13 A MULTI-CRITERIA MATHEMATICAL PROGRAMMING MODEL FOR AGILE VIRTUAL ORGANIZATION CREATION

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This paper studies virtual organization (VO) creation through an application of mathematical programming. We present a mixed integer linear programming (MILP) model to support VO configuration in a virtual organization breeding environment (VBE). The model allows for multiple criteria, of which we give several examples: One objective is to minimize total fixed and variable costs. Another objective is to maximize expected cooperative efficiency using collaboration history as reference data. We model collaboration history with bilateral cooperation indices forming a weighted graph of VBE members. Furthermore, we incorporate capacity risk-measures in the model, allowing capacity risk minimization.

1. INTRODUCTION

Collaborative network organizations (CNO's) have become a necessity for companies who have decided to concentrate on core competencies, outsourcing other operations. This trend has attracted researchers to study the phenomena related to CNO's, and more specific virtual organizations and virtual enterprises (Camarinha-Matos and Afsarmanesh, 2005). In their earlier paper, Camarinha-Matos and Afsarmanesh (2003) introduced the concept of virtual organization breeding environment (VBE), being the platform for agile virtual organization (VO) creation. The idea of a VBE is that a set of organizations has commonly agreed cooperation structure, including for instance common ICT infrastructure, strategy, and processes for agile VO creation.

Our paper focuses on supporting decision-making concerning VO partner selection. When a VBE identifies a business opportunity, it faces the problem of finding a good VO configuration that is able to meet the needs of the customer. In this paper we approach this optimization problem through a mixed integer linear programming (MILP) model. This approach has been chosen for two reasons. First, a reasonable size MILP model is computationally solvable and second, it is flexible to modifications.

Earlier literature presents several optimization-based approaches to VO configuration. For instance, Wu et al. (1999), Ko et al. (2001), Ip et al. (2004), and

Wu and Su (2005) have applied integer programming in supporting VO selection. All of them use total costs as optimality criterion. Extending the solitary cost-criterion, e.g. Mikhailov (2002), Fischer et al. (2004), Lin and Chen (2004), and Sha and Che (2005) present models that account for multiple criteria, such as organizational competitiveness and social relationships. This paper contributes to the existing literature by, firstly modeling inter-organizational relationships as a selection criterion, and secondly assessing failure risk of VO operation.

Inter-organizational relations, such as collaboration history, distinguish the case of VBE based VO creation from traditional supplier selection. This is explained by the long-term agreements between the VBE members, which enable the collection of collaboration data. In contrast, in the non-controlled global supplier markets the collection of such data is practically impossible.

The rest of this paper is structured as follows. Section 2 presents a mixed integer linear programming model for VO creation. Sections 3 and 4 extend the basic model catering for inter-organizational relationships and risk measurement. Section 5 concludes and suggests directions for further research.

2. A MIXED INTEGER LINEAR PROGRAMMING MODEL

Our main concern is in VO partner selection. We consider this as a work-allocation problem, for which we present a mixed integer linear programming (MILP) model. Sections 2.1-2.3 present the basic model, which Sections 3 and 4 extend by taking collaboration history and risk measurement into account.

2.1 Variables

Let M = {1,...,m} denote the set of organizations, i.e. the members of a VBE. Assume the VBE identifies a business opportunity, which will be performed as a project to a customer (for more discussion on VO coordination, see e.g. Camarinha-Matos and Pantoja-Lima, 2001). We model the project with a set of tasks, denoted by N = {1,..., n}. Tasks 1,...,n constitute the whole project to be carried out. Each task $j \in N$ has a work load w_j, which describes the amount of work required (e.g. person months) in order to perform that task.

In VO creation, the information gathered from VBE members $i \in M$ and from databases includes the following parameters:

- $c_{i,j}^k$ = capacity, or amount of work that member $i \in M$ can perform on task j (e.g. person months), with probability $p_{i,j}(k)$
- $p_{i,j}$ = probability measure on set $C_{i,j}$, which includes $c_{i,j}^k$'s for given i and j
- $v_{i,j}$ = variable costs of member $i \in M$ working on task j (e.g. €/person month)
- f_i = fixed cost of member i becoming part of the VO, i.e. working on at least one task of the project
- $f_{i,j}$ = fixed cost of member i starting to work on task j.

