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RELIABILITY EVALUATION OF FAILURE DELAYED ENGINEERING SYSTEMS

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This paper introduces an analytical approach for the evaluation of multi-user engineering systems presenting a failure delayed behaviour pattern, that is, systems whose performance decays progressively after the failures, due to internal fault tolerance mechanisms or to the complacency of the users regarding the temporarily unavailability of the services. The approach is based on the determination of analytical expressions for the reliability measures, e.g. frequency and probability of failure states, which may then be evaluated using general purpose mathematical tools. The paper discusses the rationale and the fundamental algorithms of the approach and presents a set of illustrative examples.

1. INTRODUCTION

This paper presents the results of a research project aiming to develop a systematic approach for the reliability evaluation of systems containing multiple concurrent processes with generalized distributions. The approach was primarily developed to assist the steady-state analysis of failure delayed systems (FDS), i.e. systems whose performance decays progressively in the sequence of a failure. The paper presents a definition of these systems and shows that they present a non-Markovian behavior pattern and that the existing methodologies present a number of shortcomings regarding the evaluation of FDS systems. Then, the paper introduces the fundamental aspects of the new approach and presents a set of numerical results in order to illustrate its practical application and usefulness.

2. FAILURE DELAYED ENGINEERING SYSTEMS

In many situations, the users of an engineering system are complacent about a temporary unavailability of the service provided to them by the system. This means that, at first, the disturbances of a system failure are often negligible. However, if the failure persists for a long time, the system will enter into successive degraded operational modes where its quality of service decays progressively, until a successful repair action is undertaken and the system restores its normal operation, or a catastrophic failure occurs.

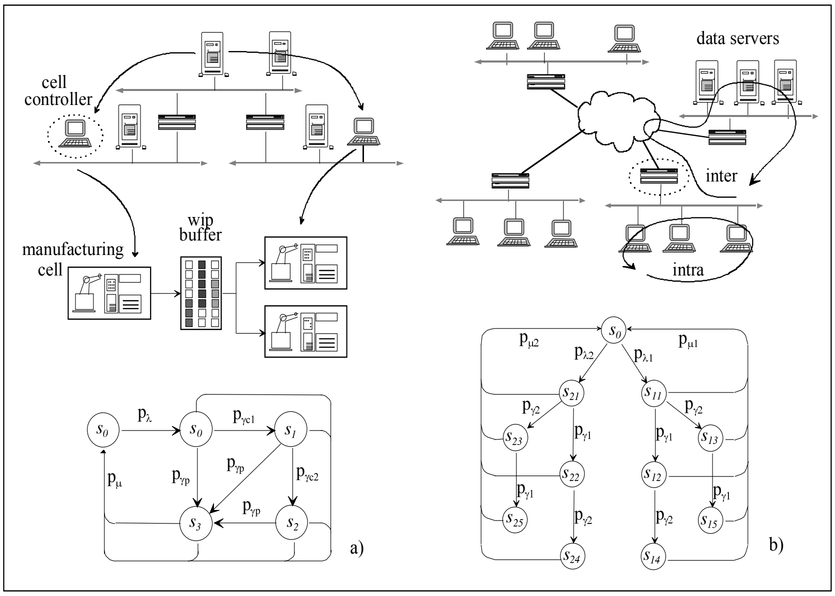

Figure 1. Failure delayed systems examples

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models, s_0 corresponds to the normal operating state of the system, and s_{ji} to the failure states. The failure, repair and propagation (or delay) processes are represented respectively by $p_{\lambda i}$, $p_{\mu i}$ and $p_{\gamma i}$. The model of figure 1.a represents a production system with intermediate work-in-process (wip) buffers between the manufacturing cells. The cells and plant controllers of the manufacturing system get their data from a plant data server. If this server becomes unavailable (process p_{λ}), the plant will be able to continue producing, because the cell and plant level production plans are frozen some time in advance of the physical production (processes $p_{\gamma c1}$ and $p_{\gamma p}$). However, the plant will enter a sub-optimal mode because it will not be possible to react to production events, such as new urgent orders. If an upstream cell halts its operation, the downstream cells will continue to be fed by the intermediate work in process buffer (processes $p_{\gamma b}$). Only when there is a shortage of products at the output of this buffer, will the consequences of the failure propagate downstream. If this production system belongs to a just-in-time supply chain, the severity of the damages is likely to increase dramatically.

