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ON FORMAL THEORIES AND FORMALISMS FOR VIRTUAL ENTERPRISES

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One of the issues in development and implementation of Virtual Enterprise (VE) is a formal theory, or formal theories, of VE. There are serious problems concerning VE formal theory(ies) development. Besides the ones which are commonly known and easily perceived, one of the biggest problems is a general misunderstanding about formalisms and formal theories (FTs) that occurs too often, substituting the formal theory by the formalism. The paper clarifies in an informal way the differences and some implications between formal theory and formalisms, expecting that it will contribute to a better perception of the problem as well as to the directions and approaches concerning VE formal theory development.

1. INTRODUCTION

One of the issues in development and implementation of Virtual Enterprise (VE) is a formal theory, or formal theories, of VE. There is a number of questions concerning a formal theory (FT), or theories, of VE, ranging from its definition, implementation, use, and similar, to the questions like "why a formal theory of VE?". These questions, and respective responses, are similar to the questions and responses on a formal theory, or theories, of other related concepts as organizations, production or manufacturing systems, enterprises, etc.

Formal theories (FTs) have proved their usefulness in "traditional" engineering areas as mechanical, civil and electrical engineering, and (relatively) more recently in computer sciences (e.g. in the 70s the telecommunications area has identified the use of formal approaches as the only mean to deal with the ever-growing complexity of standards and OSI (Open Systems Interconnection) (Turner, 1993)). Consequently, we could say the lack of a FT of VE is a serious obstacle to effective and efficient development and application of a VE concept. However, concerning the "state-of-the-art" of the development of a FT of VE, the authors are not aware of any consistent and rigorous approach towards the FT of VE. This situation could be explained by the fact that there are serious problems concerning VE formal theory(ies) development. Besides the ones which are commonly known and easily perceived (we will mention some of them later), one of the biggest problems is a general misunderstanding about formalisms and formal theories (FTs) that occurs too often, leading to the substitution of a formal theory by a formalism. This led to the fact that in spite of a number of VE subsystems formal specifications, we do not have, in fact, a FT of VE. Few contributions towards a FT of VE were found by the authors, e.g. (Gruninger and Fox, 1994) and (Janowski et al.; 1998). Within the TOVE (TOronto Virtual Enterprise) project Gruninger and Fox (1994) present a micro-theory for representing "the constraints over the objects and predicates in the ontology".

The objective of this paper is to contribute to the research, development and implementation of VE formal theory, or formal theories (and *not* to present a formal theory of VE – actually, only one example of a formal theory of VE will be presented as a part of discussion), through discussion on some relevant issues. The paper clarifies in an informal way the differences and some implications between Formal Theory and formalisms. In this way, it is expected that the paper will contribute to a better perception of the problem as well as to the directions and approaches concerning VE formal theory.

The results presented in the paper are the results from one of the strategic projects in the Centre for Production Systems Engineering at the University of Minho. The project on Formal Theories of Manufacturing Systems and Enterprises (including the FT of VE) has started on 1997.

2. WHY A VE FORMAL THEORY (OR THEORIES): SOME EXAMPLES OF COMMON MISUNDERSTANDINGS

The need for a FT of VE is mainly due to known reasons as:

- Ambiguities and inaccuracies on the used terminology;
- Inconsistencies and errors on systems specification;
- Conflicting results;
- Implementations not corresponding to specifications;
- The use of a formal language does not imply a formal theory behind;
- The FT allows automation of some phases of VE design process;
- Controlling the growing complexity of standards (Turner, 1993); etc.

Some examples of ambiguities and inaccuracies on the used terminology could be the following. There is not a common interpretation of the terms, i.e. what are we thinking on when we say, "network", "networked (enterprise)", networked organization", or "virtual enterprise", "virtual organization", Figure 1. That is, when we say "network" does it address the networked enterprise or just a networked enterprise domain, i.e. the domain over which the networked enterprise is integrated? The networked enterprise is a subset or a special case of the networked organization or the opposite? Also, could a hierarchy (Figure 2.a) be interpreted as a special case of a network (Figure 2.b)[1] or not. The network is characterized by

[1] The Figures 2.a and 2.b represent hierarchical and network structures, respectively, only under a certain interpretation (implying the interpretation of each arc as a specific relation). As Figures 2.a and 2.b do not have that arc interpretation, somebody could claim that they do not represent hierarchical and network structures. From the graph theory point of view, Figures 2.a and 2.b could be seen as two types of graphs.

relation or by operation? What is the exact meaning of “collaborative”? In an enterprise, even with sequentially performed processes, Figure 3.a, we would say that, anyway, they collaborate. Or, “collaboration” means only that any process is linked to all the other processes and they perform simultaneously, Figure 3.b. Are “hybrid” structures permitted, in the context of “collaborative”, Figure 3.c? Is the networked enterprise, or VE, subset, or a special case, of “traditional” enterprise, Figure 4.a, or vice versa, Figure 4.b? What is the (rigorous, of course) criteria? What is the (rigorous, of course) definition of a (VE) Reference Model[2]? Is VE necessarily a networked structure? Etc.

