


Fig. 2. Problem scene diagram.

ground, and is represented by the symbol $r_{i,j}$. The 3D distance between the user and the drone is expressed as:

$$d_{i,j} = \sqrt{r_{i,j}^2 + h^2} \quad (1)$$

The wireless channel model between the aerial-BS and the ground mobile user is established according to the line-of-sight link probability model, where the probability of forming a line-of-sight link between the AERIAL-BS and the ground user depends on the elevation angle (ω) of the transmission link. The model considers two propagation modes to derive the air-to-ground path loss equation. The first is the line-of-sight link (LoS), and the second is the non-line-of-sight link (NLoS). The line-of-sight link probability function can be expressed as:

$$p^{LoS}(i,j) = \frac{1}{1 + \alpha \exp(-\beta[\omega - \alpha])} \quad (2)$$

Where α and β are constants determined by the environment, the elevation angle ω is equal to $\arctan(h/r_{i,j})$, and h is the height of the AERIAL-BS. The probability of forming a non-line-of-sight link between a AERIAL-BS and a ground user is:

$$p^{NLoS}(i,j) = 1 - p^{LoS}(i,j) \quad (3)$$

Therefore, the path loss of the system can be expressed as:

$$\eta_{path}(i,j) = A_{path} + 10\delta_{path} \log_{10}(d_{i,j}) \quad (4)$$

The path in the above equation is divided into two cases: the line-of-sight link and the non-line-of-sight link. In addition, A_{path} represents the path loss within the reference distance (1 m), and δ_{path} represents the path loss parameter. These values are determined by the environment [8].

B. Resource Allocation Model

The bandwidth B (Hertz) of the aerial-BS needs to be assigned to all users. In this paper, we consider two different resource allocation models, the average allocation model, and the channel-based quality allocation model. The average allocation method is to simply distribute the total bandwidth evenly to all users. The channel quality allocation method maximizes the spectral efficiency of the network at the expense of the user's fairness. This allocation method allocates the desired bandwidth to one user with the highest channel quality. In this paper, we chose the average allocation model, and the bandwidth allocated to each user is $b_i = \frac{B}{n}$ [9].

C. Optimization Problem

In this paper, to define the spectral efficiency, we first need to define the received signal power. The received signal power of the active user i performing network service by the aerial-BS j is expressed as $S^{path}(i, j)$ (watt), which can be calculated by the following equation [10]:

$$\begin{aligned} S^{path}(i, j) &= \frac{b_i}{B} \times p_{tx} \times 10^{\frac{-\eta_{path}(i, j)}{10}} \\ &= \frac{b_i}{B} \times p_{tx} \times A'_{path} \times d_{i, j}^{-\delta_{path}} \end{aligned} \quad (5)$$

Where $A'_{path} = 10^{\frac{-A_{path}}{10}}$, b_i is the part of the total bandwidth B allocated to the user i .

In addition, the total noise power N_i (watt) received by a user i consists of two parts: thermal noise power and user equipment noise power, which can be expressed as:

$$N_i = 10^{\frac{-174 + \rho_{ue}}{10}} \times b_i \times 10^{-3} \quad (6)$$

Where ρ_{ue} (dB) is the user equipment noise figure.

Therefore, the signal-to-noise ratio (SINR) of the user i served by the aerial-BS can be expressed as[11]:

$$SINR^{path}(i, j) = \frac{S^{path}(i, j)}{N_i} \quad (7)$$

According to Shannon's theorem, the spectral efficiency (SE) (bps/Hz) of the user i served by the aerial-BS j can be expressed as:

$$\Phi^{path}(i, j) = \log_2(1 + SINR^{path}(i, j)) \quad (8)$$

According to the average allocation model, the average spectral efficiency of user i can be expressed as:

$$\begin{aligned} \Phi(i, j) &= \Phi^{Los}(i, j) + \Phi^{NLos}(i, j) \\ &= P^{Los}(i, j)(\log_2(1 + SINR^{Los}(i, j))) \\ &\quad + P^{NLos}(i, j)(\log_2(1 + SINR^{NLos}(i, j))) \end{aligned} \quad (9)$$

The average spectral efficiency $\Phi(j)$ of the aerial-BS is the average of the average spectral efficiency of all users. The goal of the algorithm is to move the aerial-BS to a location or route that has the largest average spectral efficiency $\Phi(j)$.