The model in figure 1.b sketches the information system of a business company from the retail sector. End users execute intra and inter-site transactions (which both depend on the availability of a number of remote data servers) and may tolerate a temporary unavailability of the information services. However, this complacency is different regarding intra and inter-sites transactions, and regarding the operations executed in each site (end consumers' point of sales, or logistical support). This behaviour is represented in the model by two concurrent failure propagation processes $p\gamma_1$ and $p\gamma_2$.

These two examples show that a progressive decay of performance after a failure, due to an internal temporal redundancy mechanism, or to the complacency of the users regarding the temporary unavailability of the services provided to them, is a common behaviour pattern in engineering systems. The analysis of these systems also shows that FDS systems present a number of common features that directly impact on their reliability and performance evaluation. Suppose that S is a repairable failure delayed system and M is its behaviour model (figure 2).

In this case, the following assumptions regarding *S* and *M* will be considered in the context of this paper:

- *S* provides services to multiple users (e.g. downstream manufacturing cells, electrical consumers or information systems users) each of which presents its own complacency regarding the unavailability of the services of *S*.
- *S* has a regenerative state which is represented in *M* as s_{up}.
- In s_{up}, one or more failure processes are active. Each one of those processes corresponds to a particular failure.
- The execution of a failure process leads the state of *S* to one of the initial failure states

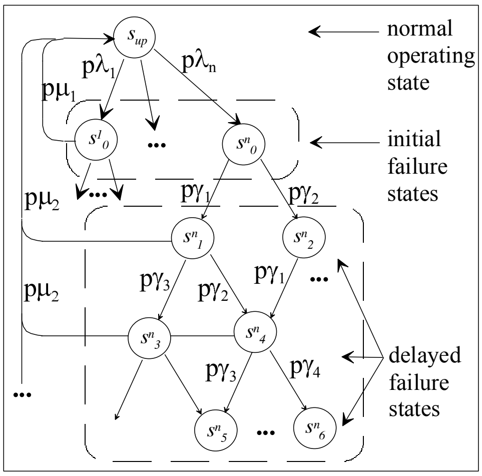

Figure 2. Failure delayed system models

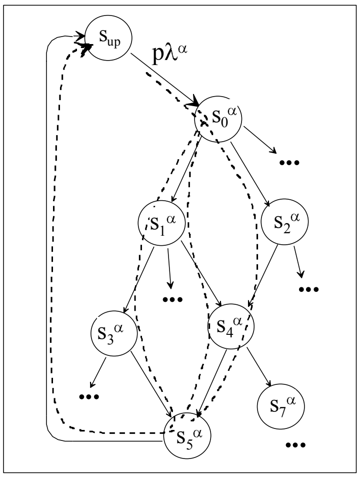

Figure 3. Alternative trajectories

where the disturbances for the users will typically be negligible.
- In each failure state, several concurrent delay processes, $p\gamma_\beta$, may be active. Each one of them corresponds to the complacency of a particular type of user regarding the failures of the system.
- The execution of a delay process leads the system to a delayed failure state, e.g. s_n^α with $n \geq 0$, where the severity of the damage will typically increase.
- In each initial or delayed failure state, a repair process $p\mu_\eta$ may be active. The execution of this process leads the system to the s_{up}. In other words, it is assumed that repair is a regenerative process that completely restores the normal operating condition (the extension of the model to non-regenerative repair will be discussed in Section 6).
- Failure, delay and repair processes may present arbitrary distributions (deterministic or stochastic).
- When a transition occurs from a failure state, the other processes that were simultaneously active in that state may be deactivated, reinitialized or remain active (keeping their firing time). Simultaneously, other repair or delay processes may be activated on the arrival at the new state.

3. REVIEW OF EXISTING METHODS

The assessment of non-Markovian systems remains a largely open issue in reliability analysis, despite the significant progress achieved in the last two decades, mostly based on stochastic Petri nets. The device of stages is one of the well proved techniques for the evaluation of non-Markovian systems, which makes it possible to model a large range of experimental probability density functions. For example, a log-normal distribution often found in repair processes may be represented through a combination of a series of states with two states in parallel, as shown in (Singh 77) and (Pages 80).

