A Basic Study of Auction-based Planning and Scheduling for Cell Manufacturing

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Abstract. Cell manufacturing is widely introduced to cope with the dynamically changing market demands. This study considers a scheduling method for a cell assembly system for the products that are characterized by the type and specification. A major set up is required at a cell for the change of types and a minor one for the change of specification within the same types. The scheduling will be made in three folds. Firstly, the scheduling for the orders with due dates of the day is made before the processing of the day starts. If cells have slack times for the production of the orders of the day, express orders to be completed within the day will be accepted in a prespecified time period or until no cells become available for the production. If slack times still exist, advanced production of orders with future due dates will be considered. Auction-based algorithms are proposed for the scheduling and their effectiveness is investigated by simulation studies.

1 Introduction

Cell manufacturing is widely introduced in assembly shops especially in the assembly of electronic devices enabling to keep the agility and the flexibility of the system and to respond the dynamically changing market demands [1]. A cell manufacturing system is constituted of several cells each of which may be also constituted of some stations. One product is assembled in a cell moving from station to station devised with appropriate tools and/or equipments. The product kinds can be changed relatively easily by changing the setups at each station. To attain the high performance out of the system, it is important not only to provide hardware with high flexibility but also to have a planning and scheduling systems which is also easy to cope with the dynamic changes of the demands in the market.

In this study, an assembly system for electronic devices is considered to be redesigned into cell manufacturing. The products are largely classified into kinds, each of which is further classified into types. The objective system needs to process a large number of orders per day. Most of orders are ordinary orders received before the day starts, but some express orders requiring the shipping in the same day are also received and accepted if possible. To improve the customers' satisfaction, the primary objective of the new system is set to ship out all ordinary orders without any delay with a limited number of cells and then to accept express orders as many as possible as the secondary objective. To cope with these objectives, the planning and scheduling procedures based on the auction are proposed for agile manufacturing.

This paper describes the outline of the objective manufacturing system and the frame of the auction based planning and scheduling procedures. Then the basic design of the planning and scheduling procedures is discussed using a numerical example. The effectiveness of the proposed system will be also demonstrated in conjunction with the designing procedure of the frame.

2 Outline of Objective Manufacturing System

The conceptual configuration of the cell manufacturing system in this study is shown in Fig. 1. The system has N cells and each cell is constituted by some stations, six stations in this study. The products are largely classified into some kinds and products in a same kind are further classified into some types. All products are processed at each station for a pre-determined fixed cycle time regardless to the kinds and types, 15 seconds in this study. For the change in the product kind, each cell needs to stop for a large setup change, 10 minutes in this study, and for the change in the type within the same kind, the setup at one station is made station to station, and one cycle time at a station, 15 seconds in this study, will be required for the setup change.

The orders usually received in advance specifying the product kind and type, the number of products (order size) and the due date for delivery. These orders are called ordinary orders in this study. The shipping date is estimated based on the distance and the due date for delivery and the shipping date becomes the due date at the factory. In this study an order whose completion time is before T, 17:00 in this study, can be shipped out by the last truck scheduled on the day and the order is considered to be in time or without delay. Because of a short processing time of the product in this study, most of the ordinary orders are scheduled for manufacturing on the day of shipping. Such ordinary orders whose shipping date is same as the manufacturing date are called today's order hereafter. In addition, special orders, express orders in this study, will be accepted by the sales department and are also expected to be completed for the shipping of the day. Since today's orders have the first priority, express orders will only be accepted up to the total capacity of the system.

