Study of Spread of Harmonics in an Electric Grid

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Abstract. This paper presents an algorithm for estimating the Total Harmonic Distortion (THD) in the electrical power grid nodes, which is based on a load flow analysis by frequency component. The aim of this algorithm is, on the one hand, to model the linear and non-linear loads which would be part of an electrical network and, on the other hand, to estimate the THD which would appear in the network nodes, in order to evaluate its effects and consequences, as well as to choose the best alternative for solving these problems. The proposed algorithm is able to show the grid's buses in which the THD achieve a maximum, and could help to choose the most appropriate one to put the electronic devices up, as active power filters, optimizing the system design, permitting the reduction of the system losses and an increase of the energy transmission effectiveness into the grid.

Keywords: Non-linear Loads, Total Harmonic Distortion, Frequency Components, Power Flow, Power Losses.

1 Introduction

The electric energy consumers have a wide variety of electrical and electronic equipment that pollute the power grid, generating currents and / or voltage harmonics. In consequence, the operation of other users' equipment would be affected, due to the requirements of high quality power supply for proper operation (critical loads).

Harmonics are distortions or deformations of sinusoidal waves of voltage and / or current in electrical systems, mainly due to the use of non-linear loads (computers, televisions, variable speed drives, rectifiers, arc furnaces, fluorescent lamps, starters electronics, etc..), the use of ferromagnetic materials in electrical machines, switching operations in substations and in general the operation of switching equipment .

The effects of harmonics in power grids have been studied in previous literatures [1], [2]. To begin with, the appearance and the circulation of currents and / or additional voltages on the electrical system causes problems such as the increasing of active power loss, overloads in capacitors, measurement errors, malfunction protection, insulation damage, deterioration of dielectrics and decrease in the life of equipment, among others.

There are several alternatives for solving the pollution on the power supply through electrical or electronic equipments. Among these alternatives are active power filters. These are electronic devices that have been studied and applied in recent years due to the advantages over other alternative solutions. It is generally assumed that the active filters will always work under the same specifications. This is a good consideration whether in reality the values of the parameters do not vary very atypical, so the filter will never show a pattern different from design.

2 Contribution to sustainability

This paper presents an Harmonic Load Flow Algorithm to evaluate the Total Harmonic Distortion (THD) that appears in a electrical grid due to electrical and electronic components, obtaining the mathematical model of loads (linear and nonlinear) under the influence of harmonics, and could be use for evaluating the best alternative to put up the equipments (for example, active power filters) for solving the electrical pollution looking for the option that achieve the same goals but minimizing the total losses produced in the transmission and distribution grids. Also the algorithm determines the losses produced in the grid, that is a key parameter when operating and managing the grid and is one of the most important objective to be minimized for build a sustainable distribution grid.
This algorithm will help in future studies to characterize and model the demeanor of active filters, determining the nodes where these devices will succeed in improving the performance of the electric grid. Besides, this algorithm will help to evaluate the effects of these elements on mitigating the harmonics produced by non-linear loads and to estimate the decrease of electric grid losses. Only by using an algorithm of this kind it will be possible to analyze different active power filter strategies for comparing them and select the best depending on the electric grid characteristics.

3 Frequency Component Load Flow

The elements of an electrical system can be represented by linear or nonlinear impedances. The first case corresponds to those elements in which there is a proportional relationship between voltage and current to the same frequency range; on the other hand, non-linear elements do not have this proportional relationship across the spectrum. Among the elements that can be represented by linear impedances, are the lines, transformers, electrical machines and certain charges. On the contrary, the components which are considered as non-linear elements are mainly electronic equipment, like rectifiers.

3.1 Linear Load Modeling in the presence of Harmonics

Transmission Lines have different mathematical models depending on their length, voltage and frequency. According to their length they are classified as short, medium and long lines.

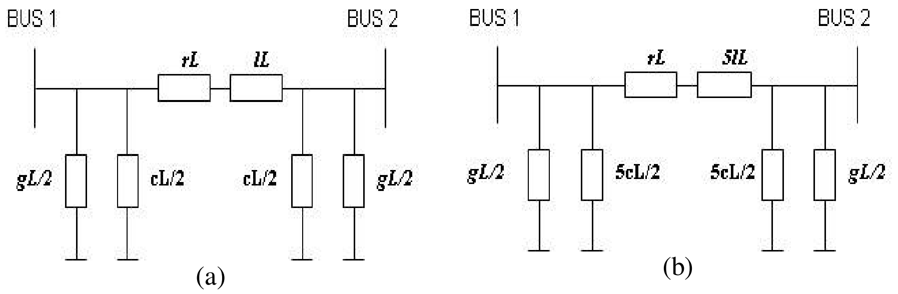


Fig. 1. (a) Equivalent single-phase model, (b) Fifth Harmonic line model

They are fully described in [4], including their mathematical models. Thereby, the model which has been taken into account in the proposed algorithm is the π nominal one represented in Fig. 1a, where, *r, l, c y g* are, respectively, the resistance, inductance, capacitance and conductance per length unit. According with [3], if the line is considered as a short line, *c y g* will be equal to 0. Figure 1b shows the equivalent single-phase model taking as an example the fifth harmonic.