First introduced in (Cox 65), it has been applied to the reliability evaluation of fault tolerant computer systems (Laprie 75), and to the reliability analysis of electrical power systems (Singh 77). An extension of the method has been proposed in (Haverkort 93), to allow the assignment of a memory policy to any timed transition. One of the important features of the method is the possibility of designing automated tools to support its application, as presented in (Cumani 85). This tool uses Petri nets as the modelling tool and converts the reachability set of the net into a continuous time Markov chain defined over an extended state space. Although very flexible, this method restricts the firing times of the stochastic processes so that they are PH distributed (Neuts 81). Consequently, it presents a major limitation when the systems under analysis contain deterministic or quasi-deterministic processes, because the number n of additional states rises quadratically with the ratio between the standard deviation and the mean of the distribution.

In the past two decades, several evaluation techniques based on stochastic Petri nets (SPN) modelling have been developed in order to support the reliability analysis and the performance evaluation of complex systems. When SPN were first introduced (Molloy 82), all the random variables associated with the transitions were assumed to be exponentially distributed, so that the evolution of a Petri net could be mapped into a continuous Markov chain. Since then, and in order to broaden the field of application of SPN, several classes of Petri nets incorporating non-exponential features in their definition have been proposed. This is the case of the deterministic and stochastic Petri nets defined in (Marsan 87), in which a single deterministic transition may exist in each marking. Subsequently, it was observed in (Choi 95) that the underlying marking process is a Markov regenerative process. This allowed the extension of the model in order to accommodate immediate transitions, exponentially distributed timed transitions and generally distributed timed transitions but with the important restriction that at most one generally distributed timed transition be enabled in each marking (Choi 94). Two evaluation approaches were then developed: one based on the derivation of the time dependent transition probability matrix in the Laplace transform (Choi 94), the other based on the supplementary variables method (German 94). In spite of this progress, several restrictions still apply to the analytical evaluation of non-Markov systems, and no general solution is available

4. NEW APPROACH FUNDAMENTALS

This Section introduces the mathematical foundations of the procedures for the determination of the frequencies and the probabilities of a non-Markovian model M. The analytical expressions for the frequencies will be considered first in paragraph 4.1, then the states probabilities expressions will be addressed in paragraph 4.2. The procedure is based on the notion of *state trajectory*: immediately after a failure, the system occupies one of the initial failure states. Then, it returns to the normal operating state following one of the several possible trajectories, as shown in figure 3. A trajectory is an ordered set of failure states { s_n^α, s_n^α , s_n^α , … s_n^α } that starts at one of the regenerative initial failure states s_n^α , and such that, for each pair of consecutive states, s_n^α and s_n^α , there is a delay process $p\gamma_\beta$ in *M* whose execution causes the transition from s_n^α to s_n^α . In the presentation of the procedure, the following notation will be adopted:

- $\boldsymbol{\Lambda}_M$ and $\boldsymbol{P}_M$: two vectors such $\boldsymbol{\Lambda}_M(s)$ and $\boldsymbol{P}_M(s)$ contain the frequency and the probability of state *s*, respectively;
- s_{up}: the normal operating state,
- $p\lambda_\alpha$: the failure process corresponding to failure mode α;
- s_0^α : the initial failure state corresponding to failure mode α;
- $p\gamma_\beta$ and $p\mu_\eta$: the processes corresponding to the propagation delay β and the repair action η, respectively;
- s_n^α : a delayed failure state subsequent to s_0^α ($n \geq 1$);
- $f_p(t)$: the probability density function of process *p*.

4.1. Failure states frequency

Suppose that s_n^α is a failure state whose frequency is to be determined and that $\boldsymbol{\Psi}_n^\alpha$ is the set of trajectories starting at s_0^α and ending at s_n^α . The frequency of the failure state $\boldsymbol{\Lambda}(s_n^\alpha)$ results from the sum of the frequencies of each trajectory ψ of $\boldsymbol{\Psi}_n^\alpha$:

$$\Lambda(s_n^\alpha) = \sum_{\psi \in \Psi_n^\alpha} \Lambda(\psi) \tag{1}$$

The frequency of each trajectory ψ comes from the product of (i) the frequency of s_0^α and (ii) the probability that, once arrived at s_0^α , the system follows the trajectory ψ:

$$\Lambda(s_n^\alpha) = \Lambda(s_0^\alpha) \sum_{\psi \in \Psi_n^\alpha} P(\psi) \tag{2}$$

The determination of $P(\psi)$ will be addressed hereafter, whereas that of $\boldsymbol{\Lambda}(s_0^\alpha)$ will be postponed to paragraph 4.3 because it requires formulae introduced in 4.2.

