

Approaching fundamental energy consumption limits in optical communications

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Abstract—We study the fundamental energy consumption of fiber-optic communications links. We show that the quantum limit for the energy efficiency of a multi-span system deploying generalized on-off keying with photon-counting inline regeneration exceeds by orders of magnitude that of state-of-the-art systems employing inline optical amplification.

I. INTRODUCTION

The economic build-out of the Internet with its plethora of applications and services has been made possible by the great progress of fiber-optic communications technologies which have increased long-haul transmission rates by more than three orders of magnitude [1], [2]. As a consequence of an unabated exponential data traffic growth, the fraction of energy consumed by the telecom industry is growing to significant levels and calls for the investigation of energy efficient telecommunications solutions [3]. The design of future communications networks will have to carefully address the tradeoff between transmission performance (in terms of capacity and reach) and energy that is consumed per bit of end-to-end transmitted information across all involved network technologies, including wireless and wire-line access, optical transport, and packet routing. Here, we focus on *optical communications links*, which form the high-capacity backbone of today's data networks and which are continuing to penetrate further towards the end user (fiber-to-the-home, FTTH) due to their inherent energy and cost efficiency at high information rates.

Several aspects of trading energy consumption for communication rate and link distance have been covered in previous studies. The most fundamental such trade-off is Shannon's seminal work on channel capacity [4], which reveals an exponential increase in the energy per bit requirement with channel capacity. In the context of optical communications systems over free-space links (i.e., without the use of in-line amplification or in-line regeneration), Shannon's energy/capacity trade-off was extended to account for quantum effects by Gordon [5], [6] and Holevo [7], with an insightful overview related to space-based laser communications links given by Boroson [8]. In the context of fiber-based systems, Tucker has studied energy trade-offs in classical optically amplified communications systems using state-of-the-art modulation

and detection techniques [3], Essiambre et al. [9] looked at Shannon trade-offs including fundamental limitations due to fiber nonlinearity, and Winzer [10] showed that spatial multiplexing is an energy-efficient solution to the capacity scalability of optical transmission systems. However, to the best of our knowledge, no studies have yet been performed on the fundamental quantum limits for the energy consumption of a particularly relevant class of optical communications systems incorporating in-line amplification or in-line regeneration, which is the subject of this paper and extends the work of [11].

Throughout this work, we consider fundamental *photon energy efficiencies* for signal generation and amplification. As such, we ignore non-fundamental sources of excess energy consumption due to technological device inefficiencies. We acknowledge that potentially required computational energies, too, have fundamental limits [12], [13], [14], but with thermal energy being small relative to an optical photon, the fundamental energy consumption having to do with encoding and decoding, is expected to be small relative to the optical energy in a scheme of the kind considered herein [15]. We show that generalized on-off-keying (G-OOK) transmission in combination with inline photon-counting regeneration has the potential of being two orders of magnitude more energy efficient than a coherent communication system using inline amplification, which represents the systems used in networks today.

II. ANALYSIS AND RESULTS

A. Shannon's classical energy trade-off

Initial insight concerning the trade-off between the achievable information rate on a communication channel and the signal-to-noise ratio (SNR), can be gained from Shannon's famous formula for the capacity of a channel impaired by additive Gaussian noise. Assuming a rectangular (Nyquist) spectrum, the highest rate at which information can be reliably communicated over one channel mode per unit bandwidth, is given by

$$C = \log_2(1 + \text{SNR}) = \log_2\left(1 + \frac{n_s}{n_{\text{th}}}\right), \quad (1)$$

where n_s is the average received signal energy within an

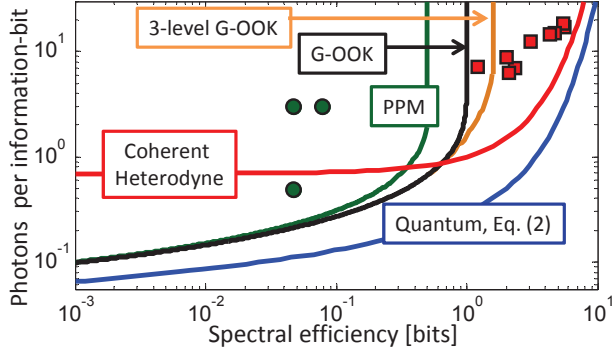


Fig. 1. Trade-off between spectral efficiency and required number of photons per information bit for various *single-span* optical communication systems (with $n_{\text{th}} = 0$). Circles and squares show recent experimental results for PPM and coherent systems respectively (see [21] and references therein).