The actual decision variable is the work-allocation matrix $X_{m\times n}$, whose element $x_{i,j}$ denotes the amount of work that VBE member i performs on task j. In addition, we define the following dummy variables, whose values depend completely on x's.

First, let

$$y_i = \begin{cases} 0, & \text{if } x_{i,j} = 0 \forall j \in N \\ 1, & \text{if } x_{i,j} > 0 \text{ for at least one } j \in N. \end{cases}$$

That is, y_i is binary, denoting whether any work in the project is allocated to VBE member i. Furthermore, let

$$y_{i,j} = \begin{cases} 0, & \text{if } x_{i,j} = 0 \\ 1, & \text{if } x_{i,j} > 0. \end{cases}$$

In words, binary $y_{i,j}$ denotes whether any work on task j is allocated to i.

2.2 Objective Function

In the basic model we present one criterion, which is cost. Therefore the objective is to minimize total costs related to each organization's variable and fixed costs:

$$\min_{X,Y} \underbrace{\sum_{i=1}^{m} f_i y_i}_{(I)} + \underbrace{\sum_{j=1}^{n} \sum_{i=1}^{m} \left(v_{i,j} x_{i,j} + f_{i,j} y_{i,j}\right)}_{(II)} \tag{1}$$

where X is m×n matrix consisting of x's and Y is m×(n+1) matrix of y's. Interpretation of the sum terms is the following.

(I) Sum of fixed costs for adding a new member to VO

(II) Sum of fixed and variable costs of each member's work on tasks

It should be noted, however, that the model is flexible in the sense that some costs can be ignored if considered irrelevant. On the other hand, the model allows accounting for completely new criteria, of which some examples are given in Sections 3 and 4.

2.3 Constraints

The constraints of the optimization problem are divided into three categories, namely 1) constraints that assure the demands of the project are met, 2) constraints that care for the feasibility of decision variables, and 3) constraints related to additional features. Beginning with the project constraints, first the work load of each task has to be covered:

$$\sum_{i=1}^{m} x_{i,j} \geq w_j \quad \forall j \in N.$$

The work allocation may not exceed expected capacities:

$$x_{i,j} \leq \sum_{k=1}^{|C_{i,j}|} p_{i,j}(k) c_{i,j}^k \quad \forall i \in M, j \in N. \tag{2}$$

Work loads are non-negative (unless capacity can be transferred inside the VBE):

$$x_{i,j} \geq 0 \quad \forall i \in M, j \in N.$$

Second, we present the feasibility constraints. The correct values for binary y_i's are assured with the following constraints:

$$y_i \geq \frac{\sum_{j\in N} x_{i,j}}{\sum_{j\in N} w_j} - \varepsilon \text{ and } y_i \leq \frac{\sum_{j\in N} x_{i,j}}{\sum_{j\in N} w_j} - \varepsilon + 1 \quad \forall i \in M,$$

where ε is a lower bound for the amount of work allocated to i in order to have i considered as an *essential* VO member.

The following constraints assure binary $y_{i,j}$'s have correct values:

$$y_{i,j} \geq \frac{x_{i,j}}{E\left[C_{i,j}\right]}, \quad \forall i \in M, j \in N \text{ s.t. } E\left[C_{i,j}\right] > 0,$$

where $E[C_{i,j}]$ is i's expected capacity for task j, defined as in (2).

Lastly, the MILP model allows numerous additions and modifications, of which we give two examples. First, *common capacity* between several tasks is captured with a simple additional constraint, e.g. $x_{i,a}+x_{i,b}\leq c_{i,ab}$.

Second, *overwork pricing* is captured with a new variable $x_{i,j}^+$ denoting work that exceeds expected capacity $E[C_{i,j}]$. The capacity constraint transforms to $x_{i,j}-x_{i,j}^+\leq E[C_{i,j}]$ and $x_{i,j}^+\leq x_{i,j}$. Objective function takes an additional cost term $v_{i,j}^+x_{i,j}^+$, where $v_{i,j}^+$ is the variable cost of overwork.