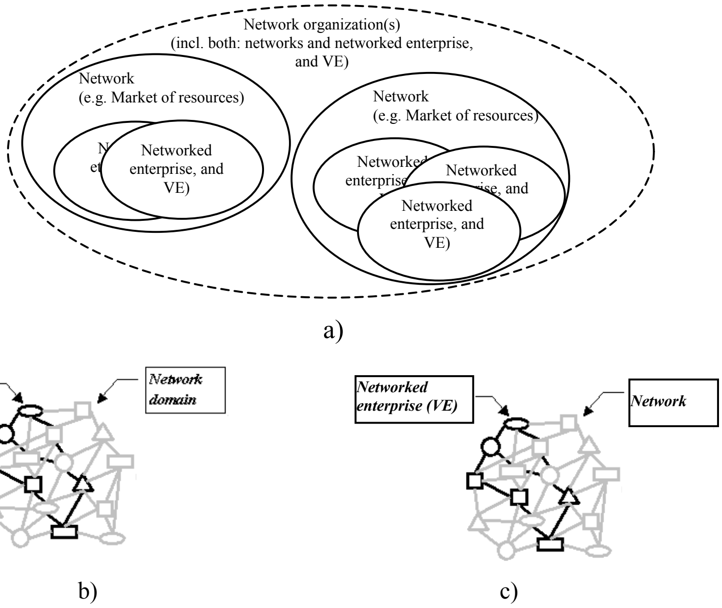
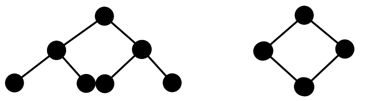


Figure 1 – Different interpretations of the terms “network”, “networked enterprise”, “networked organization”, “virtual enterprise”, “virtual organization”.



a) b)

Figure 2 – a) Hierarchical and b) networked structures – usual representation

3. FORMALISMS VS FORMAL THEORIES

It is absolutely necessary to clarify one of the biggest problems for the objective of developing a VE formal theory. That is a general misunderstanding within the engineering community about formalisms and formal theories (FTs), as it is referred above.

[2] E.g., by some authors a condition for a model to be a reference model is its acceptance by the community. By other opinions, that is not a necessary condition. This question deserves a more detailed discussion in some other paper(s).

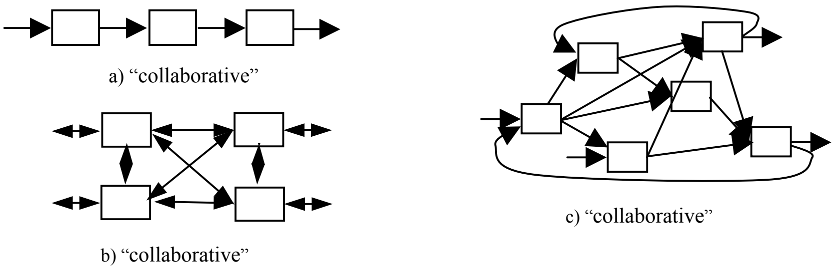


Figure 3 – Different models of the "Collaborative" systems

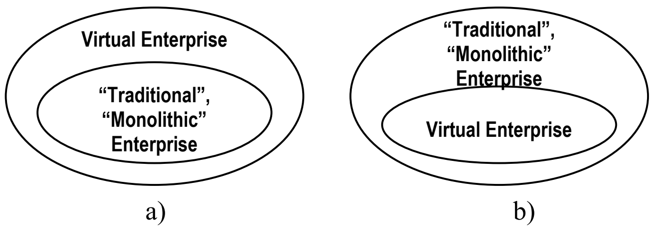


Figure 4 – Relation between VE and "traditional", "Monolithic", Enterprise

Formal Theory is build upon several concepts under well defined conditions[3]. The process of building, or defining, a formal theory could be described (informally) as follows. Based on: (1) some symbols (variables, constant symbols and function symbols) of an **alphabet** and, (2) a set of rules (formation rules, induction rules, or **calculus of terms**), we construct **terms**.