Therefore, the optimization problem is as follows:

$$\max_{h, x, y} \frac{1}{n} \sum_{i=1}^n \Phi(i, j) A_{i, j}$$

subject to:

$$\begin{aligned} \sum_{j=1}^P A_{ij} &= 1 \\ h_{min} &< h < h_{max}, \\ x_{min} &< x < x_{max}, \\ y_{min} &< y < y_{max}, \\ SINR &> SINR_{min} \\ A_{i, j} &\in \{0, 1\} \end{aligned} \quad (10)$$

Where $A_{i, j} = 1$ indicates that the i -th user is connected to the j -th BS, $[x_{min}, x_{max}]$, $[y_{min}, y_{max}]$, $[h_{min}, h_{max}]$ indicates that the 3D area of the aerial-BS can be deployed.

III. PROBLEM FORMULATION AND ALGORITHMS

In this section, we propose a 3D position planning of an aerial-BS algorithm based on DQN, which aims at maximizing the average spectral efficiency of the system. DQN is a combination of deep learning CNN and reinforcement learning Q-learning. In the reinforcement learning Q-learning, the agent continuously learns according to the rewards or punishments obtained in the interaction with the environment, so that the state of the agent is closer to the target state. $\langle A, S, R, P \rangle$ is a classic quaternion in reinforcement learning, where Action represents the action set of the agent, State is the state set of the agent, Reward is a value, representing a reward or punishment, and P represents the agent in this the probability of taking an action in the state.

In our proposed algorithm, the agent is an aerial-BS whose state space, action space and rewards are defined as follows:

The state space: $S = (x, y, z, user_number, user_{1x}, user_{1y}, user_{2x}, user_{2y}, \dots, user_{80x}, user_{80y})$

Where x, y, z represent the 3D position interval that the base station can be deployed, where x is the x-axis coordinate, y is the y-axis coordinate, h is the height of the aerial-BS, $user_number$ is the number of users in the environment, $user_{ix}, user_{iy}$, representing the x-axis and y-axis coordinates of the i -th user, respectively.

The action space: $A = \{0, 1, 2, 3, 4, 5, 6\}$, respectively, represents that the aerial-BS moves upward, moves downward, moves in the positive direction of the x-axis, moves in the negative direction of the x-axis, and is positive toward the y-axis. The direction moves, moving in the negative direction of the y-axis, and maintaining the position unchanged for a total of seven actions.

The Reward: The average spectral efficiency $\Phi(j)$ of the aerial-BS in the current state.

The goal of the algorithm is to move the aerial-BS to a location or route that has the largest average spectral efficiency $\Phi(j)$.

The basic form of the Q-learning is:

$$Q(S_t, A_t) = Q(S_t, A_t) + \Omega(R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t)) \quad (11)$$

Where $Q(S_t, A_t)$ is the reward discount obtained by the agent taking action A_t in state S_t , and Ω is the learning rate. The greater the learning rate, the less previous learning outcomes are retained, γ is the discount factor, and the discount factor is larger, the more the learning entity pays attention to the previous learning experience, the more it pays attention to the maximization of the reward value at hand. Select the action through the greedy strategy until the value function converges to get the optimal strategy:

$$\pi(s) = \operatorname{argmax}_{a \in A} Q(s, a) \quad (12)$$

This will find the best action for each state, but because the Q matrix has limited ability to store information. when the state is too much or discrete, this algorithm will naturally cause dimensional disasters. DQN model combining deep learning with reinforcement learning, which uses the value function to approximation the Q value.

$$Q(s, a) = f(s, a) \quad (13)$$

In the above formula, the functional relationship refers to learning through a neural network to obtain the Q values and the functional mapping relationship between states and actions. The neural network uses two fully-connected neural networks with the same structure and different parameters: the main network and the target network. During the training process, the first cycle randomly generates different user distribution environments, and the second cycle iterates to find the 3D position of the aerial-BS has maximum spectral efficiency. First, initialize a random state s_t , and then use the ϵ -greedy strategy to select the action. That is, an action $a_t \in A$ is randomly selected from the action set by the probability ϵ ,