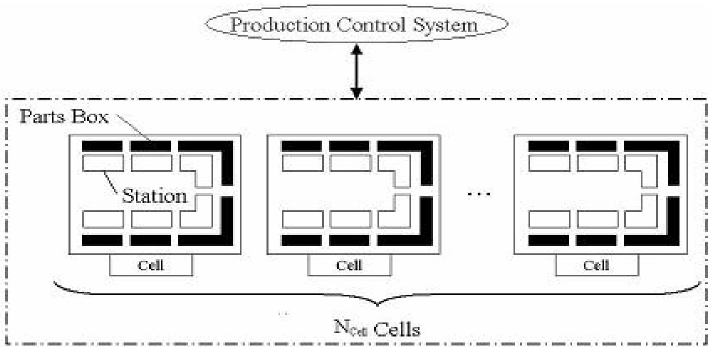

Fig. 1. Conceptual Configuration of Objective Cell System

3 Frame of Daily Planning and Scheduling

For the customers' satisfaction, the primary objective of the system is to complete all ordinary orders without any delay and the secondary one is to accept the express orders as many as possible under the constraint of given number of cells and the regular working hours. Every day, by T0 before the operation of the day starts, say 9:00, the ordinary today's orders are to be assigned to cells. This is a static scheduling of a given set of orders. At T0, all cells starts their operation based on the schedule of today's orders. The express orders received by the sales department are assigned to some cells based on some scheduling rules dynamically. If the system has some slacks even after accepting all express orders in addition to the today's orders, the system can process the future orders, whose shipping dates are tomorrow or later, in advance.

A scheduling system pursuing the optimum may be developed to produce a schedule which minimizes the setup times for today's orders in advance and can inform the available time for express orders to the sales department. It will however take a long time for scheduling a large number of orders and will not be practical to re-run the scheduling system every time the express orders are received to obtain the optimum schedule.

To solve these problems, a new auction based planning and scheduling procedures are proposed taking the advantage of cell manufacturing system into consideration. The procedures for today's, express and future orders are separately developed and then integrated as briefly described below and shown in Fig. 2. The working hour of one day starts at T0 and ends at T. The orders completed by T can be shipped out in this day.

Scheduling of today's orders (Static Scheduling): At the beginning of the day, T0, today's orders are scheduled by the auction. This is called planning auction in this study.

Scheduling of express orders (Dynamic Scheduling): Express orders will be received after the daily operation starts. As far as any cell has a slack to process them, the express order will be basically accepted. The auction is made to determine when the order is processed on which cell.

Scheduling of advanced future orders (Dynamic Scheduling): Suppose that cells have slack times even after processing all accepted express orders. Then it will be appropriate to process future orders in advance to attain a high utilization. After T1, each cell which becomes free starts to seek future orders to process by auction.

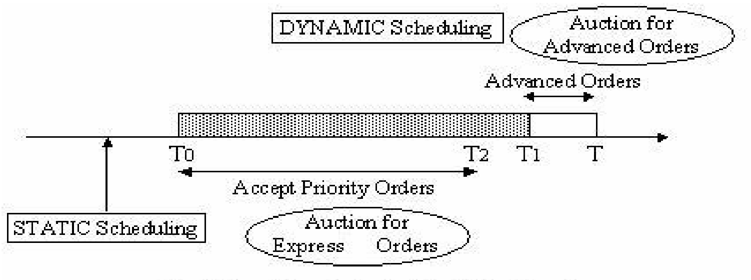

Fig. 2. Time Chart for Daily Planning and Scheduling

4 Basic Design of Objective Manufacturing System

To develop an efficient scheduling system, it is necessary to customize the basic policy and parameters to the objective manufacturing system. In this study, the scheduling system is developed for the manufacturing system with 10 cells. The products are classified into 27 kinds and 1035 types in total. The number of products in each order, i. e., order size, ranges from 1 to 100. Order sizes are grouped into three groups; G1=[1-9], G2=[10-60] and G3=[61-100], where the size in one group is assumed uniformly distributed. Orders for each day are generated so that the number of orders with order size in each group will become P1={80%:15%:5%}. For each order the product kind is firstly assigned randomly and then its type are randomly assigned. To investigate the effect of ratio of order sizes in a day, an order pattern P2={5%, 15%, 80%} is also considered.

4.1 Planning Auction for Today's Orders

The capacity of the system depends on the schedule of orders. In this system a schedule with minimum setup times is the most efficient schedule since the processing times of all products are the same. All today's orders are scheduled before the operation of the day starts and thus the auction for today's orders is named planning auction. The auction procedure is given below, where the item for auction could be one single order or a set of orders to be determined later.

Planning Auction:

Step 1: (Co) Set orders into items based on the specified rules.

Item A: A single order

Item B: A set of orders with same kind and type (no setup required)

Item C: A set of Item B with same kind (short setup time required)

Step 2: (Co) Select one item by the specified selection policy. Then the characteristics of the item are announced to cells.