4.1.1. Probability of a trajectory

The probability of a trajectory comes from the product of the probabilities of each one of its transitions. Consider, as an example, the following trajectory:

ψ = { s_0^α , s_a^α , s_b^α , … s_r^α , s_s^α }

Its probability will be:

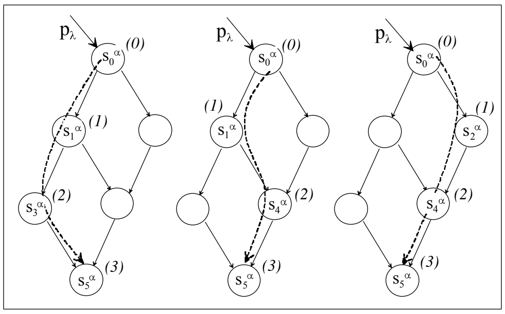

Figure 4. Renumbering of the states within each trajectory

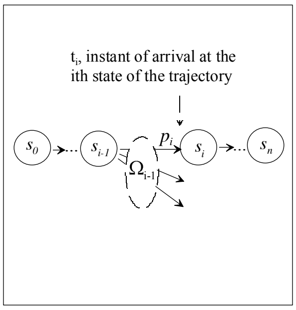

Figure 5. Arrival at the ith state of the trajectory

$= P(s_0 \rightarrow s_a) \times P(s_a \rightarrow s_b) \times \ldots \times P(s_r \rightarrow s_s)$

For the sake of simplicity of the expressions, it will be considered that, within each trajectory, the states are renumbered according to their order, as exemplified in figure 4 for the three trajectories considered above. If the random variable t_i represents the time elapsed between the arrival at the initial failure state s_0 and the arrival at the ith state s_i, the probability of a trajectory leading to the nth state, s_n, may be expressed as:

$$P(\psi) = P(s_0 \rightarrow s_1) \times P_{t_1}(s_1 \rightarrow s_2) \times .. \times P_{t_1 t_2 .. t_{n-1}}(s_{n-1} \rightarrow s_n) \quad \text{or as:}$$

$$P(\psi) = \prod_{i=1}^{n} P_{t_1..t_{i-1}}(s_{i-1} \rightarrow s_i) \tag{3}$$

where $P_{t_1..t_{i-1}}(s_{i-1} \rightarrow s_i)$ represents the conditional probability of transition from s_{i-1} to s_i given that the previous transitions of ψ have occurred at $t_1..t_{i-1}$. These conditional probabilities may, in turn, be evaluated from the following expression:

$$P_{t_1, t_2..t_{i-1}}(s_{i-1} \rightarrow s_i) = \int_0^{\infty} \int_{t_1}^{\infty} .. \int_{t_{i-1}}^{\infty} T(t_i) \, dt_i..dt_2 dt_1 \tag{4}$$

where $T(t_i)$ is the density function of the random variable t_i. This time depends, in turn, on the set of stochastic processes that are active in state s_{i-1}. If Ω_k is the set of processes that are active in a state s_k, and p_i is the process that causes the transition from i-1th to the ith state of the trajectory (figure 5), then the expression for $T(t_i)$ comes from the product of the density function of this process, $f_{p_i}(t_i)$, and the probability that the other processes p of Ω_{i-1} do not occur before t_i ($p \in \Omega_{i-1}$ and $p \neq p_i$). If p is a process of Ω_{i-1} that became active at a previous instant t_p^0, then the density function for the execution of this process is:

$$f_p'(t) = \frac{f_p(t - t_p^0)}{1 - \int_{t_p^0}^{t_{i-1}} f_p(\tau - t_p^0) \, d\tau}, \quad t > t_{i-1}$$

where τ is an auxiliary variable with local scope. Therefore, it results for $T(t_i)$:

$$T(t_i) = \frac{f_{p_i}(t_i - t^0_{p_i})}{1 - \int_{t^0_{p_i}}^{t_{i-1}} f_{p_i}(\tau - t^0_{p_i})\,d\tau} \left(\prod_{\substack{p\in\Omega_i \\ p\neq p_i}} \frac{\int_{t_i}^{\infty} f_p(\tau' - t^0_p)}{1 - \int_{t^0_p}^{t_{i-1}} f_p(\tau - t^0_p)\,d\tau)} d\tau' \right) \tag{5}$$

where:

- t^0_p is the instant of activation of process p, which will always coincide with one of the random variables t_j, with j < i -1;
- $\frac{f_{p_i}(t_i - t^0_{p_i})}{1 - \int_{t^0_{p_i}}^{t_{i-1}} f_{p_i}(\tau - t^0_{p_i})\,d\tau}$ represents the density function of the instant of transition from s_{i-1} to s_i due to p_i;
- $\frac{\int_{t_i}^{\infty} f_p(\tau' - t^0_p)}{1 - \int_{t^0_p}^{t_{i-1}} f_p(\tau - t^0_p)\,d\tau)} d\tau'$ represents the probability that another process p of Ω_{i-1} does not occur before p_i.

Now, combining (3), (4) and (5) the expression for the probability of the trajectory ψ may be obtained from:

$$P(\psi) = \int_0^{\infty} T(t_1) \int_{t_1}^{\infty} T(t_2) .. \int_{t_n - 1}^{\infty} T(t_n)\, dt_n \,..dt_2\, dt_1 \tag{6}$$

If a process p stays active from state s_k (i.e., $t^0_p = t_k$) to state s_m, its density function will participate in the expressions $T(t_j)$ for $k \leq j \leq m$. Therefore, the contribution of p to $P(\psi)$ will be:

$$\frac{\int_{t_{k+1}}^{\infty} f_p(\tau' - t_k)\,d\tau'}{1} \times \frac{\int_{t_{k+2}}^{\infty} f_p(\tau' - t_k)\,d\tau'}{1 - \int_{t_k}^{t_{k+1}} f_p(\tau - t_k)d\tau} .. \frac{\int_{t_{m+1}}^{\infty} f_p(\tau - t_k)\,d\tau'}{1 - \int_{t_k}^{t_m} f_p(\tau - t_k)d\tau}$$

Once $\int_{t_{l+1}}^{\infty} f_p(\tau - t_k)d\tau$ equals $(1 - \int_{t_l}^{t_{l+1}} f_p(\tau - t_k)d\tau)$, the global contribution of p to $T(t_k)$ will be equivalent to $\int_{t_{m+1}}^{\infty} f_p(\tau - t_k)d\tau$. This means that, if a process p is active from s_k to s_m. it is possible to consider its contribution to $T(t_i)$ only at state s_m. This fact leads to a significant simplification of the density functions:

$$T(t_i) = f_{p_i}(t_i - t^0_{p_i}) \left(\prod_{\substack{p\in\Omega_{i-1} \\ p\notin\Omega_i \\ p\neq p_i}} \int_{t_i}^{\infty} f_p(\tau - t^0_p)\,d\tau \right) \tag{7}$$

4.2. Failure states probability

Here, the procedure introduced in paragraph 4.1 will be extended in order to address the probability of the failure states. Assuming, as before, that s^{α}_n is a failure state of a model M, that $\boldsymbol{\Psi}^{\alpha}_n$ is the set of trajectories leading to s^{α}_n and that $P(\psi)$ is the probability of the trajectory ψ, then the probability of s^{α}_n may be obtained from:

$$\mathbf{P}(s_n^\alpha) = \Lambda(s_0^\alpha) \sum_{\psi \in \Psi_n^\alpha} P(\psi) \times \overline{t_n^\psi} \quad (8)$$

where the new term $\overline{t_n^\psi}$ represents the mean sojourn time in s_n^α when this state is achieved following trajectory ψ. If p is a processes of Ω_n, the mean sojourn time in state s_n^α when the transition to next state is caused by p results from the product of (i) the mean execution time of p and (ii) the probability that the other processes of $\boldsymbol{\Omega}_n$ do not occur before p, that is:

$$\int_{t_n}^{\infty} (t_{n+1} - t_n) f_p(t_{n+1} - t_{0p}) \Big(\prod_{\substack{p' \in \Omega_n \\ p' \neq p}} \int_{t_{n+1}}^{\infty} f_{p'}(t' - t_{0p'}) dt' \Big)\, dt_{n+1}$$