information symbol, and n_{th} is the corresponding noise energy. We use the subscript ‘th’ to emphasize that the additive noise is thermally distributed in our considerations. For convenience we specify n_s and n_{th} in photon-energy units, such that the actual energies are $\hbar\omega n_s$ and $\hbar\omega n_{\text{th}}$, respectively, where \hbar is the reduced Planck constant and ω is the center-frequency of the field. Note that with our assumption of Nyquist pulses, C may also be referred to as ‘spectral efficiency,’ and is hence equivalent to ‘capacity per symbol,’ or simply ‘capacity,’ terms that we use interchangeably in what follows. The energy consumption per bit of information is given by $n_s/C = n_{\text{th}}(2^C - 1)/C$ (as readily obtained by inversion of (1)), which is a monotonically increasing function of C , implying that high energy efficiency requires operation in the low spectral efficiency regime. The optimal trade-off suggested by this observation is to divide the energy used for data-transmission between multiple spatially multiplexed paths [10], such that the capacity (and hence the SNR) in each path does not exceed unity.

B. Gordon’s quantum energy trade-off and pulse position modulation (PPM)

In the limit of interest, when the number of transmitted signal photons is desirably low, quantum effects need to be taken into account. The extension of (1) to the quantum domain was first done by Gordon [5] and it is given by

$$\begin{aligned}
 C = & \log_2 \left(1 + \frac{n_s}{1 + n_{\text{th}}} \right) \\
 & + (n_s + n_{\text{th}}) \log_2 \left(1 + \frac{1}{n_s + n_{\text{th}}} \right) \\
 & - n_{\text{th}} \log_2 \left(1 + \frac{1}{n_{\text{th}}} \right). \quad (2)
 \end{aligned}$$

The design of a receiver apparatus that is capable of extracting the full capacity of (2) is still an open problem in quantum optics. If one assumes a standard telecommunications receiver that is equivalent to heterodyne detection [16], the extractable capacity reduces to that given by the first term, which is identical to Shannon’s classical formula

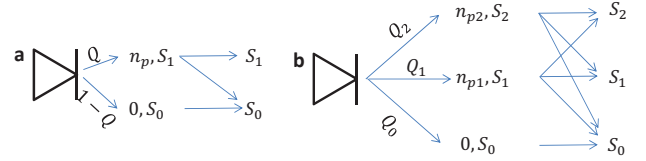


Fig. 2. (a) The G-OOK scheme as proposed by Gordon [5]: a logical one (S_1) is represented by the transmission of pulse laser with n_p photons on average, whereas a logical zero (S_0) is represented by no transmitted power. The probabilities of transmitting S_1 and S_0 are Q and $1 - Q$, respectively. (b) Three-level extension of G-OOK: the symbols S_2 , S_1 and S_0 , transmitted with probabilities Q_2 , Q_1 , and Q_0 , respectively, are represented by the transmission of a pulse laser with n_{p2} and n_{p1} photons on average, and by no transmitted power, respectively.

(1) except for the replacement of n_{th} with $n_{\text{th}} + 1$, with the extra noise photon accounting for vacuum fluctuations (manifesting as shot noise). The contributions of the second and third terms are significant only when $n_s \ll 1$ and $n_{\text{th}} \ll 1$, while in the case where $n_{\text{th}} \ll n_s \ll 1$ the second term dominates and it can be shown to equal the capacity of ideal (i.e. noiseless) classical pulse-position-modulation (PPM).