3. COOPERATIVE EFFICIENCY

Organizational collaboration is seldom completely frictionless when compared to work performed by a single organization. Since organizations have different physical and social relationships to each other, it is not justified to assume that two distinct groups of organizations would perform equally well, even with equivalent intra-organizational competencies. We denote this phenomenon by cooperative efficiency of organizations.

3.1 Cooperation Measures

We define the bilateral cooperation measure as follows.

Definition 1 *Let* M *be a set of organizations. For each* a, b, c, d ∈ M, a≠b, c≠d, *bilateral cooperation measure* $e_{a,b}$ *satisfies the following conditions*

(i) $e_{a,b} \in$ IR

(ii) $e_{a,b} = e_{b,a}$

(iii) $e_{a,b} > e_{c,d}$ ⇔ *pair* (a,b) *has higher expected cooperative efficiency than pair* (c,d).

Suppose there is a documented collaboration history between the organizations of VBE M. This can be modeled for instance as a graph, where nodes are the organizations of M and edges between the nodes represent successful/unsuccessful collaboration history. More generally, the edges are weighted corresponding to the success/failure of past collaboration, measured by a bilateral cooperation measure. Recently, Lavrač et al. (2005) have modeled trust in a VBE through an application of multi-attribute decision analysis. For instance, their work provides a means for measuring efficiency of bilateral cooperation, whereas Pearson et al. (2002) present a model for the evolution of bilateral relationships.

In the following, we suggest that the cooperative efficiency of a network can be obtained from bilateral cooperative efficiencies. Let G = (M,E) be a graph whose nodes are the members of VBE M and edges are weighted with bilateral cooperation measures between members of M. Let $E = \{e_{a,b} \mid a, b \in M, a<b\}$ denote the set of edges, i.e. the bilateral cooperation measures of each pair $(a, b) \subset M$. Furthermore, let P^M denote the power set of M, and let $G_P(P^M \setminus M, E^P)$ denote a graph whose nodes are the elements of P^M excluding set M itself, and edges exist between every two nodes $\alpha, \beta \in (P^M \setminus M)$, such that $\alpha \cup \beta = \varnothing$. Figure 1 illustrates a $G_P(P^M \setminus M, E^P)$ with M = {a, b, c}.

Definition 2 *Network cooperation measure is a mapping* γ: $G_P(P^M \setminus M, E^P) \rightarrow$ IR *with the following property. Let* $A,B \subseteq M$, $A \neq B$, *then* $\gamma(A) > \gamma(B)$ *network organization A has higher expected cooperative efficiency than B.*

Hence, the network cooperation measure accounts not only for bilateral cooperative efficiencies between individual organizations, but also for bilateral cooperative efficiencies between possible coalitions inside a network. This is necessary, for instance if a, b, and c get along well twosome, but cannot work together as a triplet.

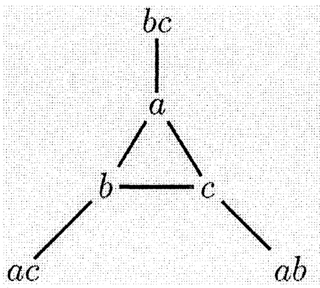

Figure 1 – Graph $G_P(P^M \setminus M, E^P)$ with M = {a, b, c}

3.2 Two Examples of Network Cooperation Measures

The following additive measure is defined for $e_{a,b} \in (-\infty, 1]$:

$$\gamma^{ADD}(A,E) = \frac{1}{\left\lceil \frac{|A|(|A|-1)}{2} \right\rceil} \sum_{\substack{(a,b)\subset A \\ a<b}} \lambda_{a,b} e_{a,b},$$

where $\lambda_{a,b} > 0$ is a weight attached to cooperative efficiency of pair (a,b). The weights sum up to one. The weighted sum is normalized with the number of edges in complete graph with |A| nodes. Hence, a network with perfect relationships would attain a value of one. This measure enables the comparison of two different size networks.

