Network Economics Approach to Data Offloading and Resource Partitioning in Two-Tier LTE HetNets

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***Abstract*—In two-tier LTE heterogeneous networks (HetNets), picocells can be offered radio resource in order to mitigate interference to picocell users in downlink transmission from high-power macrocell base station (MBS). This becomes important in order to maintain efficient operation of the network and generate benefit tradeoff between macrocell and picocells. In this paper, we propose a game based approach for joint resource partitioning and data offloading scheme to determine the amount of radio resource a MBS should offer to picocells and to determine how much traffic each picocell access point (AP) should admit from MBS. In our proposal, a two-stage Stackelberg game theory is applied to optimize the strategies of both MBS and APs in order to maximize both of their utilities and this scheme is implemented using the notion of Almost Blank Subframes (ABS) proposed in the LTE standard.**

Index Terms*—4G LTE, Heterogeneous Networks, data offloading, resource partitioning, Game Theory.

I. Introduction

In traditional HetNets, data traffic of macrocells is deliberately routed to the complementary networks, namely small cells such as picocells, femtocells, or WiFi networks in order to offload its bursty traffic to small cells [1][2], but this can lead to congestion in small cells. Thus, it is necessary to have an efficient resource partitioning mechanism in order to achieve optimal data offloading. The authors in [1][2] use game theory to model the data offloading problem. Stackelberg game is applied in [1] to help MBS (leader) offloads it data to APs (followers). However these works do not address the resource partitioning issue between MBSs and APs.

In 3GPP's Long Term Evolution (LTE) system, the standard enhanced inter-cell interference coordination (eICIC) [6] has been introduced for mitigating interference caused by high-power MBS to its underlay picocells by muting all MBS's downlink transmission to its mobile users (MUs) in certain subframes termed Almost Blank Subframes (ABS). [7] solves the joint resource partitioning and user offloading problem. However they do not consider amount of data that should be offloaded for optimal performance of network. Our motivation, model, and results are different from above mentioned studies. We jointly consider resource partitioning and amount of data to offload to small cells with different approach.

The key results and contributions of this paper are: We propose a *network economic incentive non-cooperative approach* for the joint mobile data offloading and resource partitioning problem. We use game-theoretic model to design an economic incentives scheme that encourages each individual picocell AP to admit offloaded traffic for MBS in a non-cooperative fashion along with determining its own traffic demand for optimizing its total utility. We show that the optimal solution for the proposed scheme can be attained by a two-stages Stackelberg game and the optimal solution presents a unique Nash equilibrium.

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II. System Model

A. System Description

We consider a downlink two-tier HetNet with a MBS, a set $\mathcal{P}$ of APs, and M is the size of set $\mathcal{P}$. Both MBS and APs use the same orthogonal frequency bands to transmit data. MBS and each AP serve its own group of mobile users (MUs) which are randomly distributed within the MBS and APs' coverages with infinite traffic sources. We study for one time period. The MUs' location and traffic may change over time but for simplicity they are considered fixed within each period.

Let α denote the fraction of ABS subframes per pattern time period which are reserved for interfered picocells by macrocell. In LTE-HetNets, fixed eICIC pattern could be used, which means that there is a fixed number of ABS in a certain number of subframes [6], e.g., ABS/subframes: 5/40-10/40-15/40. Specifically MBS can mute in 5, 10 or 15 ABS subframes in a period of 40 subframes, corresponding to $\alpha = 0.125$, $\alpha = 0.25$, $\alpha = 0.375$. Let l_m denote the traffic volume that AP m can admit from MBS and l_0 denote the total MBS's traffic that cannot be offloaded to any AP. The traffic profile of MBS is denote by $\boldsymbol{l} \triangleq (l_0, l_1, ..., l_m)$.

Let x_m denote the AP m's own traffic demand which changes randomly over time. An AP's incurred cost for admitting traffic for MBS depends on how loaded the AP already is, and how much traffic is offloaded for MBS. Each AP $m \in \mathcal{P}$ has an instantaneous rate c_m which we assume to be fixed over the ABS time period and hence the maximum amount of data that it can serve within a certain time period is $\alpha c_m T$:

$$x_m + l_m \leq \alpha c_m T. \tag{1}$$

Without loss of generality, we normalize the time duration to be $T = 1$.