Based on: (1) **terms** and, (2) on the other **alphabet symbols** (including relation symbols) and according to another set of rules (**calculus of formulas**), we construct **Formulas**. The **set of all the formulas**, one can build up from a given **alphabet** using the **calculus of formulas**, is a **language** denoted by L^S (S is the set of alphabet symbols associated to the object concept). Therefore, a **language** is a set of **formulas**. Among all the **formulas** that constitute a **language**, some have a special characteristic – they have no free occurrences of variables (that is, variable occurrences out of $\forall$, $\exists$ quantifiers scope). These **special formulas** are designated as **sentences** (a **formula** without quantifiers and variables is also a **sentence**). A **formula** is **satisfiable** if there is at least one **interpretation** under which that **formula** is true. An **interpretation** J is composed by a **structure** A and an **assignment** β of variables (that is, $J = (A, \beta)$). A **structure** is constituted by a domain A and a map a (that is, $A = (A, a)$). The map a assigns a relation, a function and a constant, from the domain A, to each symbol for relation, function and constant from the **alphabet** symbol set S.

If under a given **interpretation** a **formula** becomes true then that **interpretation** is a **model** of that **formula** (and the **formula** is obviously **satisfiable**). A **sentence** is **satisfiable** if there is at least one **interpretation** under which that sentence is true. When applying an **interpretation** to a **sentence** the **assignment** of variables is

[3] Examples of terms, formulas and other relevant concepts are omitted due to paper length restriction. However, they can be found in (Sousa and Putnik; 2004).

irrelevant (as the **sentence** has no variables occurring free). Thus one can say that a **sentence** is **satisfiable** if exists at least one **structure** making the **sentence** true, that is, if exists at least one **structure** which is a **model** of the **sentence**. As seen before, some of the **formulas** that constitute a **language** are **sentences**. From all those **sentences** some will eventually be **satisfiable**. The subset of **sentences** whose elements are **satisfiable sentences**, **and closed under consequence**, is a **Theory**. Because **sentences** are **satisfied** by a **structure** A the referred subset of **sentences** can be designated by **Theory of** A.

Therefore, a **set of formulas** is a **language**. Some of those **formulas** could be **sentences**. Some of those **sentences** could be **satisfiable**. The last ones are a **Theory** (Figure 5).

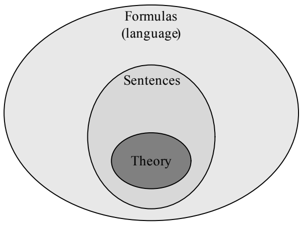


Figure 5 – Formulas, sentences and theory

A **theory** is in fact a **language** which is a subset of an **involving language**.

A **language** could be a **theory** (if exists a **model** for that subset of **sentences**).

A **language** may include none, one or more **theories**.

When applying different **interpretations** to the whole set of **sentences** each one of these **interpretations** could eventually satisfy different subsets of **sentences**. Each one of these subsets is, according to the definition, a **theory**. Thus, the same **language** could include several **theories**, Figure 6. These **theories** can be disjoint, or they can have a common part, or even one **theory** includes another **theory**. (the last situation is referred in [Mendelson, 1987, page 171] via the **finite extension of a theory** concept. In Fig. 6(c), **theory 1** will be a **finite extension of a theory 2**.)

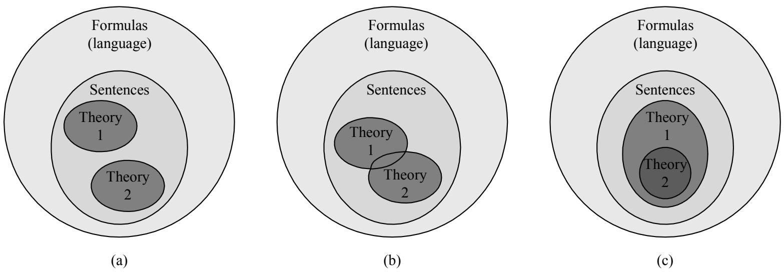


Figure 6 – Theories (a) disjoint (b) intersecting (c) included

The kind of theory described is called **First Order Theories**. Some other characteristics are presented:

- A first order theory T is ***consistent*** if and only if there is no *S-sentence* φ such that both φ and $\neg\varphi$ are theorems of T.
- A first order theory T is ***complete*** if for every *S-sentence* φ, $\varphi \in T$ or $\neg\varphi \in T$.