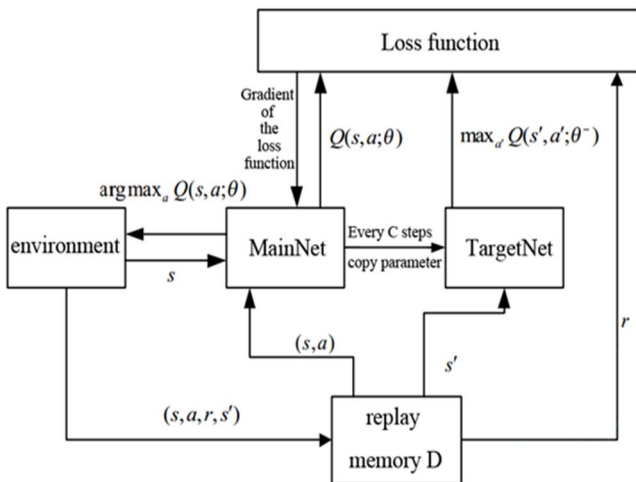


Fig. 3. DQN algorithm flow.

and the action $a_t = \max_a Q(s_t, a; \omega)$ having the highest action value is selected with the probability $1 - \epsilon$. Get the new state s_{t+1} and reward r_t , and update the current Q value, then update the target Q value every C steps, and reverse transmission with the square of the difference between the two as a loss function. The algorithm flow is shown in the Fig. 3.

For high-dimensional state space, the DQN algorithm takes the state S as an input and outputs a matrix, $[Q(s, a_1), Q(s, a_2), \dots, Q(s, a_n)]$, which is the current The reward and punishment values corresponding to all possible actions in the state, through the empirical learning, establish the mapping relationship between the state S and the matrix, and then select the optimal action from it.

The steps of the DQN for 3D placement of an aerial-BS are summarized in Algorithm 1:

Algorithm 1 DQN for 3D placement of an aerial-BS

- Input:** 3D position coordinates of the aerial-BS, number of users, and distribution of each user
Output: Optimal deployment location coordinates of the aerial-BS
- 1: Initialize replay memory D to capacity N
 - 2: Initialize action-value function Q with random weights θ
 - 3: Initialize target action-value function \hat{Q} with weights $\theta^- = \theta$
 - 4: **For** episode = 1, N **do**
 - 5: Initialize the number of user and distribution of users in sequence s
 - 6: Initialize 3D position coordinates of the aerial-BS in sequence s and preprocessed sequence $\phi_1 = \phi(s_1)$
 - 7: **For** $t = 1, T$ **do**
 - 8: With probability ϵ select a random action a_t
 - 9: Otherwise action a_t in emulator and observe reward r_t and image x_{t+1}
 - 10: Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
 - 11: Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in D
 - 12: Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from D
 - 13: Set $y_j =$

$$\begin{cases} r_j & \text{if episode terminates} \\ r_j + \gamma \max_{a'} \hat{Q}(\phi_{j+1}, a'; \theta^-) & \text{at step } j + 1 \\ \text{otherwise} & \end{cases}$$
 - 14: Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ with respect to the network parameters θ
 - 15: Every C step reset $\hat{Q} = Q$
 - 16: **End For**
 - 17: **End For**
-

IV. SIMULATION RESULTS

In our simulations, we consider $2\text{km} \times 2\text{km}$ area, in this area, the user is randomly generated, and in order to simulate the scene of Figure 1, we simulate the stadium in the lower right corner, generate a large number of users, and randomly generate a small number of users in other locations. To make the research content closer to the actual needs, a ground base station is arranged in the center. The basic parameters and spectral efficiency calculation methods of the ground base station are the same as those of the aerial-BS. When the spectral efficiency of the ground base station is greater than the spectral efficiency of the aerial-BS for a certain user, the aerial-BS chooses not to provide the wireless network service for the user. When the spectral efficiency of the ground base station is greater than the spectral efficiency of the aerial-BS for all users, the aerial-BS is not selected for use. This approach is more in line with the goal of the aerial-BS to provide services for areas with sudden high network demand.

As shown in Fig. 4, the green circle represents the coverage of the ground macro base station, the red dot represents the user, and the user in the lower right corner is

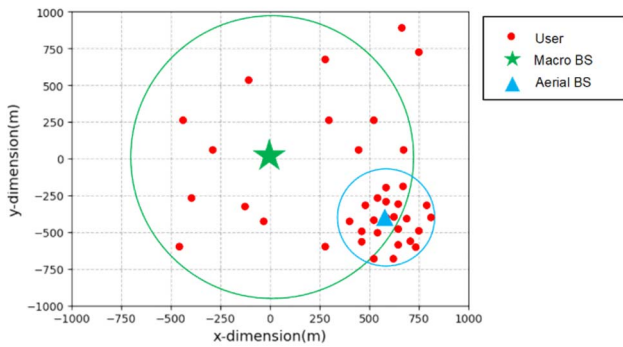


Fig. 4. Aerial-BS deployment simulation scenario

densely distributed, and the network demand suddenly increases, so we deploy the air base station here to enhance the network capacity. The blue circle in the figure represents the coverage of the air base station.