In Fig. 1 we show the tradeoff between energy consumption per bit of information and spectral efficiency in a single-span system (i.e. in the absence of thermal noise). The red curve corresponds to the use of an ideal heterodyne receiver, whereas the blue curve represents the full capacity predicted by Eq. (2). The green curve corresponds to a PPM receiver which is optimized with respect to the number of slots for every value of spectral efficiency. While heterodyne detection is clearly superior to PPM in the limit of high spectral efficiencies, its energy performance is significantly inferior at low spectral efficiencies. The black and the orange curves represent the performance of the G-OOK scheme and its three-level generalization, respectively.

C. Generalized on/off keying (G-OOK) for single-span

The G-OOK scheme has been originally proposed by Gordon in 1962 [5] and it is described schematically in Fig. 2(a). This is a binary scheme in which a logical one (denoted by S_1) is represented by the transmission of a standard laser pulse containing n_p photons on average, whereas a logical zero (S_0) is represented by no transmitted power. The probabilities of transmitting S_1 and S_0 are Q and $1 - Q$, respectively, where both Q and n_p are parameters subject to optimization. The G-OOK receiver uses a standard photo-detector (D), which in the absence of dark-count noise registers the received signal as S_1 whenever at least one photon is detected. The performance of G-OOK (black curve in Fig. 1) is identical to PPM in the low spectral efficiency limit, but, when C approaches 0.1, G-OOK outperforms PPM. When C exceeds 0.5 the single-span G-OOK scheme is outperformed by the coherent heterodyne system. The orange curve refers to our extension of G-OOK to 3 levels (see Fig. 2(b)) consisting of S_0 , S_1 and S_2 , corresponding to no transmitted power, to the transmission of a pulse with an average number

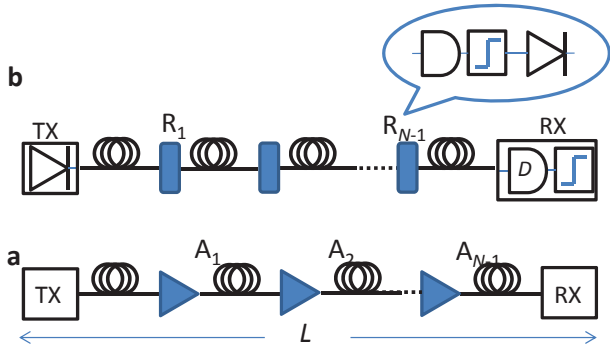


Fig. 3. (a) Schematic of a system with linear phase-insensitive inline optical amplifiers. A link of length L consists of a transmitter (TX), a receiver (RX) and N equal-length optically amplified spans. Each span is terminated by an optical amplifier (A_n , $n = 1, 2 \dots N-1$). (b) Schematic of the regenerated GOOK link. Each regenerator consists of a photo-detector (D), a threshold circuit (T) and a laser that generates coherent laser pulses according to Fig. 2.

of photons n_{p1} and to the transmission of a pulse with an average number of photons n_{p2} , respectively. This scheme is characterized by two extra parameters, which are the probability of transmitting S_2 and the threshold for differentiating S_2 from S_1 . As is evident from Fig. 1, the extra-level offers no benefit except for capacities exceeding unity, which obviously are not achievable with binary transmission.

D. Multi-span energy trade-offs for coherent heterodyne and photon-thresholding systems

The performance of the coherent detection scheme combined with inline amplification is characterized by Eq. (2) with n_{th} accounting for the accumulation of amplified spontaneous emission (ASE) noise. A schematic of such a system is depicted in Fig. 3(a) (bottom part) and its energy consumption per bit of information is shown by the dashed curves in Fig. 4, as a function of the number of spans. The top, middle, and bottom panels in Fig. 4 correspond to spectral efficiencies of $C = 0.1$, $C = 0.5$ and $C = 1.3$, respectively. The left panels corresponds to a 1,000-km link, whereas the right panels correspond to a link of 10,000-km. The proposed multi-span G-OOK scheme using conceptually simple photon-thresholding regenerators but no inline error-correcting decoders, is illustrated in Fig. 3(b) (top part). The performance of the proposed multi-span G-OOK system [17] is shown by the solid curves in Fig. 4. As expected, the three level scheme was found to have no advantage relative to two levels for $C = 0.1$ and $C = 0.5$. As is evident from the figure, the performance of the regenerated G-OOK scheme is significantly better than that of the coherent heterodyne system in all cases. In particular, the multi-span transmission energy advantage in the case of $C = 0.1$ reaches the remarkable level of 20 dB.