B. Game Model

We focus on the incentive that MBS needs to provide to APs in order to encourage cooperative data offloading. The challenge for pricing in this case is that the MBS needs to provide the right incentive so that each AP is willing to offload for MBS even though each AP serves its own MUs. The interaction between the MBS and APs can be characterized as a two-stage Stackelberg game model as shown in Fig. 1. The MBS publishes the resource partitioning scheme and offers economic incentive to APs in Stage I, and APs respond with their admission abilities in Stage II. All APs want to maximize their total utilities by optimizing the traffic offloaded they can admit according to the resource partitioning scheme. The MBS wants to maximize its utility by setting the right resource partitioning scheme to satisfy the admit abilities of APs.

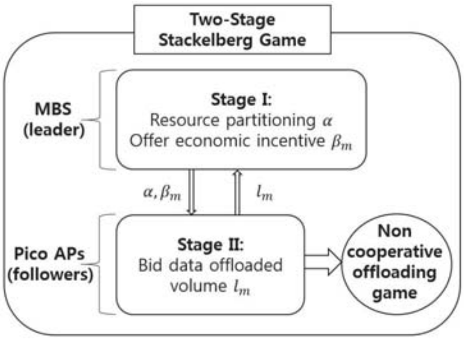


Fig. 1: Two-stage Stackelberg game: MBS's economic incentive, resource partitioning and APs' data offloading

In the following, we first analyse the potential game and best response potential function according to the payoff function of APs. We then prove the existence and uniqueness of Nash equilibrium for AP's strategy based on the best response potential function.

III. Game Theoretic Analysis for Non-cooperative Data Offloading Scheme

A. *Potential Game and Best Response Potential Function*

In the two-stage Stackelberg game we consider the traffic which each AP can admit from MBS as a "bid". In the second stage, each AP submits a "bid" to the MBS, then in the first stage, MBS accepts these submitted bids and determines the fraction of resource and economic incentive (as shown in Fig. 1). APs are then allocated fraction of resource corresponding to their "bids". We use the notation $\boldsymbol{l}_{-m}$ to denote the vector of all bids by APs except m; i.e., $\boldsymbol{l}_{-m} = (l_1, l_2, ..., l_{m-1}, l_{m+1}, ..., l_M)$. Without loss of generality, we assume that at each time period the total traffic volume that MBS desires to offload to APs is L. The value of L may change over the time periods, but within each time period we assume that this value is fixed.

The existence of Nash equilibrium is considered the main purpose of all APs' strategies in which all other APs except m are fixed such that the strategy chosen by AP m maximizes its payoff. To analyze strategies of all APs in order to achieve Nash equilibrium point we consider a payoff function of AP m as follows

$$\widetilde{P}_m(\boldsymbol{l}) = P_m(l_m) - V(\sum_{m\in\mathcal{P}} l_m - L)^2. \tag{2}$$

where $P_m(l_m)$ is payoff term and V is a non-negative weight of penalty term $(\sum_{m\in\mathcal{P}} l_m - L)^2$. We call V as a traffic offloading adjust factor which is shown in Section IV. We assume that the payoff term $P_m(l_m)$ is strictly concave and continuously differentiable.

Definition 1. A data offloading game $\mathcal{G}$ is defined as a triple $\mathcal{G} \triangleq \{\mathcal{P}, (S_m)_{m\in\mathcal{P}}, (\widetilde{P}_m)_{m\in\mathcal{P}}\}$, where $\mathcal{P}$ is the player set (set of all picocell APs), $(S_m)_{m\in\mathcal{P}}$ is the stratergy set of players $S_m \triangleq \{l_m | 0 \le l_m \le c_m\}$, and $(\widetilde{P}_m)_{m\in\mathcal{P}}$ is the payoff function set.