For our MILP model we would like to have a linear measure. For that purpose, let us first define a binary variable $z_{a,b} \in \{0,1\}$, such that

$$z_{a,b} = \begin{cases} 0, & \text{if } y_a = 0 \text{ or } y_b = 0 \\ 1, & \text{if } y_a = 1 \text{ and } y_b = 1. \end{cases}$$

In words, $z_{a,b}$ denotes whether both a and b have work allocated. For z, we need the following constraints:

$$z_{a,b} \le \frac{y_a + y_b}{2}, \text{and } z_{a,b} \ge y_a + y_b - 1 \ \forall \{a,b\} \subset M, \text{s.t. } e_{a,b} > 0.$$

Then, the following cooperation measure is linear:

$$\gamma^{LIN}(Z,E) = \sum_{\substack{a,b \in A \\ a<b}} e_{a,b}(f_a + f_b) z_{a,b},$$

where Z is m×m matrix of z's. The bilateral cooperative efficiencies are weighted with fixed participation costs.

However modeled, the purpose of collaboration data is to highlight such subsets of a VBE that, based on e.g. historical evidence, have a higher expected cooperative efficiency than those with unsuccessful or negligible past collaboration. This phenomenon can be included in our MILP model using cooperation measures.

4. CAPACITY RISK MEASURES

Apart from financial portfolio optimization problems, where risk is usually measured through fluctuations in profit, in our case it is meaningful to define risk through fluctuations in capacity. This is reasoned with two facts. Firstly, project income, i.e. payment from customer, is normally risk-free, excluding force majeure reasons, e.g. customer's bankrupt. Secondly, almost all project failures, e.g. quality defects, material shortages, and unexpected raise in demand, can be reduced to capacity shortfall.

We shall review two linear risk measures, which are therefore applicable to our MILP model. First, Eppen et al. (1989) suggest *expected downside risk* (EDR) be used for capacity-risk measurement. Applied to our model, expected downside risk of i's work on j is

$$\rho_{i,j}^{EDR} = \sum_{\forall c_{i,j}^k < x_{i,j}} p_{i,j}(k)\left(x_{i,j} - c_{i,j}^k\right).$$

In words, EDR is the expected value of downside difference between allocated work and capacity. Hence, the summation is taken over such events $c_{i,j}^k$ that imply a shortfall in capacity, if work $x_{i,j}$ is allocated.

Explicitly, EDR is incorporated in our model as follows. For each $c_{i,j}^k \in C_{i,j}$, let $c_{i,j}^{k+} \ge 0$ and $c_{i,j}^{k-} \ge 0$, respectively, denote the positive and negative difference of $c_{i,j}^k - x_{i,j}$. The correct values of $c_{i,j}^{k+}$ and $c_{i,j}^{k-}$ are assured with constraints:

$$x_{i,j} - c_{i,j}^{k-} + c_{i,j}^{k+} = c_{i,j}^k \forall i \in M, j \in N, c_{i,j}^k \in C_{i,j}.$$

Formula for EDR becomes

$$\rho_{i,j}^{EDR} = \sum_{\forall c_{i,j}^k \in C_{i,j}} p_{i,j}(k) c_{i,j}^{k-}.$$

The other risk measure we shall review is *lower semi-absolute deviation* (LSAD), which was presented by Gustafsson and Salo (2005) in the context of financial portfolio selection. For our model, LSAD of i's work on j is calculated as follows:

$$\rho_{i,j}^{LSAD} = E\left[C_{i,j}\right] - \sum_{\substack{k \\ c_{i,j}^k < E\left[C_{i,j}\right]}} c_{i,j}^k.$$

Hence, LSAD is the expected downside deviation from expected capacity.

Both EDR and LSAD are interpreted as expected shortfalls from a target value; in EDR the target value is allocated work, whereas in LSAD the target value is expected capacity. Both can be incorporated in our MILP model either as linear constraints, e.g. EDR$\leq r_1$, LSAD$\leq r_2$, or as additional costs in the objective function. However, both approaches require parameter estimations; in the former the accepted risk-levels (r's) are to be defined, and in the latter the cost of capacity risk is to be determined.

