On Integral Sum Numbers of Cycles

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Abstract. In this paper we determine that integral sum number of graph C_n, namely for any integer $n \geq 5$, then $\xi(C_n) = 0$, therefore we prove that the graph $C_n (n \geq 5)$ is an integral sum graph.

Key Words: Integral sum number, Integral sum graph, Graph C_n

1 Introduction

The graph in this paper discussed are undirected、no multiple edges and simple graph, the unorganized state of definitions and terminology and the symbols in this paper referred to reference [1],[2].

F.Harary[3] introduce the concept of integral sum graphs. The integral sum graph $G^+(S)$ of a finite subset $S \subset Z$ is the graph (V, E), where $V = S$ and $uv \in E$ if and only if $u + v \in S$. A graph G is an integral sum graph if it is isomorphic to the integral sum graph number of $G^+(S)$ of some $S \subset Z$. The integral sum number of a given graph G, denoted by $\xi(G)$, is defined as the smallest nonnegative integer S such that $G \bigcup sk_1$ is an integral sum graph. For convincing, an integral sum graph is written as an integral sum graph in references [3, 4]. Obviously, graph G is an integral sum graph iff $\xi(G)=0$.

It is very difficult to determine $\xi(G)$ for a given graph G in general. All paths and matchings are verified to be integral sum graph in references [3], and we see from references [4] that $\xi(C_n)\le 1$ for all $n\ne 4$. And further, an open conjecture was posed in references [4] as followss:

Conjecture[4]: Is it ture that any old cycle is an integral sum graph?

Definition 1.1. If a graph is isomorphism graph $G^+(S)$, then we call graph G is Integral sum graph, denoted by $G\cong G^+(S)$.

Definition 1.2. For graph G, if it exists nonnegative integer S such that $G\bigcup sk_1$ is an integral sum graph, then we call number s is integral sum number of G, denoted by $\xi(G)=s$.

2 Main Results and certification

Theorem 2.1. For any integer $n\ge 3$, then

$$\xi(C_n)=\begin{cases}3, & \text{when } n=4;\\ 0, & \text{when } n\ne 4.\end{cases}$$

Proof. It is immediate from references [3, 4] that $\xi(C_3)=0$ and $\xi(C_4)=3$. And it is clear that $C_5\cong G^+\{2,1,-2,3,-1\}$ and $C_7\cong G^+\{1,2,-5,7,-3,4,3\}$.

Next we consider two cases: For all $C_{2j}(j\ge 3)$ and $C_{2j+1}(j\ge 4)$, we will show that the two classes of cycle are integral sum graph.

Let the vertices of $C_{2j}(j\ge 3)$ and $C_{2j+1}(j\ge 4)$ be marked as the methods in Fig.1 and Fig.2.

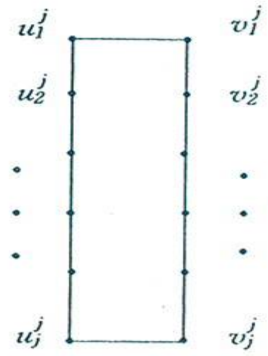

Fig.1. marking of C_{2j}

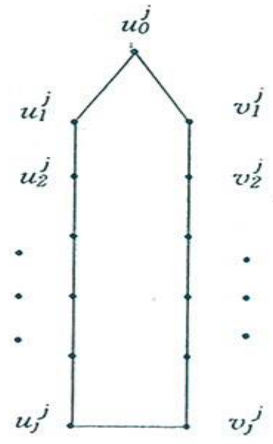

Fig.2. marking of C_{2j+1}

Case 1 When $n = 2j\ (j \geq 3)$

We first give the labels of C_6 and C_8 as follows:

Let $u_1^3 = 4,\ u_2^3 = -1,\ u_3^3 = 5;\quad v_1^3 = 1,\ v_2^3 = 3,\ v_3^3 = -2$.

and

$u_1^4 = 7,\ u_2^4 = -2,\ u_3^4 = 9,\ u_4^4 = -11;\ v_1^4 = 2,\ v_2^4 = 5,\ v_3^4 = -3,\ v_4^4 = 8$.

Then $C_6 \cong G^+\{4, -1, 5, -2, 3, 1\}$ and

$C_8 \cong G^+\{7, -2, 9, -11, 8, -3, 5, 2\}$, therefore $\xi(C_6) = 0$ and $\xi(C_8) = 0$.

When $j \geq 5$, we give the labels of C_{2j} as follows:

Let $u_1^j = u_1^{j-1} + u_1^{j-2}\quad (j \geq 5);\quad u_2^j = u_2^{j-1} + u_2^{j-2}\quad (j \geq 5);$

$v_1^j = v_1^{j-1} + v_1^{j-2}\quad (j \geq 5);\quad v_2^j = v_2^{j-1} + v_2^{j-2}\quad (j \geq 5).$

and

$$u_k^j = u_{k-2}^j - u_{k-1}^j \quad (k = 3, 4, \cdots, j);$$

$$v_k^j = v_{k-2}^j - v_{k-1}^j \quad (k = 3, 4, \cdots, j).$$

By labeling of above, we know $\xi(C_{2j}) = 0\,(j \ge 5)$.