Environment parameters are shown in Table I and DQN algorithm parameters are shown in Table II.

TABLE I. ENVIRONMENT PARAMETERS

Parameter	Value
Bandwidth (B)	10MHz
The center carrier frequency (f)	2GHz
aerial-BS transmitted power P_{tx}	24dBm
Path loss parameter (LoS/NLoS) δ	2.09/3.75
Reference distance (1 m) path loss (LoS/NLoS) (A)	41.1/33
Urban environmental parameters (α, β)	11.95,0.136

TABLE II. DQN ALGORITHM PARAMETERS

Parameter	Value
Learning rate (Ω)	0.0001
Discount factor (γ)	0.9
Starting value of greedy Parameter (ϵ)	0.9
Final value of greedy Parameter (ϵ)	0.01
Memory update iterations (t)	100
Initial experience step (s)	1000
Memory unit size (D)	2000
Batch Size (b)	32
Round number (T)	5000
Episode of different users distribution (N)	100000

First, a 3D position is randomly selected as the starting point of the aerial-BS. In Fig. 5, as the learning process progresses, the aerial-BS gradually moves to a position where maximizes the average spectral efficiency of the system and remains substantially stationary, with blue representing the user, red representing the starting point of the aerial-BS, and green representing the moving process. The corresponding Q value changes during the training process are shown in Fig. 6.

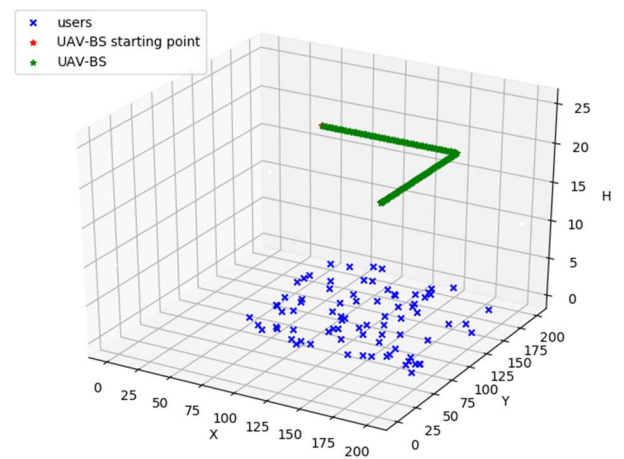


Fig. 5. Aerial-BS location moving process using DQN algorithm.

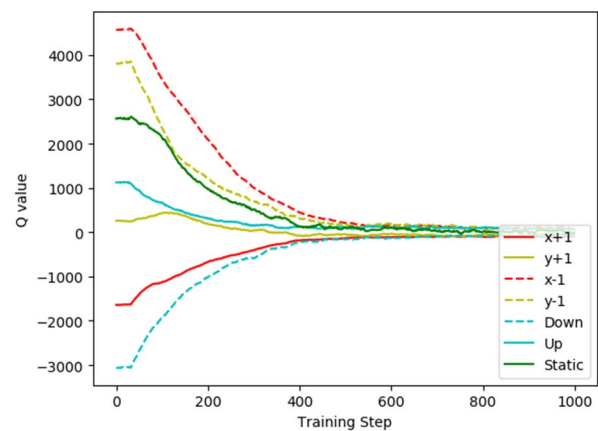


Fig. 6. Reward in the algorithm iteration process.

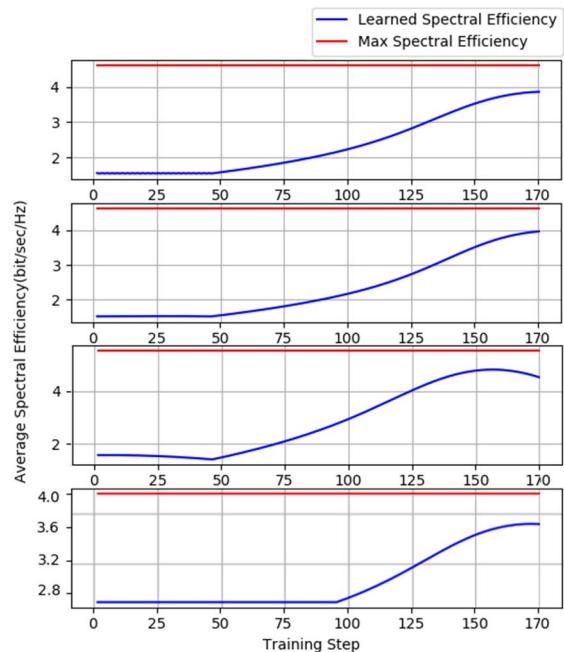


Fig. 7. The average spectral efficiency of systems with different 4 user distributions.