Loads. In the study of harmonic flow, low-power loads are not represented individually; however, they are combined into equivalent circuits that represent the impedance characteristics of all charges. It is possible to consider variations in the impedance of the system due to the frequency or the chargeability level, both for domestic and industrial consumers. Nevertheless, as industrial loads are usually those that use capacitors for making up for the power factor, they are the ones that have more possibilities for contributing to the appearance of series and /or parallel resonance into the electrical system, [5]. They are fully described in [4], including their mathematical models.

Capacitors are modelled by their equivalent capacitance, providing a unique model that can be incorporated into a series or parallel circuit; these capacitive reactances must be multiplied by h [3], for taking into account the harmonic flow effects.

Non-controlled rectifiers (Fig. 2) are one of the sources of emission of harmonics in a power system. For modelling these loads it is necessary to take into account the distortion in the waveform in order to achieve a better description of the interaction with the network. According to the current injection model [3], the current wave is decomposed in Fourier series and each harmonic component is injected into the system as a power source; thus, it is possible to determine the system nodes harmonic voltages if a frequency sweep is made on the network. Equation (1) expresses the current injection model:

$$I_h = Y_{1h} \times V_1 , \tag{1}$$

where Y_{1h} is interpreted as the relationship between each harmonic current and the fundamental component of tension, and is not necessarily linear.

Therefore, the nonlinear loads are modelled as constant current sources for each harmonic frequency and are calculated regarding to the fundamental frequency current.

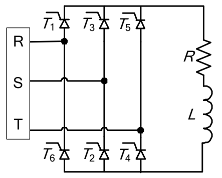


Fig. 2. Three-phase 6-pulse non-controlled rectifier diagram.

These injections are based on the Fourier series, [4]. The Fourier series of AC input of a typical 6-pulse rectifier, when the output current is almost constant, fed by a transformer YY are listed below [3]:

$$I_{R,h} = I_{R,1} \times \left\{ \cos(\omega t) - \frac{1}{5} \cdot \cos(5\omega t) + \frac{1}{7} \cdot \cos(7\omega t) - \frac{1}{11} \cdot \cos(11\omega t) + \frac{1}{13} \cdot \cos(13\omega t) + \ldots \right\} \quad (2)$$

where $I_{R,1}$ is the current drawn by the rectifier at the fundamental component, whose expression is obtained from:

$$I_{R,1} = \frac{4}{\pi} \int_{\pi/6}^{\pi/2} I_o \cdot sen(\theta) \cdot d\theta \cdot \quad (3)$$

By integrating the previous expression, the next one is obtained:

$$I_{R,1} = \frac{4}{\pi} I_0 [\cos\theta]_{\pi/2}^{\pi/6} = \frac{4}{\pi} I_0 \frac{\sqrt{3}}{2}, \quad (4)$$

and consequently:

$$I_{R,1} = \frac{2 \cdot \sqrt{3}}{\pi} I_0, \quad (5)$$

being

$$I_0 = \frac{V_0}{R} = \frac{1.35 V_{LL}}{R} = \frac{1.35\sqrt{3} V_{FN}}{R}. \quad (6)$$

Finally, it can be shown from the expression (2):

$$|I_{R,h}| = \frac{I_{R,1}}{h}. \quad (7)$$

On the other hand, the equivalent resistance of rectifier for the purposes of modelling its behaviour for the fundamental component can be expressed by:

$$R_{eq,Y} = \frac{V_{FN,1}}{I_{R,1}}. \quad (8)$$

If the expressions (5) and (6) are used into expression (8), it is finally obtained:

$$R_{eq,Y} = R \times F_D \quad (9)$$

where:

$$R = \frac{(1.35 \times V_{LL})^2}{S_{RECTIF.}}; \; F_D = \frac{\pi}{1.35 \times 6} \quad (10)$$

3.2 Frequency Component Load Flow Algorithm

The frequency component load flow, like the conventional load flow, has different purposes. Firstly, to establish the state of the system taking into account the parameter of linear elements that shapes it. Secondly, to obtain information about the demanded power at the charge nodes and the generated power and, finally, to draw the topology of the system and the characteristics of nonlinear elements which cause the harmonics of voltages and currents that are multiples of the fundamental frequency in the system. The linear and nonlinear elements must be modelled considering the variation suffered with the frequency, according to previous epigraphs.