Policy 1: Randomly select Item A

Policy 2: In the decreasing order of total processing time of Item B

Policy 3: In the decreasing order of total processing time of Item C

Step 3: (Cell) Assume to process the item after the last item assigned to the cell. Calculate the setup time and completion time of the item. If the completion time is later than T, the item will be delayed to the shipping time and thus the bid is "not available". If "available", the bids are "setup time" and "present load (=completion time of the last order except the item)".

Step 4: (Co) Select cells with shortest setup time and then select a cell with the lightest present load among the selected cells, i. e., the earliest completion time. If more than one cell remains, select one randomly. The item is awarded to the selected cell. Go to *Step 2* if items for auction remain. Otherwise, stop.

4.2 Efficiency of Planning Auction and System Capacity

To investigate the efficiency of the planning auction, preliminary experiments are performed for selection policies of items. The results for one set of orders are shown in Fig. 3. From the figure the effectiveness of the policy 3 is obvious, and Policy 3 is adopted in the planning auction in this study.

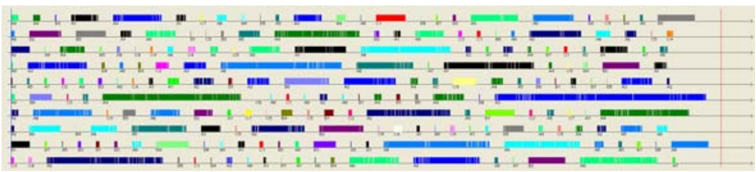

Fig. 3 (a). Policy 1: Item A, Random

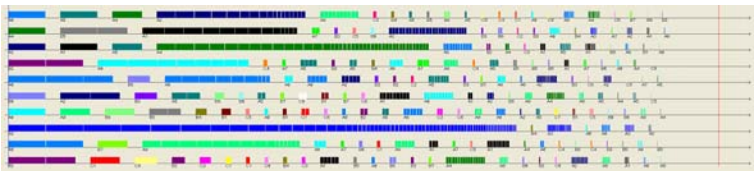

Fig. 3 (b). Policy 2: Item B, Decreasing Order of Processing Time

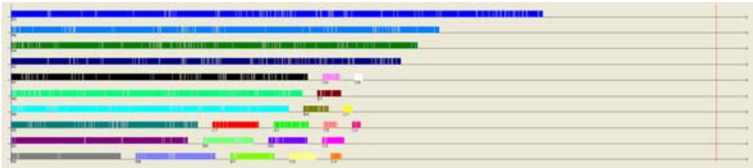

Fig. 3 (c). Policy 3: Item C, Decreasing Order of Processing Time

The capacity of the system can be defined as the maximum number of orders to be processed in one day without any delay. Increasing the number of orders, the system utilization in 8 hours operation and the number of tardy orders which delayed to the shipping are evaluated by running 10 simulations for each number of orders. The averaged results for each number of orders are given in Table 1. Based on the results, the maximum capacity of the system is considered to be around 1300.

Table 1. System Performance for given Number of Orders

Number of Orders	Utilization (%)	Number of Delayed Orders
1000	68.07	0
1100	74.41	0
1200	79.20	0
1300	86.89	6.9
1400	91.69	49.5
1500	92.53	111.1

4.3 Auction for Express Orders

The auction for express orders is performed at the time of reception of an express order. This implies that *Steps 1* and *2* of the planning auction will be set to Item A and Policy 1, respectively, in the auction for express orders. The bids provided by cells depend on the timing when the express orders are proessed. Two scheduling patterns are considered. If the orders including the express order under consideration cannot be completed in time by any cell, the order will not be accepted.

Pattern 1: Express orders are processed after the order with minimum setup time, provided that the last order on the cell can be completed in time. This means that the schedule is dynamically changed when an express order is accepted. A cell with the lightest load or the earliest completion time of the last order on the cell at the time of auction is selected among those with same minimum setup time. The bids of cells in *Step 3* are then "availability (can or cannot complete the all orders in time)", "minimum setup time on the cell" and "present load"

Pattern 2: Express orders are processed after completing the last order assigned to the selected cell. This means the schedule for today's order is fixed. The cell is selected by the lightest load rule. The bids of cells in *Step 3* are then "availability" and "completion time of last order assigned by the time auction".