Definition 2. $\mathcal{G} \triangleq \{\mathcal{P}, (S_m)_{m\in\mathcal{P}}, (\widetilde{P}_m)_{m\in\mathcal{P}}\}$ is a best-response potential game if there exists a function $\widehat{P}(\boldsymbol{l})$ such that

$$\arg\max_{l_m\in S_m} \widetilde{P}_m(\boldsymbol{l}) = \arg\max_{l_m\in S_m} \widehat{P}(\boldsymbol{l}), \quad \forall m\in\mathcal{P}, \forall l_{-m}\in S_{-m}. \tag{3}$$

The function $\widehat{P}(\boldsymbol{l})$ is called a *best-response potential function* for the game $\mathcal{G}$ [8].

Proposition 1: The game $\widetilde{\mathcal{G}} \triangleq \{\mathcal{P}, (S_m)_{m\in\mathcal{P}}, \widehat{P}(\boldsymbol{l})\}$ is a potential game. The corresponding best-response potential function is given by

$$\widehat{P}(\boldsymbol{l}) = \sum_{m\in\mathcal{P}} P_m(l_m) - V(\sum_{m\in\mathcal{P}} l_m - L)^2. \tag{4}$$

Proof. This follows using the characterization of potential games in [4], i.e., $\frac{\partial \widehat{P}(\boldsymbol{l})}{\partial l_m} = \frac{\partial \widetilde{P}_m(\boldsymbol{l})}{\partial l_m}, \forall m\in\mathcal{P}$. □

B. *Existence and Uniqueness of Nash Equilibrium for APs' strategy*

In this subsection we derive the bid for the potential game $\widetilde{\mathcal{G}}$ such that the unique equilibrium of $\widetilde{\mathcal{G}}$ will coincide with the desired operating point $\boldsymbol{l}^*$.

Theorem 1. A Nash equilibrium exist in the game $\widetilde{\mathcal{G}} \triangleq \{\mathcal{P}, (S_m)_{m\in\mathcal{P}}, \widehat{P}(\boldsymbol{l})\}$ and this Nash equilibrium is unique.

Proof. The following result is obtain from [3].

Proposition 2:. A Nash equilibrium exists in game $\widetilde{\mathcal{G}} \triangleq \{\mathcal{P}, (S_m)_{m\in\mathcal{P}}, \widehat{P}(\boldsymbol{l})\}$, if for all $m\in\mathcal{P}$:

1) $(S_m)_{m\in\mathcal{P}}$ is a nonempty, convex, and compact subset of some Euclidean space $\mathbb{R}^M$.

2) $\widehat{P}(\boldsymbol{l})$ is continuous and concave in $\boldsymbol{l}$.

Strategy space is defined to be $\mathcal{S} = \{[S_m]_{m\in\mathcal{P}} : l_m | 0 \le l_m \le c_m\ \}$. So it is a nonempty, convex and compact subset of the Euclidean space $\mathbb{R}^M$.

According to the assumption that *best-response potential function* $\widehat{P}(\boldsymbol{l})$ is continuous and concave, a Nash equilibrium exists in game $\widetilde{\mathcal{G}}$, and the uniqueness of this Nash equilibrium can be proved similarly as in [5]. □

Recall that all APs have the same utility function so that APs are homogeneous. If the APs have different utility functions, the network is said to have heterogeneous APs. The motivation for studying networks of heterogeneous APs is to provide differentiated services to different APs (e.g., picocell or femtocell access points). Toward this end, we consider homogeneous APs and symmetric equilibrium.

Definition 3. A Nash equilibrium $\boldsymbol{l}^*$ is said to be a symmetric equilibrium if $l_i^* = l_j^*$ for all $i, j\in\mathcal{P}$, and an asymmetric equilibrium otherwise.

Corollary 1. For a network of homogeneous APs, if the game $\widetilde{\mathcal{G}}$ has a Nash equilibrium, it must be unique and symmetric.

Corollary 1 guarantees the uniqueness of Nash equilibrium, and it guarantees the fair resource partitioning based on the "bid" of data offloading among homogeneous APs.

IV. Data Offloading and Resource Partitioning Scheme

In the following section, we first model the strategy of picocell AP m in the stage II of the Stackelberg game. We focus on how to obtain the unique Nash equilibrium by using potential game and best response potential function in a non-cooperative fashion. As a result, we derive the *data offloading rule* in stage II of the two-stage Stackelberg game in a close form. Finally, we model the *resource partitioning scheme* in stage I of the game accordingly.