Once the conventional load flow is finished, consequently the models of linear and nonlinear loads have been defined. In order to find the harmonic voltages, the Y_{BUS} matrix should be built for each frequency and the next equation should be solved:

$$I_{(h)} = Y_{BUS}^{(h)} \cdot V^{(h)} . \tag{11}$$

In the above expression current injections are known due to their dependence on the nonlinear loads which are considered.

Once the mathematical model of the network elements are obtained, it is possible to make the flow of loads on the grid and, in consequence, to obtain the THD at each node, which is caused by the propagation of harmonics in the network due to the non-linear loads, as well as to assess the power losses due to these harmonic currents.

Thus, the THD is the square root of the sum of the harmonic voltage amplitudes squared between the amplitude of the fundamental voltage component, which is expressed as a percentage. In mathematical terms, the THD is defined by the next expression:

$$THD(\%) = 100 \frac{\sqrt{\sum_{h=2}^{H} V_h^2}}{V_1} . \tag{12}$$

Where V_1 is the RMS fundamental voltage component, V_h is the RMS h-harmonic voltage component and H is the number of harmonic to evaluate.

Fig. 3 reflects the flow chart of the frequency component power flow algorithm.

4 Simulation Results

The proposed algorithm is applied to a radial power distribution line (Fig.4), which is based in the case presented in [7], in order to demonstrate its effectiveness for determining the THD in a power system. The circuit model of the radial distribution grid is illustrated in Fig. 4. The parameters in the simulation are included as follow:

- Power system: 220 V (line to line), 60 Hz. The transmission line parameters are L_1= 0.2 mH, R_1= 0.05 Ω, C= 150 μF, L_2= 0.4 mH and R_2= 0.1 Ω.
- Nonlinear loads: two rectifiers rated at 2760 VA and 3328 VA are installed at bus 2 and bus 6, respectively. The dc side of the rectifiers consists of an inductor and a load resistor that model a load which produces harmonics.

Network parameters expressed in the per unit system

The base power and the base voltage which have been taken into account for the per unit system are U_b= 220 V and S_b= 6 kVA. As a consequence, the base impedance of the circuit is Z_b= 8.0666Ω.

The parameters of the radial network expressed in the per unit system are included inside the Table I.

Mathematical models of the lineal and non-lineal elements

Each of the elements of this system is modeled as follows:

- *Transmission line*: It is modeled using the Midline Model,[4] ,(see Fig.1). The conductance G is not considered in this network.

Rectifiers: They are considered as the non-linear elements of the network, which are located on Buses 2 and 6 (Case I), and buses 4 and 9 (Case II) respectively. According with epigraph 3.1, the parameters that define each rectifiers´ mathematical model are the ones included in Table II.

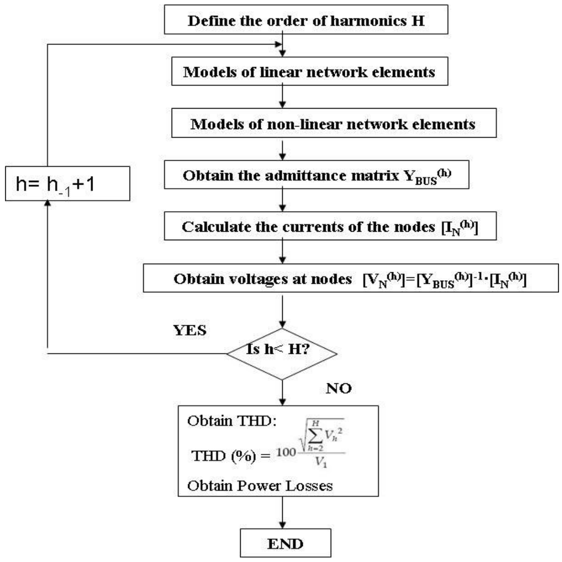
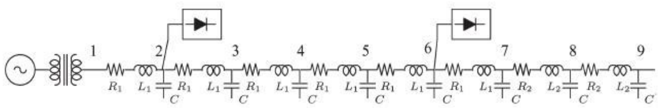
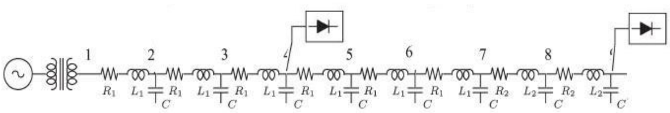


Fig. 3. Harmonic Load Flow Algorithm



(a)



(b)

Fig. 4. Radial power distribution line: (a) Case I, (b) Case II.