The effectiveness of the pattern is investigated under the condition that today's orders generated will require 7 hours to process on the average. Typical schedules for these patterns are shown in Fig. 4. It is easily observed that more orders are scheduled when the schedule is dynamically changed inserting the express orders in the schedule of today's orders. Number of express orders generated for 6 hours (= T2- T0) is increased from 80 to 140 and the system utilization and the number of tardy orders are obtained by simulation. The results are shown in Fig. 5 and Pattern 1 shows better performance with 130 to 140 acceptable express orders with less than one tardy orders and higher utilization than that of Pattern 2.

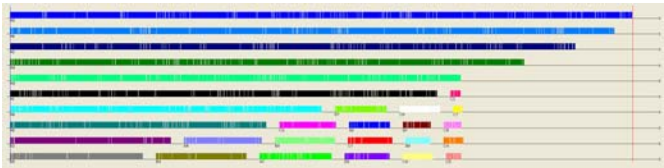

Fig. 4 (a). Schedule of Today's Orders

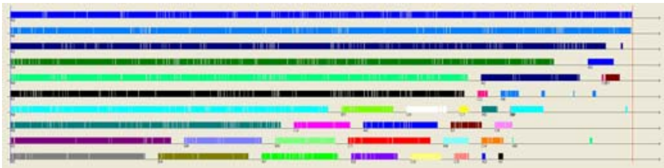

Fig. 4 (b). Schedule of Express Orders with Pattern 1

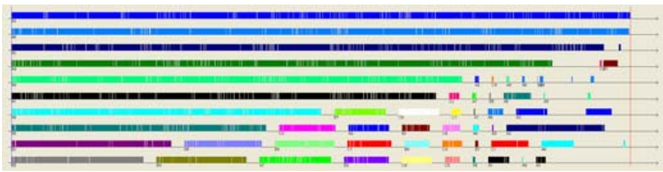

Fig. 4 (c). Schedule of Express orders with Pattern 2

4.4 Reverse Auction for Advanced Future Orders

If cells complete all assigned orders before the end time of the day, orders with future shipping dates can be processed. This will be preferable not only to improve the utilization of the system but also to absorb the daily variation of the production volume incurred by the fluctuation in the number of ordinary orders. The auction for such future orders is quite different from the planning auction and auction for express order. The auction is initiated by a cell which has a slack time and thus the cell becomes the coordinator and the future orders are the participants. Since the relation between cell and order are reversed from the planning auction, the auction for future orders is named a reverse auction in this study.

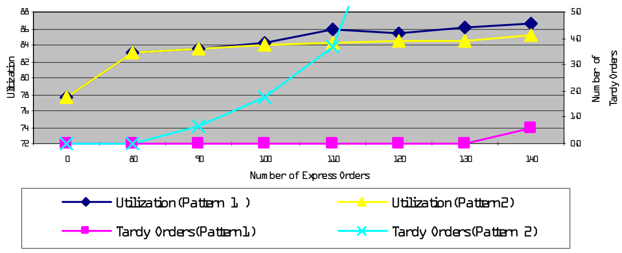

Fig. 5. Effects of Number of Express Orders and Scheduling Patterns

In the reverse auction, the effectiveness is defined to be evaluated by the number of orders or the total production volume manufactured in the slack time. These measures will be affected by the number of orders to be included in the auction. In other words, it will be important to determine the planning horizon. Simulations are run after scheduling the today's orders for 7 working hours, where no express orders are considered. The numbers of future orders processed in the slack time for cases with the planning horizons of one day and two days were 158 and 165, respectively, and the system utilizations were 86.2 and 86.6. Longer planning horizon shows that more future orders can be processed yielding higher utilization. This is explained by the fact that the possibility to have orders of same type or at least of same kind to the last order on the cell will be higher in larger number of orders.

5 Conclusions

This paper considers a cell manufacturing system of assembling electronic devises. In addition to a large number of orders to process in a day, express orders are also to be accepted as many as possible. To cope with these requirements, auction based scheduling procedures are proposed as a dynamic and flexible planning and scheduling system. Planning auction is presented for scheduling orders before the daily operation starts. By simulation studies, it was shown that the auction should be made as a group of orders with a same kind in the decreasing order of total group-wise processing time. For express orders, an auction is performed when it is received. High performance will be attained by a scheduling such that orders are processed just after an order of the same type or at lest in the same kind is completed. A reverse auction is proposed for scheduling future orders to be processed in slack times of cells. The experiments showed that higher performance will be attained when longer planning horizon is set for future orders.