- A first order theory T is ***axiomatic*** if there is a decidable set Φ of *S-sentence* φ such that $T = \Phi^{\models}$.
- A first order theory T is ***decidable*** if there is an **effective procedure** to determine if every *S-sentence* φ is a theorem of T.
- A first order theory T is ***enumerable*** if there is an effective procedure to list all theorems of T.

Therefore, we could say (informally) that a Formal Theory is "a set of statements closed under certain rules of inference." (Wikipedia, 2006). Some of the statements are "initial" statements i.e. axioms, and the statements are composed by a finite set of symbols, i.e. finite "alphabet". We could say now, concerning formalism vs. formal theory of some *object* concept, that formalism means use of any formal language, or even some formal theory (e.g. Set theory, graph theory, etc.) for the concept representation, but, we can not say that we have a (theorem of the) formal theory of the object concept, or that we contribute to the formal theory of the object concept. The sentences from the theory, we can call them a *representational* or *involving* "theory", and correspondent "representational" or "involving" language, which we used for the object concept representation, are the theorems of the *involving* "theory" but they are not, by default, theorems of the object concept formal theory. They could be the theorems of the object concept only by chance or the theorems of an unknown underlying object concept formal theory.

In other words, considering that the formal theory is a language, an object **theory** is in fact a **language** which is a subset of an **involving language** (of the theory used for representation of the object theory). Consequently, we may say, Figure 7.:

Formalism $\neq$ Formal Theory
Formalism $\supseteq$ Formal Theory
Formal Theory $\Rightarrow$ Formalism
but
$\neg$ (Formalism $\Rightarrow$ Formal Theory)

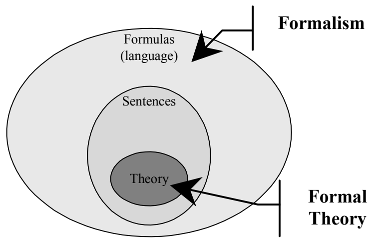


Figure 7 – Formalism and Formal Theory

Thus, the next question is: What does it mean, then, when we use some formalism, i.e. a formal representation for VE (VO – Virtual Organization, NO – Network Organization (e.g. set theory, graph theory, Petri nets, "multi-agents", "metaheuristics", "dynamic programming", C, C++, SDL, RSL, PROLOG, ...) **?** This question is already responded above implicitly, but let us be more "object oriented". Regarding VE, or VO, or NO, formalism means use of any formal language, or even some formal theory (e.g. Set theory, logic theory, or graph theory, etc.) for VE, or NO, representation, but, we can not say that we have a (theorem of the) formal theory of VE (VO, NO), or that we contribute to the formal theory of VE. Each formal specification of VE (VO, NO) instance, or "model", that use an involving language of some other theory (e.g. set theory, logic theory, graph theory, Petri nets, "multi-agents", "metaheuristics", "dynamic programming", C, C++, SDL, RSL, PROLOG, ...), represents, if it is assumed it is a formula, **a "formula" of an unknown underlying formal theory of VE**, Figure 8. The underlying formal theory is "unknown" because we can assume that the formula is a theorem, or even an axiom, but the derivation rules are unknown. In special cases we could assume that the formula is the only theorem (axiom) representing, thus, a trivial case. This case we would call "**a trivial theory**" (this is because **there is no usefulness**, and **there is no science!,** of that case in terms of a VE FT), Figure 9.

