Nonlinear Optimization of GM(1,1)Model based on Multi-Parameter Background Value

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Abstract. By studying the existing algorithms for background value in GM(1,1), a nonlinear optimization model of GM(1,1) based on multi-parameter background value is given. The paper uses the invertible matrix of the parameter to optimize and estimate the parameters $\hat{a}$; in addition, the parameter estimate $\hat{a}$ obtained from the multi-parameter background value has higher prediction accuracy, thus overcoming the restriction on the prediction based on the fixed average background value in other literatures. the simulated values obtained by the optimized model (NOGM(1,1)) are more precise, and the maximum error is reduced by 15%. The nonlinear optimization model of GM(1,1) based on multi-parameter background value provides algorithms for further study of GM(1,1) model.

Keywords.background value; parameter; nonlinear optimization; ratio

1 INTRODUCTION

The grey model GM(1,1) is a forecast model in grey system. It is important to the application of grey system theory. Successful examples of using GM(1,1) for prediction can be found in many fields [1-2]. Much research has been done on this model, and a number of methods have been proposed to improve it [3-9]. Fundamentally, the effect of the change of the background value on the fitted value has been analyzed. However, when solving GM(1,1), the area is partitioned and the background value is obtained through approximate iteration, or the solution can be obtained through a parameter expression derived from a special background value model.

As for the background value, the $(1-\delta:\delta)$ ratio established between the background value and the two endpoint prototype values restricts the expression and estimation of grey differential equation, which causes the estimated parameters to produce a greater prediction error. The model solution is probably imprecise. This paper discusses the GM(1,1) model of the background value with two or more parameters obtained through the extension of the background value, the nonlinear optimization of the analog value with parameters called the nonlinear optimization model of GM(1,1) based on the background value with multi-parameters (NOGM(1,1)).

2 THE NONLINEAR OPTIMIZATION MODEL OF GM(1,1) BASED ON THE MULTI-PARAMETER BACKGROUND VALUE

2.1 The grey model GM(1,1) of background value with two parameters

Denote the original sequence by $X^{(0)} = (X^{(0)}_{(1)}, X^{(0)}_{(2)}, \ldots, X^{(0)}_{(N)})$, calculate an AGO $X^{(1)}_{(k)} = \sum_{i=1}^{k} X^{(0)}_{(i)}$,get the new sequence $X^{(1)} = (X^{(1)}_{(1)}, X^{(1)}_{(2)}, \ldots, X^{(1)}_{(N)})$,albinism differential form of GM(1,1) is $\frac{dX^{(1)}}{dt} + aX^{(1)} = b$ (1)

Difference form of GM(1,1) is $X^{(0)}_{(k)} + a * z^{(1)}_{(k)} = b$ (k=2, 3, 4, …, N) (2)

There is $z^{(1)}_{(k+1)} = \mathrm{r_k} * X^{(1)}_{(k)} + s_k * X^{(1)}_{(k+1)}$, solving (2) and getting parameter

$\hat{a} = [a, b]^T = (B^T B)^{-1} B^T Y_N$ (3)

Where $Y_N = (X^{(0)}_{(2)}, X^{(0)}_{(3)}, \ldots, X^{(0)}_{(N)})$,

B= $\begin{bmatrix} -z^{(1)}_{(2)} & -z^{(1)}_{(3)} & \cdots & -z^{(1)}_{(N)} \\ 1 & 1 & \ldots\ldots & 1 \end{bmatrix}^T$, the model of the background value $z^{(1)}_{(k+1)}$ with parameters r_k and s_k is obtained, and it is called the grey model GM(1,1) of the background value with two parameters.

2.2 The nonlinear optimization model of GM(1,1) for the multi-parameter background value

In order to achieve parameter estimation based on the multi-parameter background value, inverse matrix must be directly expressed by easy other matrix. The following is the process to get the parameter estimate through invertible matrix.

Theorem 1: If F= $\sum_{k=2}^{N}(z_{(k)}^{(1)})^2$, and C= $\sum_{k=2}^{n} z_{(k)}^{(1)}$,

$$(B^T B)^{-1} = \frac{1}{(N-1)F - C^2}\begin{bmatrix} N-1 & C \\ C & F \end{bmatrix}$$

Proof: Because $B^T B = \begin{bmatrix} \sum_{k=2}^{N}(z_{(k)}^{(1)})^2 & -\sum_{k=2}^{N} z_{(k)}^{(1)} \\ -\sum_{k=2}^{N} z_{(k)}^{(1)} & N-1 \end{bmatrix} = \begin{bmatrix} F & -C \\ -C & N-1 \end{bmatrix}$,

$$(B^T B)^{-1} = \frac{(B^T B)^*}{\left|B^T B\right|} = \frac{1}{(N-1)F - C^2}\begin{bmatrix} N-1 & C \\ C & F \end{bmatrix}.$$

Theorem 2: If D= $\sum_{k=2}^{N} X_{(k)}^{(0)}$ and E= $\sum_{k=2}^{N} X_{(k)}^{(0)}.z_{(k)}^{(1)}$, $B^T Y_N = \begin{bmatrix} -E \\ D \end{bmatrix}$

Theorem 2 is easily established.