To implement these procedures in a system, the period to receive the express orders, the time to start reverse auction and other parameters are to be determined, but they are left for further studies..

References

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Sizing of Heijunka-controlled Production Systems with Unreliable Production Processes

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Heijunka is the notion to level a production system by removing ups and downs in volume caused by batch processing and customer order fluctuation in order to reach a mixed model production system with a constant flow of parts. We show how to implement lean production principles in systems with unreliable production processes. Process unreliabilities occur because tool machines may have small overall equipment effectiveness. Our present results were derived during performed implementation projects, where supermarket-pull-systems had to be dimensioned. In particular, the calculation of required inventory levels is presented which uses analytical mathematical models on the basis of discrete time queuing systems. By considering variable capacities we essentially extend the content of reference [1]. The application of our model is demonstrated by an example.

1 Introduction

Production unreliability is a frequent problem when lean production systems have to be installed. The problems occur because unreliability leads to variability in processing times and consumption, which must be compensated by inventory buffers. Production orders for the relevant process, which would guarantee appropriate refill of the process exit buffer supplying goods for the next process step are behind schedule and the next step is starved. This effect is more noticeable with increasing process utilization.

In cases where all processes are highly reliable with low variability, the standard Toyota formulas can be used for sizing the buffers. Otherwise the relevant literature suggests concentrating first on the process itself (point kaizen) prior to implementing kanban cycles. Unreliable processes are causing constraints. Special tool machines often have an overall equipment effectiveness (OEE) lower than 70%. OEE comprehensively indicates the relative productivity of a piece of equipment compared to its theoretical performance. Low reliability may be caused by technological complexity, which is not under control. Therefore it is the question whether lean production can only be implemented and the production be levelled if all production processes are highly reliable or if it is possible to begin with levelling before making improvements. [2] dealt with the problem of determining the number of circulating kanbans for a manufacturing system with machine breakdowns utilizing perturbation analysis. In order to find the optimal number he estimated gradients and preformed stochastic approximations.

We approach the problem by developing a method for the calculation of buffer sizes. A path towards an implementation of kanban cycles and levelled production for cases where the production processes are unreliable is shown by calculating the required inventory levels which are necessary to guarantee a certain service level. The following sections provide insights into the impacts of variability in production systems and into production levelling and present the derived model and its assumption as well as the analytical method to compute the required inventory levels. We finish the text with an illustrative example.

2 Basic concepts

2.1 The effect of variability on Lean Manufacturing

Lean Manufacturing is a management philosophy focusing on reduction of the seven types of waste (over-production, waiting time, transportation, processing, inventory, motion and scrap) in manufacturing or any type of business [3]. [4] note that is possible to achieve the same throughput either with long cycle times and large work or short cycle times and small work in process. The difference between both cases is variability. It exists in all production systems and affects significantly production throughput, delivery, quantity, costs and customer satisfaction [5]. The most common causes of variability in manufacturing environments according [4] are: natural variability (including minor fluctuations in process time due to differences in operators, machines, and material), random outages, setups, operator availability, rework and scrap. [6] distinguish between process and flow variability. After [4] both kinds of variability in production systems will be buffered by some combination of inventory, capacity and time and we can conclude that variability is the root cause for waste. Because in a perfect balanced system no buffering is needed, increasing variability always degrades the performance of production systems [4].

2.2 Production levelling strategy

The basis of a lean or just-in-time production is to level the work flow for optimizing the manufacturing line [7]. Customer orders may arrive relatively constantly in the long run but they appear to be inconstant and unpredictable in short intervals. The aim of levelling is the reduction of the customer order variability by analysis of the orders in a given time span resulting in a pattern which fits into a smaller time scheme. It creates a constant flow of parts in a mixed model production and reduces or eliminates the need for spare capacity or stocks to cope with peaks of demand. [8] distinguishes two phases of levelling: I. Of the total production volume, II. Of the product mix. Phase I specifies both kind and quantity of product variants (model types) which are to be produced in an individual manufacturing shift. The computation starts with the monthly order of each product variant. The quantity of a given levelling horizon per variant is divided by the number of available manufacturing shifts. Thus the levelled outputs per shift of each variant and the cycle times for all products are found. The aim of the continuous improvement is to produce every-part every-day or better every-shift [9]. Because in today's production systems the number of variants can be very high, [10] suggests to level in a first step only the large-volume variants (high runners), which are to be produced following the average demand in a given time interval (every-part-every-interval, EPEI). The length of the EPEI is an indicator for the capability of the production process. Its determination is described in detail in [9].