As the output transition from state s_n^α may be caused by any of the processes belonging to $\boldsymbol{\Omega}_n$, the total sojourn time $\overline{t_n^\psi}$ may be obtained from:

$$\overline{t_n^\psi} = \sum_{p \in \Omega_n} \int_{t_n}^{\infty} (t_{n+1} - t_n) f_p(t_{n+1} - t_p^0) \Big(\prod_{\substack{p' \in \Omega_n \\ p' \neq p}} \int_{t_{n+1}}^{\infty} f_{p'}(\tau - t_{p'}^0) d\tau \Big)\, dt_{n+1} \quad (9)$$

The expression of $\overline{t_n^\psi}$ depends on the instants of the previous transitions of ψ (due to the instants of activation t_p^0 and $t_{p'}^0$ of the processes belonging to $\boldsymbol{\Omega}_n$. Therefore, this expression should be combined the probability of ψ (6), yielding:

$$\boldsymbol{P}(s_n^\alpha) = \Lambda(s_0^\alpha) \sum_{\psi \in \Psi_n^\alpha} \int_0^{\infty} T(t_1) \ldots \int_{t_{n-1}}^{\infty} T(t_n) \times$$

$$\times \left[\sum_{p \in \Omega_n} \int_{t_n}^{\infty} (t_{n+1} - t_n) f_p(t_{n+1} - t_p^0) \Big(\prod_{\substack{p' \in \Omega_n \\ p' \neq p}} \int_{t_{n+1}}^{\infty} f_{p'}(\tau - t_{p'}^0) d\tau \Big)\, dt_{n+1} \right] dt_{n-1} .. dt_1 \quad (10)$$

The expressions for the states probabilities (as the previous expressions for the states frequencies) depend on the frequency of arrival at the initial failure state, $\Lambda(s_0^\alpha)$, which is addressed in the next paragraph.

4.3. Initial failure states frequency

Depending on the distributions of the failure and the repair processes, four situations regarding the determination of frequencies of the initial failure states have to be considered: (i) exponential failure processes, and a common repair process, (ii) exponential failure processes, and several repair processes, (iii) non-exponential failure processes, and a common repair process and (iv) non-exponential failure processes, and several repair processes.

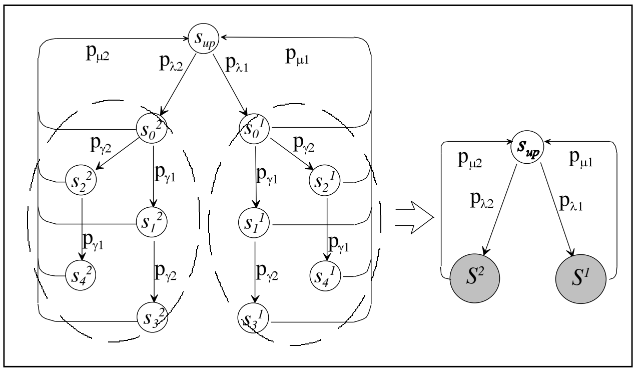

Figure 6. Macro-failure states

Hereafter, just the first one of these situations will be considered. This is the simpler and more common situation found in practical applications regarding FDS systems: the failure processes present exponential distributions; the repair processes are enabled immediately after the occurrence of the failures; and they remain active until the system re-enters the normal operating state s_{up}. In this case, the set of failure states corresponding to a particular failure mode may be grouped in a single macro-state because all of them share the same repair process (figure 6). The mean sojourn time in the macro-state corresponding to failure mode α is:

$$\overline{t^{\alpha}} = \int_0^{\infty} t\, f_{\mu_\alpha}(t)\, dt$$

where $f_{\mu_\alpha}(t)$ is the density function of the repair process. Once the failure rates λ_α are constant and the state probabilities verify:

$$P(s_{up}) + \sum_{s \in \boldsymbol{F}_M} P(s) = 1$$

where $\boldsymbol{F}_M$ is the set of failure states of M, the probability of the normal operating state may be obtained from:

$$\mathbf{P}(s_{up}) = \frac{1}{1 + \sum_{\alpha} \lambda_\alpha \int_0^{\infty} t\, f_{\mu_\alpha}(t)\, dt}$$