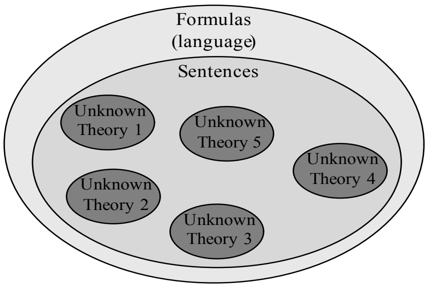


Figure 8 – Sentences as representatives. of "unknown" theories

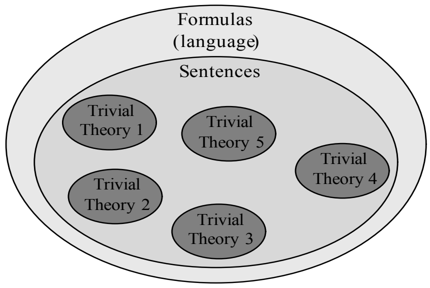


Figure 9 – Sentences as "trivial" theories

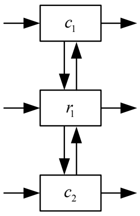
The derivation rules of the involving formal language used could lead to inconsistencies and other "errors" (in the sense of the formal theory, but in practical sense as well), regarding the VE model and theory intended. This is because we can be "led" to claim, or to assume, that the theorems of the "involving" theory, e.g., graph theory, are the theorems of our object VE theory. The consequence of this kind of assumption is that we could derivate another theorems using the production rules of the "involving" theory, e.g. graph theory, for which we would believe represents the theorems of the object VE theory while in practice they are not[4]. Otherwise, the "involving" theory, or language, e.g. graph theory, would be equivalent to the object VE theory. In consequence of this eventual equivalency the question is why we talk at all about VE.

4. AN EXAMPLE – THE CASE OF BM_VE FORMALISMS AND FORMAL THEORY

To exemplify the above, we present two instances of BM_Virtual Enterprise[5] (BM_VE) organizational structures, Figure 10.a and Figure 11.a. Their description formulas, or their formal descriptions, are presented on Figure 10.b and Figure 11.b respectively. Presenting these specifications only, even if they are formal and rigorous, we can not say that they are produced rigorously in compliance with eventual BM_VE (formal) theory because we are not sure about the production rules of that BM_VE (formal) theory.

[4] See the footnote 1. The comment in footnote 1 illustrates the above.
[5] BM_Virtual Enterprise (BM_VE) is a VE in total or partial compliance with the BM_Virtual Enterprise Architecture Reference Model (BM_VEARM), (Putnik, 2001), (Sousa, 2003), (Putnik et al., 2005).

Without knowing these rules, if we need to specify another BM_VE organizational structure instance we can make an error, i.e. we can specify a structure that does not follow the (derivation) rules by which the structures in Figure 10 and Figure 11 are constructed (actually, although we can claim that both structures in Figure 10 and Figure 11 are constructed by the same rules, rigorously speaking, only apparently they follow the same rules).

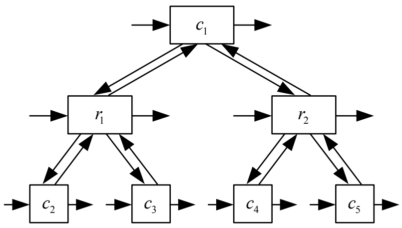


a)

$$c_1\left(\downarrow\uparrow r_1\left(\downarrow\uparrow c_2\right)\right)\equiv s_{eq}$$

b)

Figure 10 – An instance of BM_VE organizational structure



a)

$$c_1\left(\downarrow\uparrow r_1\left(\downarrow\uparrow c_2 c_3\right) r_2\left(\downarrow\uparrow c_4 c_5\right)\right)$$

b)

Figure 11 – An instance of BM_VE organizational structure

But let us analyze the language defined bellow, represented by the grammar G_{BM}:

$$G_{BM} = (V_T, V_N, S, R) \quad \text{where}$$

$$V_T = \left\{c_1,\ldots,c_{n_c}, r_1,\ldots,r_{n_r}, s_{eq}, \equiv, \downarrow\uparrow,), (\right\}$$

$$V_N = \left\{S, A, B\right\}$$

$$\begin{aligned} R = \{\; & S \rightarrow c_i\left(\downarrow\uparrow A\right) \equiv s_{eq}, \\ & A \rightarrow r_i\left(\downarrow\uparrow B\right), \\ & A \rightarrow AA, \\ & B \rightarrow c_i\left(\downarrow\uparrow A\right), \\ & B \rightarrow BB, \\ & B \rightarrow c_i \;\; \} \end{aligned}$$

The BM_VE organizational structure instances in Figure 10 and Figure 11 are generated by the grammar G_{BM}. Moreover, using the grammar G_{BM} we can produce another structure instances, e.g. the structure instance in Figure 12.a, with the correspondent "textual" representation, or "formula", in Figure 12.b.

If we consider all the words generated by the grammar G_{BM} as the instances of the BM_VE organizational structures, then we will say that the grammar G_{BM}[6] is a formal theory of BM_VE organizational structures.

To deal with other aspects of BM_VE, or VE in general, other grammars, as the FT models, would be necessary. At the moment, this work is under development.