After a certain scale of learning, the network structure parameters in the DQN algorithm are obtained and saved as a model. The model is directly applied during the testing, the aerial-BS will stay in the position where is maximum spectral Efficiency in the system. Fig. 7 shows the test results of four different user distributions, as can be seen, that on average, the learned spectral efficiency of the system can reach 91.3% of the maximum spectral efficiency under ideal conditions.

Reward in the algorithm iteration process is shown in Fig. 8, this figure shows the gradual improvement of the system as the reward reduces in each iteration, it also shows system approaches the optimum position after sufficient iterations and become stable as the reward becomes zero.

The advantage of the DQN algorithm is that after learning and getting the model, it can be directly applied, and the application time is very short, so the efficiency is very high. In this paper, we compare the running time of the DQN algorithm with the traditional mobility management algorithm (climbing algorithm, simulated annealing algorithm) and Q-learning algorithm. We test under the same certain user distri-

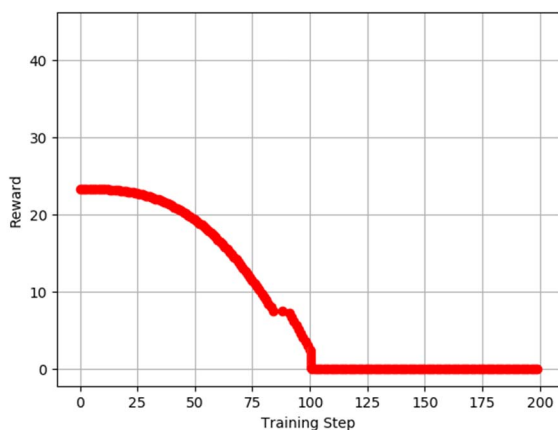


Fig. 8. Reward in the algorithm iteration process.

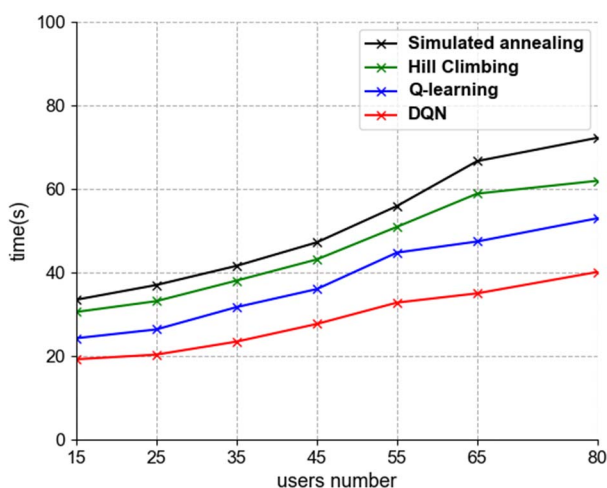


Fig. 9. Time performance comparison of different algorithms.

bution with the number of users as variables, and get the running time of the four algorithms. As shown in Fig. 9, the

algorithm time of DQN is the shortest, and as the scale of the problem increases, the gap between the algorithms will become larger and larger.

V. CONCLUSION

In this paper, the DQN algorithm is applied to the optimal deployment location of the aerial-BS in the system. Simulation results show that under certain constraints, if the learning entity has a sufficiently long learning time, after enough iterations, the learning entity will learn the environmental characteristics and save the learning model, find the best deployment location in a very short time when applying the model. The learned spectral efficiency of the system can reach 91.3% of the maximum spectral efficiency under ideal conditions.

In this paper, the running time of DQN algorithm is compared with the traditional mobility management algorithm (climbing algorithm, simulated annealing algorithm) and Q learning algorithm. The algorithm time of DQN is the shortest. As the scale of the problem increases, the algorithm The gap between the two will increase. Therefore, it is a very practical and effective algorithm that can effectively enhance the 5G network capacity.

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