Now, the frequency of the initial failure state corresponding to a particular failure mode α may be readily obtained from:

$$\Lambda(s_0^{\alpha}) = \lambda_\alpha \boldsymbol{P}(s_{up}) \qquad (13)$$

5. NUMERICAL RESULTS

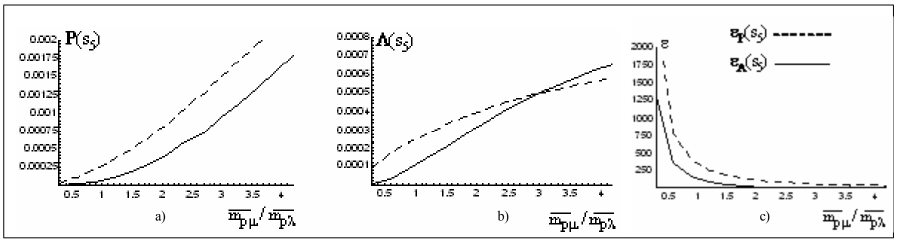

Figure 7. Numerical results

1.a. It is assumed that s_5 is a catastrophic failure state and that its probability and frequency are to be evaluated. The analytical expressions for these two measures were already introduced in paragraphs 4.1 and 4.2 Two scenarios will be considered here for illustrative purposes: scenario 1 where all the processes present exponential distributions and scenario 2 where the repair and delay processes present 3rd order Erlang distributions.

For the sake of simplicity, it is also be assumed that the three delay processes are identical and that their mean $\overline{m_{p\gamma}}$ is 3 hours. For the mean of the repair processes, several values will be considered for the mean ranging from $\overline{m_{p\gamma}}/4$ to $4\overline{m_{p\gamma}}$. Figure 7.a and 7.b represent the evolution of the probability and of the frequency of the catastrophic failure state with the ratio $\rho = \overline{m_{p\gamma}} / \overline{m_{p\gamma}}$, for the two scenarios.

Figure 7.c provides another important result. It shows the error that will be introduced in the evaluation of a system presenting the non-Markovian behaviour corresponding to scenario 2, using the Markovian model of scenario 1 (which is something often done in reliability analysis). The error ε in a reliability measure $\Re$ is calculated from:

$$\varepsilon = \frac{\Re_1 - \Re_2}{\Re_2}$$

where $\Re_1$ *and* $\Re_2$ are the values corresponding to the two scenarios. These results reinforce the idea that, when a model contains concurrent processes having non-exponential distributions, the use of non-Markovian techniques becomes mandatory. In fact, even with this simple system, the error may be high then 1000%.

6. DISCUSSON AND CONCLUSIONS

The paper has presented an approach for the reliability and performance evaluation of ergodic repairable systems containing a Markov regenerative state (corresponding to normal operation) and multiple concurrent processes with generalized distributions. There are well established analytical solutions for the transient and steady state evaluation of regenerative Markov systems. These solutions allow immediate, exponentially distributed and generally distributed timed transitions to be considered but they require that all the non-exponential processes be enabled at the same instant.

As it has been shown, the approach presented here does not impose this important restriction. Other approaches for the evaluation of non-Markovian systems require the consideration of additional variables, whose number increases quickly when the model contains several concurrent processes with narrow hyper-exponential distributions, i.e.

deterministic or quasi-deterministic processes, as happens with the device of stages. In these conditions, the approach presented here may offer a more straightforward solution. In fact, the analytical expressions for the relevant reliability measures may be obtained through a systematic procedure directly from the structure of the model and the distributions of the stochastic processes. There is no need for auxiliary variables, and the expressions may be evaluated using general purpose mathematical tools. The approach has been successfully applied to the study of non-Markov industrial manufacturing systems, distributed information systems and electrical power systems, and it constitutes an effective alternative to simulation based techniques. For relatively small models, containing just a few states and processes, the analytical expressions can be evaluated directly using general purpose mathematical tools. For the evaluation of larger models, the use of these general purpose tools may become ineffective, but it is possible to develop specialized evaluation tools.

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