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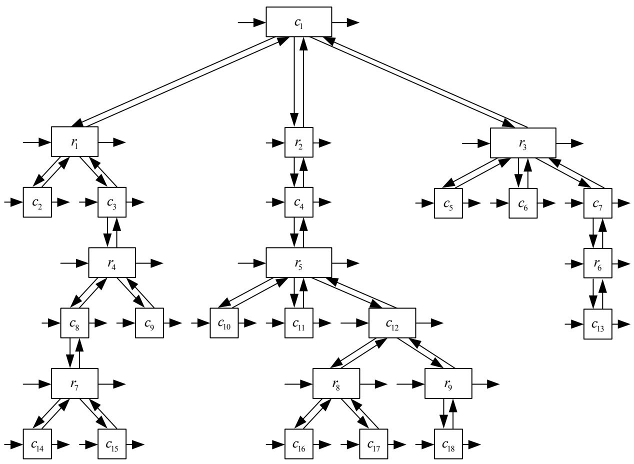
[6] Rigorously, the grammar G_{BM} is an attributed grammar. For the purpose of this paper, the grammar attributes presentation is omitted as it does not influence qualitatively the paper's discourse.



a)

$$c_1\left(\downarrow\uparrow r_1\left(\downarrow\uparrow c_2 c_3\left(\downarrow\uparrow r_4\left(\downarrow\uparrow c_8\left(\downarrow\uparrow r_7\left(\downarrow\uparrow c_{14}c_{15}\right)\right)c_9\right)\right)\right)\right)r_2\left(\downarrow\uparrow c_4\left(\downarrow\uparrow r_5\left(\downarrow\uparrow c_{10}c_{11}c_{12}\left(\downarrow\uparrow r_8\left(\downarrow\uparrow c_{16}c_{17}\right)r_9\left(\downarrow\uparrow c_{18}\right)\right)\right)\right)\right)r_3\left(\downarrow\uparrow c_5 c_6 c_7\left(\downarrow\uparrow r_6\left(\downarrow\uparrow c_{13}\right)\right)\right)$$

b)

Figure 12 – An instance of BM_VE organizational structure

5. WHY NOT A VE FORMAL THEORY (OR THEORIES)

The FT approach has many critics. Four (selected) critics of the FT approach are:

- **It is difficult to understand** (from the cognitive point of view), difficult to learn, difficult to develop;
- **It is difficult** by itself, difficult to develop (objectively; analytical or formal approach - highly complex problem (in terms of complexity theory)).
- Difficult, or impossible, to cover all necessary practical requirements, i.e. user's needs.
- May invalidate practical results if these are not obtained from the theory, or may lead to results without practical importance, etc.

6. CONCLUSIONS

The usefulness of the formal theory, e.g. the formal theory of BM_VE, is obvious. The majority of the problems referred in the section 2 of this paper could be much more easily managed, if not resolved at all. Especially, it is obvious the contribution of a formal theory for engineering tasks, that is, in design, implementation, maintenance, etc., providing the so much desired efficiency and effectiveness (for many engineering tasks). As we have said above, formal theories have proved their usefulness in "traditional" engineering areas. Somebody said: "There is nothing as practical as a good theory".

We would interpret it saying that having a FT of VE means to have **an excellent tool** for VE efficient development, implementation and control.

Naturally, aiming at development of a FT of VE, there is a number of questions to be addressed. Some of them are:

1) Existence of a FT of VE,
2) The theory's "external" problems, i.e. the theory object's nature, i.e. VE nature (which is the main purpose of developing the FT of VE)
3) The theory's "internal" problems, i.e. the properties of the FT of VE itself
4) Relation between FT of VE and FT of PS/MS (Production System/Manufacturing System),
5) Models of FT of VE
6) Development "strategy", etc.

The authors have started the project on FT for VE development, within the research on general issues of formal theories of production systems. Actually, it is developed a FT model for the "canonical" model of BM_VE – the grammar G_{BM} presented in the section 4 (in accordance with the BM_VEARM, under development on the University of Minho, (Putnik, 2001), (Putnik et al., 2005), (Sousa, 2003)).

Concerning the relation between VE formal theory and Production/ Manufacturing Systems (PS/MS) formal theory, the phenomena of the issue is apparently the same (or at least very similar). However, it is an open question if the VE formal theory ia a sub-theory of a PS/MS theory or vice versa, or there is some another relation. This question could have a practical implication in terms of how could we plan development of a VE formal theory, i.e., starting with development of PS/MS formal theory and apply it to VE or starting an independent development, etc. Finally, the lack of the FT of VE doesn't mean that we do not have the Theory of VE. It only means that we do not have a Formal Theory of VE.

7. REFERENCES

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