5. CONCLUSIONS AND FURTHER RESEARCH

In this paper we have presented a mathematical programming approach to virtual organization creation. The use of such models helps obtain better understanding of prevailing conditions and helps answering questions such as "What would be the structure of an optimal VO?", "Should the project be performed in a centralist rather than in a distributed manner?", "What are the risks related to distributed manufacturing?".

We have formulated a mixed integer linear programming model for the optimal VO creation problem. The objective of the model is to match the core competencies of different VBE members with the requirements of a project and thereby select the optimal VO to serve the customer. The most important contributions of our model are first the modeling of inter-organizational relationships and second, measuring risk of VO failure.

Topics for future research are manifold. First, our optimization model could be improved by several features. These include for instance dynamic decision-making and uncertainties, interdependent risks, hedging against capacity risk, etc. Second, the effect of incentives, e.g. profit sharing rules, on VO creation should be studied. Third, VBE member performance measurement models are needed in order to most efficiently use operative models. For instance, our model raises the need to measure factors related to cooperative efficiency.

ACKNOWLEDGEMENTS

The European Integrated Project ECOLEAD, and the funding of National Technology Agency of Finland (Tekes) and the National Workplace Development Programme (Tykes) through COBTEC project are acknowledged.

REFERENCES

1. Camarinha-Matos LM, Afsarmanesh H. Elements of a base VE infrastructure. Computers in Industry 2003; 51: 139–63.
2. Camarinha-Matos LM, Afsarmanesh H. Collaborative Networks: A New Scientific Discipline. Journal of Intelligent Manufacturing 2005; to appear.
3. Camarinha-Matos LM, Pantoja-Lima C. Cooperation coordination in virtual enterprises. Journal of Intelligent Manufacturing 2001; 12: 133–50.
4. Eppen GD, Martin RK, Schrage L. A Scenario Approach to Capacity Planning. Operations Research 1989; 37(4): 517–27.
5. Fischer M, Jähn H, Teich T. Optimizing the selection of partners in production networks. Robotics and Computer-Integrated Manufacturing 2004; 20: 593–601.
6. Gustafsson J, Salo A. Contingent Portfolio Programming for the Management of Risky Projects. Operations Research 2005; to appear.
7. Ip WH, Yung KL, Wang D. A branch and bound algorithm for subcontractor selection in agile manufacturing environment. International Journal of Production Economics 2004; 87: 195–205.
8. Ko CS, Kim T, Hwang H. External partner selection using tabu search heuristics in distributed manufacturing. International Journal of Production Research 2001; 39(17): 3959–74.
9. Lavrač N, Ljubič P, Jermol M, Papa G. A Decision Support Approach to Modeling Trust in Networked Organizations. In Proceedings of the 18th International Conference on Industrial & Engineering Applications of Artificial Intelligence & Expert Systems, Bari, June 22-24, 2005, Springer Verlag.
10. Lin C-WR, Chen H-YS. A fuzzy strategic alliance selection framework for supply chain partnering under limited evaluation resources. Computers in Industry 2004; 55: 159–79.
11. Mikhailov L. Fuzzy analytical approach to partnership selection in formation of virtual enterprises. Omega 2002; 30: 393–401.
12. Pearson DW, Albert P, Besombes B, Boudarel M-R, Marcon E, Mnemoi G. Modeling enterprise networks: A master equation approach. European Journal of Operational Research 2002; 138(3): 663–70.
13. Sha DY, Che ZH. Virtual integration with a multi-criteria partner selection model for the multi-echelon manufacturing system. The International Journal of Advanced Manufacturing Technology 2005; 25(7-8): 793-802.
14. Wu N, Mao N, Qian Y. An approach to partner selection in agile manufacturing. Journal of Intelligent Manufacturing 1999; 10(6): 519–29.
15. Wu N, Su P. Selection of partners in virtual enterprise paradigm. Robotics and Computer-Integrated Manufacturing 2005; 21: 119–31.