The many small-volume products (low runners) are scheduled upon need in a reserved period of the day. If a production order for a low runner cannot be fulfilled completely within a reserved time block, then it is scheduled into the next cycle [11]. The reduction of flow variability by smoothing the orders leads to a decoupling of production and demand which must be compensated by an inventory buffer for finished goods. A short levelling horizon facilitates the implementation and minimizes the required inventory buffer for finished goods, however the desired levelling effect is lost, if the period is chosen too short. The optimal level depends on the fluctuation of the customer orders and the price of finished goods.

In phase II the production sequence of the individual orders per shift is determined, which leads to a finer levelling of the product mix. Numerous relevant algorithms exist in the literature [e.g. 8, 12]. The final result of levelling is a production sequence with a continuous flow adjusted to the customer demand and leading to an even utilization of the production stages.

3 Buffer sizing model for unreliable systems

3.1 System Model and Control Policy

A manufacturing structure, where each process has only one predecessor and one successor [12] is the one with the lowest possible complexity. The complexity increases with the number of the possible preceding and following processes. Evidently an even utilization of all resources is much more difficult in a system with high material flow complexity. To improve transparency and to reduce system variability, lean manufacturing tools introduce and steadily improve flow production. The flow production method arranges the production processes complying with the material flow of the products. The parts flow after each production process to the subsequent process without buffering and are processed immediately. There is no over production, because each stage produces exactly what the next stage needs at a given time. Abandoning of buffers results in short throughput times and low work-in-process inventory levels. To realise pure flow production, the production times must be balanced perfectly. This is often only a vision but nevertheless the aim of the continuous improvement process. A useful tool to improve flow is value-stream design initiated by [8]. One important design principle is to arrange directly connected production processes in a flow and manage the flow between the sections by continuous flow with pull control loops [8].

Each stage consists of a manufacturing process, which is a sub-part of the production system, and an output buffer (supermarket). The manufacturing process contains parts that are currently being processed in the stage (either waiting for, or receiving service at the different machines), referred to as the work-in-process (WIP) of the stage. The output buffer contains the finished parts of the stage, referred to as the finished good inventory of the stage.

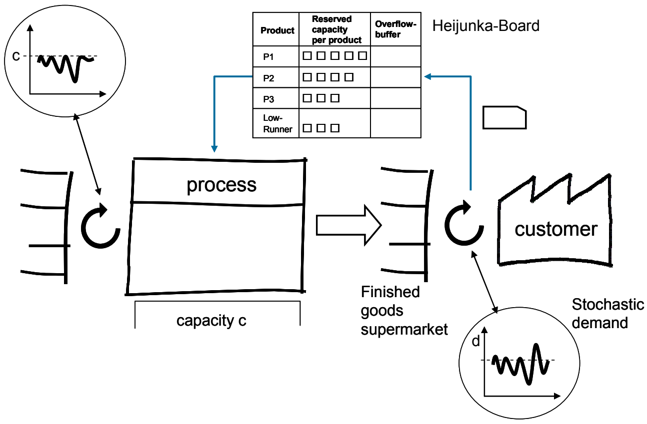

Fig. 1. Scheme of a Kanban cycle: Single stage Heijunka-controlled production

The coordination between the stages is achieved by a pull mechanism. Each material withdrawal generates a production order for the previous process with the goal of filling up the gap as soon as possible. As information medium between consumers and the previous production process in practice a kanban is used.