A. ***Stage II: Picocell AP's Strategy and Data Offloading Scheme***

The strategy of each picocell AP m is to optimize its own traffic and offloading traffic in order to maximize the total utility under the given fraction of resource and incentive proposed by MBS. Let β_m denote the economic incentive proposed by MBS to encourage AP m to offload l_m traffic for MBS. The picocell AP m's optimization problem can be rewritten as follows:

$$\max_{x_m, l_m} \quad P_m(l_m) = \log(x_m + l_m) - \frac{1}{2}\alpha l_m^2 + \beta_m l_m \tag{5}$$

$$\text{s.t.} \quad l_m + \sum_{m' \in \mathcal{P}, m' \neq m} l_{m'} = L, \tag{6}$$

$$x_m + l_m \leq \alpha c_m, \tag{7}$$

$$x_m \geq 0, \quad l_m \geq 0. \tag{8}$$

The objective function is motivated by the proportional fairness in allocation rule (the first term), with the additional convex component capturing the convex increasing cost function of the AP m (the second term). We also utilize the linear economic incentive function (the third term in the objective function) which means serving an additional offloading traffic unit results in an additional unit of economics incentive. The objective function is strictly concave and the constraint set is compact and convex. Hence, the AP's problem can be solved by the Karush-Kuhn-Tucker (KKT) conditions [9]. However, the desired optimal value l_m^* which yielded from KKT is generally not an equilibrium.

From Theorem 1, we replace the AP's payoff function by the potential function in order to achieve Nash equilibria for all APs as follows

$$\begin{aligned} \max_{\boldsymbol{x}, \boldsymbol{l}} \quad & \sum_{m \in \mathcal{P}} \log(x_m + l_m) - \frac{1}{2} \sum_{m \in \mathcal{P}} \alpha l_m^2 \\ & + \sum_{m \in \mathcal{P}} \beta_m l_m - V(\sum_{m \in \mathcal{P}} l_m - L)^2 \\ \text{s.t.} \quad & x_m + l_m \leq \alpha c_m, \quad \forall m \in \mathcal{P} \\ & x_m \geq 0, \quad l_m \geq 0, \quad \forall m \in \mathcal{P}. \end{aligned} \tag{9}$$

We define the Lagrangian corresponding to problem (18) as follows

$$\begin{aligned} \mathcal{L}(\rho, \lambda, x_m, l_m) = & \sum_{m \in \mathcal{P}} \log(x_m + l_m) - \frac{1}{2} \sum_{m \in \mathcal{P}} \alpha l_m^2 \\ & + \sum_{m \in \mathcal{P}} \beta_m l_m - V(\sum_{m \in \mathcal{P}} l_m - L)^2 \\ & - \sum_{m \in \mathcal{P}} \lambda_m (x_m + l_m - \alpha c_m), \end{aligned} \tag{10}$$

where λ_m is the Lagrange multipliers. By solving the KKT conditions we obtain the following results,

Lemma 1. The Nash equilibrium $\boldsymbol{l}^*$ of the game $\widetilde{\mathcal{G}}$ is achieved as follows:

$$l_m^* = \frac{b_m}{a_m}, \tag{11}$$

where $a_m = \alpha + 2VM$ and $b_m = \beta_m + 2VL$

Remark. It is important to note that (11) defines the **data offloading rule** of our proposed scheme. As shown in next subsection, the economic incentive β_m is equally for all AP m. This is in accordance with the symmetry of the Nash equilibria. Hence in practical deployment, (11) can be implemented in a distributed fashion since there is no exchanged information among APs and MBS broadcasts fraction of resource α, economic incentive β_m, penalty weight V and its desired offloaded traffic L. Each AP reports traffic volume which it can serve l_m.

The Nash equilibrium defined in (11) has the following properties.