Derived from Theorem 1and Theorem 2 is equation (3), the parameter estimate $\hat{a} = [a,b]^T = (B^T B)^{-1} B^T Y_N$, where

$$a = \frac{C*D-(N-1)E}{(N-1)*F-C^2}, b = \frac{D*F-C*E}{(N-1)F-C^2} \quad (4)$$

Substituting a and b in the discrete responsive type

$$\hat{X}_{(k+1)}^{(1)} = (X_{(1)}^{(0)} - \frac{b}{a})*e^{-a*k} + \frac{b}{a} \qquad (5)$$

Because matrix B and estimation parameter $\hat{a}$ include parameters r_k and s_k , parameters a and b cannot be directly calculated by (3), but will be calculated by the nonlinear optimization model as follows:

Step1: Get a and b with parameters r_k and s_k and the expression of the fitted value.

Get the fitted value $\hat{X}^{(0)}_{(k+1)}$ of the original data $X^{(0)}_{(k+1)}$ from form (5),

and $\hat{X}^{(0)}_{(k+1)} = \hat{X}^{(1)}_{(k+1)} - \hat{X}^{(1)}_{(k)}$ ($k =$

0,1,…,N-1), $\hat{X}^{(0)}_{(1)} = X^{(0)}_{(1)}$

Step2: Get and solve the expression of the minimum object function:

min $f(r_i, s_i) = \sum_{k=2}^{N} (\hat{X}^{(0)}_{(k)} - X^{(0)}_{(k)})^2$ (6)

Use MATLAB to program and calculate r_k and s_k .

Step3: Substitute r and s in (3) to calculate a and b, substitute r_k and s_k in (4) to calculate the fitted value $\hat{X}^{(1)}_{(k+1)}$, and get the error values.

3 APPLICATION OF THE NONLINEAR OPTIMIZATION MODEL OF GM(1,1) FOR THE BACKGROUND VALUE WITH PARAMETERS

Use the GDPs of china from 1991 to 2003 to set background values with two parameters r and s in NOGM(1,1), and get r=0.0267, s=0.8945,but r+s $\neq 1$ through (3), (4), and (5). The results show that the model that expresses the background value better is not based on the $(1-\delta : \delta)$ ratio between the two endpoint values. The values simulated in the NOGM(1,1) are compared with those simulated in NGM(1,1)[7] in the following table 1.

Table 1. Comparison between values simulated in NOGM(1,1) and those in NGM(1,1)

year	Original data	Simulated data in DGM(1,1)	Simulated error in DGM(1,1) %	Simulated data in NOGM(1,1) with two parameters	Simulated error in NOGM(1,1) with two parameters %
				a=-0.1040,b=34468	
1991	21617.8	21617.8	0	21617.8	0
1992	26638.1	40500	52.04	36548	37.21
1993	34634.4	44750	29.22	42754	23.4437
1994	46759.4	49450	5.76	47419	1.41063
1995	58478.1	54640	6.56	52594	10.0621
1996	67884.6	60380	11.05	58332	14.0718
1997	74462.6	66720	10.4	64697	13.1148
1998	78345.2	73720	5.9	71756	8.4107
1999	82067.5	81460	0.73	79586	3.02373
2000	89468.1	90020	0.61	88269	1.34025
2001	97314.8	99470	2.21	97901	0.602375
2002	105172.3	109910	4.51	108580	3.24011
2003	117251.9	121450	3.58	120430	2.71049
Average error			10.1977		9.167

4 CONCLUSION

GM(1,1) model has its adaptation range[6]. By optimizing and making background values parametric, the paper uses the invertible matrix of the parameter to optimize and estimate the parameters $\hat{a}$,the improved model overcomes the dependence of the background value on the $(1-\delta:\delta)$ ratio between the two endpoint values in other literatures. It provides a better prediction model for the conversion from a continual form to a discrete form. The examples indicate that the simulated value obtained by the model NOGM(1,1) is more precise, and the maximum error is reduced by 15%. However, because the simulated value is expressed by an exponential function, the simulating error is bigger at some points. Then, how to reduce the overall error will be a concern of the future study.

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