Table I. Network parameters in the per unit system.

Parameter	***Real Value***	***p.u. Value***
V_1	220 V	1
X_{L1}	j·0.075398 Ω	j·0.0093468
R_1	0.05 Ω	0.0061983
X_C	-j·17.68388 Ω	-j·2.192216
X_{L2}	j·0.150796 Ω	j·0.0186937
R_2	0.1 Ω	0.01239669

Table II. Rectifiers in buses 2,4 and 6,9: Parameters in the per unit system

LOAD: RECTIFIER ON BUS 2 (I) and 4 (II)		**LOAD: RECTIFIER ON BUS 6 (I) and 9 (II)**	
PARAMETER	**VALUE (p.u.)**	**PARAMETER**	**VALUE (p.u.)**
S	0.46	S	0.554
R	3,9619	R	3,2897
F_D	0.27425	F_D	0.27425
R_{eqY}	1,5356	R_{eqY}	1,2759
$I_{Nn,1}$	0.3757	$I_{Nn,1}$	0.4525
$I_{Nn,h}$	$I_{Nn,1}/h$	$I_{Nn,h}$	$I_{Nn,1}/h$

Network analysis and THD calculation at each node.

For this power system the simulations were performed using the MATLAB software. The results of the THD at each node for each case I and II are presented in Table III.

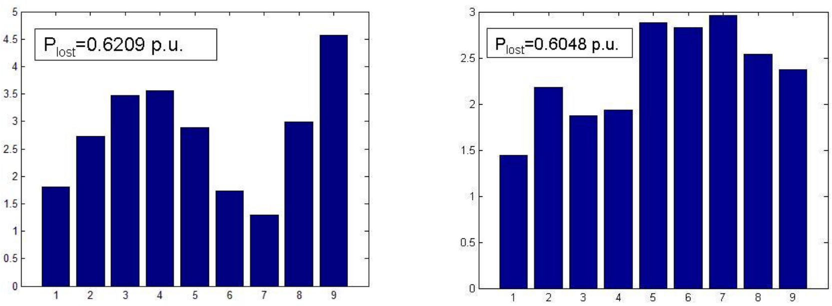
Table III. Total Harmonic Distortion in the network's buses.

BUS	**THD(%)**	
	Case I	**Case II**
Bus 1	1.8038	1.4416
Bus 2	2.7262	2.1832
Bus 3	3.4746	1.8766
Bus 4	3.5541	1.9312
Bus 5	2.8873	2.8794
Bus 6	1.7368	2.8327
Bus 7	1.2930	2.9575
Bus 8	2.9950	2.5417
Bus 9	4.5654	2.3711

The Figure 5 shows the evolution of the THD on each node, which is based on the data collected in Table III, and also show the losses of the transmission lines of the studied system, being 0.6209 p.u. for case I and 0.6048 p.u. for case II.

5 Conclusion

In this paper an algorithm for estimating the Total Harmonic Distortion (THD) in an electrical power grid is presented. This algorithm is based on the harmonic load flow in a radial network, due to non-linear loads.



(a) (b)

Fig. 5. Grid nodes THD for the system of Fig.4: a) case I, b) case II.

It has been shown by simulation that the proposed algorithm is able to show the grid's buses in which the THD is maximum, helping to choose the most appropriate one to put the electronic devices up, and optimizes the system design, permitting the reduction of the system losses and an increase of the energy effectively injected into the grid.

First of all, the different elements that could appear on an electrical network have been mathematically modeled, including transmission lines, linear and non-linear loads. Secondly, the Harmonic Load flow algorithm has been proposed, being able to calculate the Total Harmonic Distortion which appears on the network due to the non-linear loads. Moreover, the proposed algorithm has been applied to a radial power distribution line in order to demonstrate it effectiveness in predicting the THD in a power line. As a result, it could be established the system's buses in which the THD is maximum, helping to evaluate the best alternative to put up the equipments for solving the electrical pollution. Also this algorithm determines the losses in the distribution grid, and will allow, in future applications, to evaluate the influence of the presence of active power filter and the nodes where they are connected.

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