(1) With a given fraction of resource α value, offloaded traffic l_m which each AP can serve is inversely proportional to the number of picocells M. However, the total offloaded traffic which all APs can serve will increase when there are more picocells implemented in the macrocell's coverage.
(2) Choosing weight V can adjust the volume of traffic which MBS desires to offload to APs. However, the value of penalty weight V can impact to the fraction of resource α value.

Therefore, the choice of penalty weight V entails careful consideration of the trade-offs between the MBS's offloaded traffic and fraction resource remaining to itself. These properties will be illustrated more clearly in Section V.

B. ***Stage I: MBS's Strategy and Resource Partitioning Scheme***

Based on the analytical results of the picocell APs, the MBS can optimize its strategy in order to maximize its utility, the MBS's optimization problem can be formulated as follows:

$$\max_{\alpha, \boldsymbol{\beta}} \quad \log(1-\alpha) + \alpha \sum_{m \in \mathcal{P}} l_m - \sum_{m \in \mathcal{P}} l_m \beta_m^2 \tag{12}$$

$$\text{s.t.} \quad 0 \leq \alpha \leq 1 - \epsilon \tag{13}$$

$$0 \leq \beta_m, \quad \forall m \in \mathcal{P}. \tag{14}$$

The first term of the objective function is the *utility* which MBS attains when it servers its remaining traffic at fraction of resource $(1-\alpha)$. We utilize logarithm utility function for positive, increasing and strictly concave utility. The second term is the *saving cost* earned by MBS for offloading traffic to all APs. We use linear saving cost function which mean offloading an additional traffic unit results in additional unit of saving cost. We also capture the quadratic *economic incentive cost* term into objective function for convex increasing cost when MBS pays economic incentive to APs for offloading traffic.

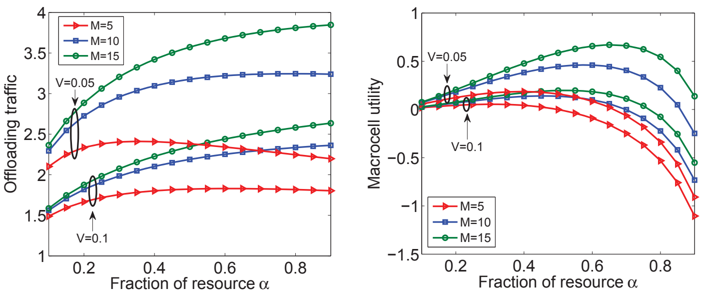
Substituting (11) from Lemma 1 into MBS's problem we can further show that the MBS's problem above is a convex optimization problem. We can solve MBS's problem by sequentially solving two sub-problem which are named as follows

i) the *economic incentive problem* (EI): for a fixed α, maximize the objective over $\boldsymbol{\beta}$.
ii) the *resource partitioning problem* (RP): plug the solution of economic incentive problem, and maximize the objective over α.

Given resource partitioning decision, we consider the economic incentive problem of determining the optimal $\boldsymbol{\beta}^*$, which can be solve based on the results of solving the RP problem. The

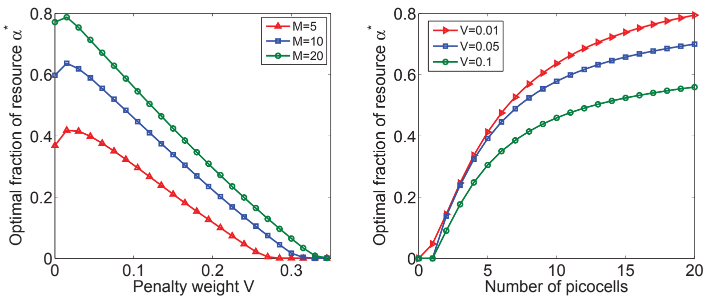
TABLE I
RESOURCE PARTITIONING AND DATA OFFLOADING SCHEME

Proposed Resource Partitioning and Data Offloading Scheme
1) MBS calculates resource portion α and economic incentive β_m by solving the EI and RP problems and send these information to all APs (e.g., via the backhaul links)
2) Each AP m computes its optimal offloaded traffic it can admit l_m^* based on the received α and β_m, and transmits data within the ABS subframes corresponding to α.