There is a flow of parts moving from upstream to downstream stages, and a flow of demands is going from downstream to upstream stages. Fig. 1 illustrates a single stage production unit as a segment of the whole process chain. The customers request parts in each period n according to a stochastic density function d. The kanbans attached to the requested products move to the Heijunka Board and are loaded into the corresponding hole to create a production order. For each product a certain capacity of the overall capacity is reserved according to the average demand. If the amount of kanbans for a given product is less than the reserved capacity, the kanban is put in the next empty hole for the product. Otherwise, the kanban is put in the overflow box. The overflow box contains backorders that will be produced as soon as the demand during one period is lower than the capacity. The peaks of the stochastic demand are thus cut and the production is levelled. If the production process with capacity c is unreliable, a stochastic maximum production output C_n of the stage exists. Because of machine breakdowns and other unexpected difficulties the real output may deviate from the planned output. For modelling the unreliability of the relevant process, its temporal losses of capacity are evaluated retrospectively and the probable daily distribution of working capacity c_i is calculated and is assumed to be identically each day. On the basis of this distribution the process exit buffer (supermarket) is filled with parts in a way that the following process is always supplied.

Reserved capacity which can not be filled remains empty in order to avoid producing of goods, which are not requested. The amount of consumed parts in one period is equivalent to the stochastic demand of the stage given in form of a kanban to the pervious stage.

3.2 Mathematical model formulation

A methodology for sizing the buffers (supermarkets) by using the G|G|1-queueing system on discrete time is developed. The number of items in the supermarket (buffer) results from the superposition of two stochastic processes: Filling up by the (limited) production with capacity C_n and emptying by selling the quantity D_n. The inventory level X_n changes in each interval with:

$$X_n = Q_n - C_n.$$

W_n, the number of kanbans (cards) on the Heijunka board is given by:

$$W_{n+1} = \max\{ (W_n + X_n) ; 0 \} \qquad (1)$$

The number of items in the supermarket-buffer is the difference between the number of kanbans in the system and the last value of W_n. Equation (1) is equivalent to Lindley's equation for queuing systems in discrete time. The production capacity corresponds with the arrival vector, the demand rate with the service time vector. Methods of [15] can be used to determine the numbers of items in the supermarket and of kanbans on the Heijunka board. [15] determine waiting time and idle time distributions of a G|G|1-queue where the inter-arrival time A is distributed by a_i and the service time S is distributed by s_i. The inter-arrival time between two customer orders specifies the time period with decreasing workload, which corresponds to the production capacity in our model. The service time of an arriving customer order increases the workload, which is equivalent to the sold quantity, because this quantity must be reproduced.

The algorithms of [15], which are based on a Wiener-Hopf factorization using ladder height distributions, can be used to solve equation (1). Therefore, the same methods which are used to compute the distribution of the waiting time, can be used to compute the distribution of the number of waiting orders at the end of a period (allocation of the overflow box), which is equivalent to the number of kanbans on the Heijunka board. This results in a vector $\vec{w}$, where w_i is the probability that the number of waiting orders in the overflow box is exactly i. Additionally, the idle capacity distribution can be computed by the algorithm of [15], because it is equivalent to the idle time distribution.

We assume that the replenishment time is one period (EPEI = 1). Then the probability distribution $\vec{h}$ of the number of items not yet replaced in the finished goods stock, which is equivalent to the allocation of the Heijunka board at the end of a period, is given by

$$\vec{h} = \vec{d} \otimes \vec{w} \tag{2}$$

In order to reach the desired alpha service level SLα, the base stock of finished goods and hence the number of required kanbans at the considered production stage k is the absolute value of the smallest integer k, where the relation

$$SL_\alpha \geq \sum_{i=-\infty}^{k} h_i \tag{3}$$

is satisfied. The number of waiting production orders at the end of a period is equivalent to the amount of kanbans in the overflow box. For each kanban in the overflow box, there must be one part in the finished goods supermarket to fulfil the customer order. The allocation probability of the supermarket sj depends on the constant maximal overall inventory I (WIP cap), which is the sum of items in the supermarket and amount of kanbans on the Heijunka board. Thus the allocation of the supermarket at the end of a period is

$$s_{I-j} = h_j \qquad 0 \leq j \leq I \tag{4}$$

If the EPEI is longer than one period, we suggest to add for each additional period the amount of one maximal request per period extra in the supermarket to guarantee the desired service level.