III. DISCUSSION

Some insight into the reason for the fact that the regenerated GOOK scheme outperforms the coherent optically-amplified link with heterodyne detection can be gained

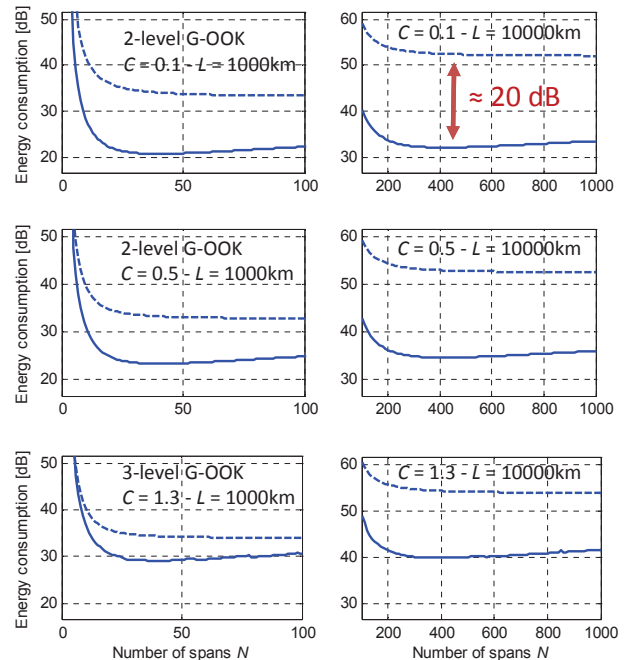


Fig. 4. Energy per bit vs. number of spans for the displayed system lengths L and spectral efficiencies C . Dashed curves represent coherent amplified transmission, solid curves correspond to G-OOK.

from noting that ASE noise is unavoidable in linear amplifiers, whereas in the case of intensity regenerators (which can be viewed as “nonlinear amplifiers”), the presence of noise is not fundamental. In addition, in the low spectral efficiency (and correspondingly low signal-to-noise ratio) regime, the ability of the coherent heterodyne scheme to support multi-level signaling is of negligible value, and hence the advantage of regenerated GOOK becomes particularly noticeable. This also explains the fact that the only appreciable benefit of the three-level extension of the regenerated GOOK is the capability of achieving capacities that exceed unity.

A natural question that comes to mind concerns the fairness of the comparison between an optically amplified scheme and a scheme that contains regeneration. Would a coherent heterodyne system operate better if instead of optical amplifiers one would perform regeneration? The answer to this question depends on the type of regeneration that one has in mind. If one restricts oneself to memoryless symbol-by-symbol regeneration of the kind that we perform in the regenerated GOOK case, then in terms of the effect on capacity, optical amplification is rigorously equivalent to heterodyne detection and optical retransmission when a continuum of signal levels is used [18], [19], [16]. Moreover, the energy consumption of such a regenerator cannot be lower than that of an amplifier, implying that this kind of regeneration will not improve the energy performance of the coherent heterodyne scheme with respect to the results shown in Fig. 4.

A different situation arises when regenerators are allowed to contain a full-scale receiver that is capable of decoding the received signal (using error-correcting codes)

and extracting the information prior to retransmission. In this situation, which we address in [17], the capacity of the link becomes identical to that of a single regenerated span (because in the Shannon limit of infinite codewords no error accumulation occurs) and thus a system capable of extracting the full capacity of Eq. (2) in each span will clearly be preferable. However, methods for extracting the full Holevo capacity in coherently modulated systems are yet to be found [20], and standard heterodyne detection is highly inferior.

IV. CONCLUSION

We have addressed the fundamental limits of energy consumption in multi-span optical communications links. We have shown that the energy consumption of G-OOK can be lower by two orders of magnitude than that of a standard optically-amplified coherent system.

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