(a) (b)

Fig. 2: MBS's offloaded traffic, utility Vs fraction of resource α



(a) (b)

Fig. 3: Effect of penalty weight V and number of picocells on optimal fraction of resource α^*

economic incentive problem is defined as follows:

$$\boldsymbol{EI}: \quad \max_{\boldsymbol{\beta}}. \quad \alpha \sum_{m\in\mathcal{P}} \frac{b_m}{a_m} - \sum_{m\in\mathcal{P}} \frac{b_m}{a_m}\beta_m^2 \tag{15}$$

$$\text{s.t.} \quad 0 \leq \beta_m, \qquad \forall m \in \mathcal{P} \tag{16}$$

Lemma 2: The optimal solution of the economic incentive sub problem $\boldsymbol{EI}$ is:

$$\beta_m^* = \frac{1}{3}(\sqrt{4V^2L^2 + 3\alpha} - 2VL), \quad \forall m. \tag{17}$$

Based on the results of solving the economic incentive problem from Lemma 2, substituting β_m^* into MBS's problem then the *resource partitioning problem* is defined as follows:

$$\boldsymbol{RP}: \quad \max_{\alpha}. P_{MBS} = \log(1-\alpha) + \alpha \sum_{m\in\mathcal{P}} \frac{b_m}{a_m} - \sum_{m\in\mathcal{P}} \frac{b_m}{a_m}\beta_m^{*2} \tag{18}$$

$$\text{s.t.} \quad 0 \leq \alpha \leq 1 - \epsilon. \tag{19}$$

The last term in objective function can be considered as a cost which MBS incurs when it pays the economics incentive to picocells. We can prove that P_{MBS} is a concave function of α, since $\frac{\partial^2 P_{MBS}}{\partial \alpha^2} < 0$. Hence, $\boldsymbol{RP}$ is a convex problem and it can be solved by some standard convex optimization algorithms, such as interior-point methods [9], or existing solvers such as CVX [10]. Our propose scheme is shown in Table I.

V. Numerical Results

We have conducted numerical simulations to evaluate the performance of our proposed scheme. We first show the variation of offloading traffic volume and macrocell utility with respect to fraction resource α. After that we investigate the trade-off for fraction of resource α and penalty weight V on the performance of network. Finally, we assess the effective number of picocells on the performance of network.

The optimal fraction of resource α^* can be solved efficiently by standard numerical solvers. In this simulation we use the active-set algorithm with the *fmincon* function in the MATLAB software. The threshold ϵ can be set to an arbitrary value, we in our simulation set the ϵ to 0.1. APs' capacities c_m are chosen randomly, from a uniform distribution on [0,1].

In Fig. 2, we show the offloading traffic and macrocell utility α with respect to fraction of resource under our proposed scheme. We consider three cases for number of picocells in the network represented by M, i.e., $M = 5$, $= 10$ and $M = 15$. We have fixed the penalty weight for two cases represented by V, i.e., $V = 0.0.5$ and $V = 0.1$. It can be seen that the offloading traffic increases with the increase of fraction of resource α, and intuitively, more number of APs in MBS's coverage more traffic can be offloaded. The second inference is lower value of trade-off penalty weight V more traffic offload. We can increase offloading traffic by decreasing V, however, this will lead to the increase of fraction of resource α, which MBS must sacrifice to APs, consequently this will decrease the fraction of resource available to the remaining macrocell users as shown in Fig. 3a. In practice, MBS can adjust the value of penalty weight V based on its demand traffic and offloading traffic to obtain the balance of resource partitioning. Fig. 3b. depicts the relationship between optimal fraction of resource α^* that can be allocated to picocells and number of picocells M which underlay in MBS's coverage. It can be seen that the fraction of resource that macrocell should offer to picocells is about a half when there are 5 picocells in macrocell's coverage.

VI. Conclusion

In this paper, we have developed a Stackelberg game model for joint data offloading and resource partitioning problems in co-channel two-tier LTE heterogeneous networks. We did a number of simulation by varying the number of picocells in the HetNet to find the optimal fraction of resource and traffic offloaded to the picocells by MBS. Simulation validate that our proposal achieves Nash equilibrium by the best response potential function.

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