Further, we conclude that the capacity will be completely used, if the number of items not yet replaced in the finished goods stock, given by $\vec{h}$, is at least the available capacity; Otherwise, exactly the capacity will be used to fill up the items in the finished goods supermarket. The production vector is denoted by $\vec{q}$ and represents the amount of required material in one period. Therefore $\vec{q}$ is the demand vector to the preceding production level. We obtain

$$q_i = c_i \cdot \sum_{j=i}^{\infty} h_j + h_i \cdot \sum_{j=i}^{c^{\max}} c_j \qquad for\, c^{\min} \leq i \leq c^{\max} \tag{5}$$

To explain the usage of the modelled production system and the G|G|1-queue in discrete time we give an example in the next section.

3.3 Explicatory application

The required amount of kanbans in the kanban cycle to ensure a given service level has to be calculated. At first, the data for the calculation has to be provided. The demand function can be determined easily by taking the customer orders or forecasted demands. The effective production capacity is the difference between total capacity and the sum of all breakdown times of one shift or day. Often only the mean length of a downtime of a machine (MTTR) and the mean time between two successive down times (MTBF) are used to describe the availability A of a machine (A = MTBF/(MTTR + MTBF)). But the performance of a production system is heavily depending on the higher moments of the distribution. It is Important, whether the breakdowns are frequent and short-lived, or seldom and long-lasting. First of all, the cause of the output-gap has to be determined and classified as e.g. breakdowns or changeover. The effective production capacity needed to get a discrete density function can be determined as an excerpt from the shift-book and displayed as illustrated in Fig. 2.

With the algorithms from [15] the allocation of the Heijunka Board, the overflow-box and the demand function of the process are calculated. Table 1 illustrates the performance of the calculation from data input to result. The result is the service level with a given amount of kanbans, which is taken from the record.

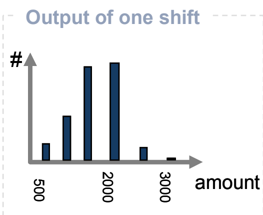

Fig. 2. Example of a histogram with shift-output

Table 1. Illustration of the computational steps of the model

Demand distribution	
0	0%
1	17%
2	17%
3	17%
4	17%
5	17%
6	17%
7	0%
8	0%
9	0%
10	0%

Production capacity	
0	0,0%
1	0,0%
2	0,0%
3	30,0%
4	40,0%
5	30,0%
6	0,0%
7	0,0%
8	0,0%
9	0,0%
10	0,0%

Number of	allocation of		
kanbans	Overflow-Box	Heijunka-Board	Servicelevel SL
0	37,8%	0,0%	0,0%
1	13,2%	6,5%	6,5%
2	12,2%	8,4%	14,9%
3	9,6%	10,5%	25,5%
4	6,8%	12,1%	37,5%
5	5,2%	13,0%	50,6%
6	3,9%	14,2%	64,8%
7	2,9%	8,4%	73,2%
8	2,2%	6,7%	80,0%
9	1,6%	5,1%	85,0%
10	1,2%	3,7%	88,8%
11	0,9%	2,8%	91,6%
12	0,7%	2,1%	93,7%
13	0,5%	1,6%	95,3%
14	0,4%	1,2%	96,5%
15	0,3%	0,9%	97,3%
16	0,2%	0,7%	98,0%
17	0,2%	0,5%	98,5%
18	0,1%	0,4%	98,9%
19	0,1%	0,3%	99,1%
20	0,1%	0,2%	99,3%

Effektive Production	
0	0,0%
1	6,5%
2	8,4%
3	32,7%
4	33,8%
5	19,3%
6	0,0%
7	0,0%
8	0,0%

4 Summary and outlook

Our result is a new analytical calculating method for the required inventory levels in the buffers (supermarkets) of a manufacturing system and is applicable when unreliable machines are involved. For that purpose, the manufacturing system is treated as a multi-stage production system. The application of standard formulas for lean production systems is limited because they require reliable processes. The benefit of the new approach is that the buffers can be sized without performing laborious and time consuming simulations, which were used so far.

Uncertainty of the results may be due to inevitable limitations set by assumptions: The availability of raw material has to be assumed to be 100%, the distribution of daily demand has to be independent and identical, and the demand pattern cannot be modelled. Hopefully further research will relieve these restrictions.

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