The labeling of above is illustrated in Fig.3.

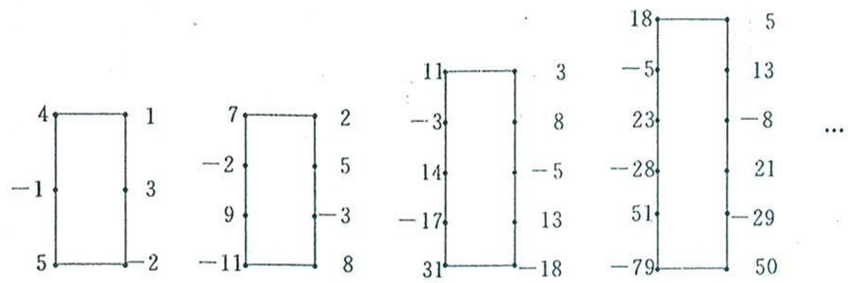

Fig.3. labeling of $C_{2j}\,(j \ge 3)$

Case 2 When $n = 2j + 1\,(j \ge 4)$

We first give the labels of C_9 and C_{11} as follows:

Let $u_0^4 = 2,\ u_1^4 = 5,\ u_k^4 = u_{k-2}^4 - u_{k-1}^4\ (k = 2, 3, 4)$;

$$v_1^4 = -5,\ v_2^4 = 7,\ v_k^4 = v_{k-2}^4 - v_{k-1}^4\ (k = 3, 4).$$

and

$$u_0^5 = 3,\ u_1^5 = 8,\ u_k^5 = u_{k-2}^5 - u_{k-1}^5\ (k = 2, 3, 4, 5);$$

$$v_1^5 = -8,\ v_2^5 = 11,\ v_k^5 = v_{k-2}^5 - v_{k-1}^5\ (k = 3, 4, 5).$$

Then $C_9 \cong G^+\{2, 5, -3, 8, -11, 19, -12, 7, -5\}$ and

$C_{11} \cong G^+\{3, 8, -5, 13, -18, 31, -49, 30, -19, 11, -8\}$, therefore $\xi(C_9) = 0$

and $\xi(C_{11})=0$.

When $j \geq 6$, we give the labels of C_{2j+1} as follows:

Let $u_0^j = u_0^{j-1} + u_0^{j-2} \quad (j \geq 6)$; $u_1^j = u_1^{j-1} + u_1^{j-2} \quad (j \geq 6)$;

$$v_1^j = v_1^{j-1} + v_1^{j-2} \quad (j \geq 6);$$

and

$$u_k^j = u_{k-2}^j - u_{k-1}^j \quad (k = 2, 3, \cdots, j);$$

$$v_k^j = v_{k-2}^j - v_{k-1}^j \quad (k = 2, 3, \cdots, j).$$

Where $v_0^j = u_0^j$ for $j \geq 6$.

By labeling of above, we know $\xi(C_{2j+1}) = 0\,(j \geq 6)$.

The labeling of above is illustrated in Fig.4.

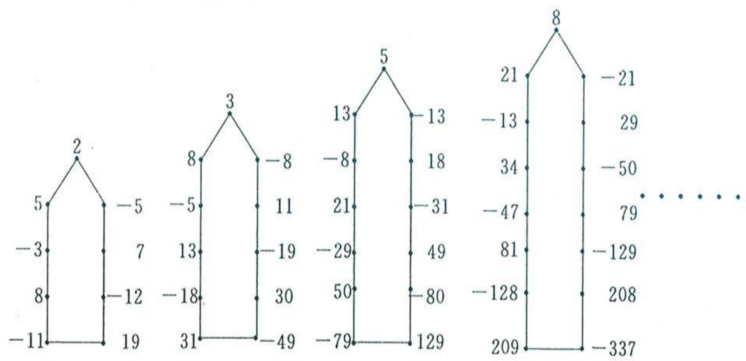

Fig.4. labeling of $C_{2j+1}(j \geq 6)$

Theorem 2.2. For any integer $n \geq 5$, the graph C_n is integral sum graph.

Proof. From theorem2.1, we know $\xi(C_n)=0\,(n\geq 5)$, therefore for any integer $n\geq 5$, the graph C_n is integral sum graph.

References

1. F. Harary. Graph Theory[M]. Addison Wesley. Reading MA, 1969.
2. Bondy J A,Murty U S R.Graph Theory with Applications[M].Elsevier North-Holland, 1976.
3. F. Harary. Sum graphs over all the integers[J]. Discrete Math. 1994(124):99~105.
4. Baogen Xu. On integral sum graphs[J]. Discrete Math. 1999(194):285~294.
5. Achaya B D and Hegde S M.Arithmetic graphs[J]. Journal of Graph Theory,1990, 18(3):275-299.
6. Hui Yu.The Windmill W_n^* is the intergral sum Graph and Mod intergral sum Graph[J]. Journal of Heze University. 2006, 28(3):18-19.
7. Gao Xiu-lian.Several kinds Interal Sum Tree , Journal of Dezhou University. 2011, 17(2):18-22.