Study on the GPS Data De-noising Method Based on Wavelet Analysis*

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Abstract：Signal de-noising is one of the classical problems in the field of signal processing. As a new signal processing tools, wavelet analysis, which has excellent noise performance, has caused growing concern and attention. The wavelet threshold de-noising has been researched systematacially, and the wavelet de-noising method is used on the GPS signal, which has achieved very good results.

Key words: wavelet; threshold; de-noising; GPS signal

1 Introduction

Analysis of wavelets theory is part of the branch of mathematics-----functional analysis, which is developed on the basis Fourier analysis theory, and it can provide more information that Fourier analysis theory can not provide. So in terms of theoretical study and engineering application it is of great value and practical significance. The past ten years, wavelet analysis has rapidly developed, and it has been widely used in graphics, image processing and analysis, CT Imaging, radar, seismic exploration, edge phase cycle slips detection, error analysis, waveform decomposition and combination (refactoring), data compression and so on. Wavelet analysis not only in time domain but also in frequency domain has good localization properties, its window size does not change its shape, and it is a time-frequency localization analysis method that can be changed by time window and frequency window. It has the characteristics of multi-resolution analysis, and the density of different frequency components spatial sampling in time-domain as the signals in the process of thinning can be adjusted automatically. Therefore, the wavelet analysis has the properties that it can reflect the details of observation function and analysis. Based on this characteristic, it was known as mathematics microscope.

In the process of testing signal， Signals are often under the influence of interference and noise signal, such as data collection and transmission may introduce noise. Therefore in practice, de-noising must be carried out before measurement data analysis.

At present, there are traditional methods of achieving noise filtering and wavelet de-noising methods, in the actual test different de-noising method should be chosen depending on the different noise and signal. De-noise filter is only suitable for stationary random signals and signal separation,

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and entropy of a signal will be higher after changes, thus signal correlation can not be get. Wavelet analysis method has developed nearly a dozen years; it has an obvious advantage in de-noising because it has the characteristics of multi-resolution analysis, it can be used for non - stationary signal de-noising, it is not only effective to remove the noise, but also better to retain signal of mutation section.

2 Principle of wavelet threshold de-noising

General principle of signal de-noising is that the frequency distribution of noise and signal are different. In the traditional signal de-noising method based on Fourier transformation, the band overlapped part of the signal and noise is always made as small as possible, thus in the frequency domain by time - invariant filter, it can be distinguished between signal and noise. But if the two overlapping area is large, it is unlikely to make a noise removal effect.

With white Gaussian noise signal Wavelet transform, known by the characteristics of Wavelet transform, Gaussian noise by Wavelet transform is still a Gaussian distribution, and it is evenly distributed in parts of the frequency scale space. Due to their limited nature, wavelet coefficients of signal are just concentrated in limited parts of the frequency on the scale-space.

Principle of threshold de-noising: After wavelet decomposition, each coefficient model larger or smaller than a threshold should be processed separately, then process wavelet coefficients to inverse transform, and reconstruction after the signal de-noising, thus to achieve the purpose of de-nosing. The main steps are as follows:

(1) Decompose signal using Wavelet to obtain Wavelet coefficients;

(2) Process the threshold of wavelet coefficient to get new Wavelet coefficients;

(3) Reconstruction de-noised signal by the new Wavelet coefficient;

2. 1 Select threshold function

Donoho has divided threshold into the threshold function into soft threshold and hard threshold, Assume that w is the size of wavelet coefficients, ^w is the size of wavelet coefficients, λ is the threshold value.

(1) Hard threshold

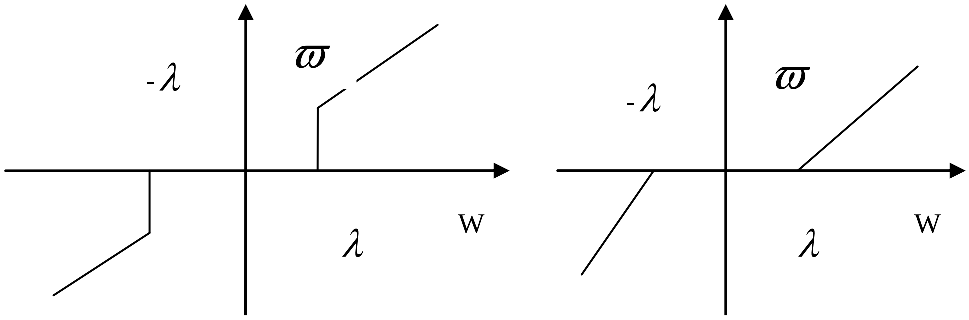
When the absolute value is less than the given threshold Wavelet coefficient value, assume Wavelet coefficient as 0, otherwise, the Wavelet coefficient is invariant, which is showed in Fig 1(a).

$$\varpi = \begin{cases} w, |w| \geq \lambda \\ 0, |w| \prec \lambda \end{cases} \quad (1)$$

(2) Soft threshold

When the absolute value is less than the given threshold Wavelet coefficient value, assume Wavelet coefficient as 0, otherwise, minus the threshold, which is showed in Fig 1(b).

$$\varpi \equiv \begin{cases} \mathrm{sgn}(w)(|w| - \lambda), |w| \geq \lambda \\ 0, |w| \prec \lambda \end{cases} \quad (2)$$



（a）Hard threshold （b）Soft threshold

Figure1. the soft and hard threshold method of estimated Wavelet coefficient

2. 2 Select threshold

How to choose a threshold value in the threshold function is a key problem that is related to de-noising directly. If the threshold is chosen too small, some noise will be saved after the signal de-noising; If the threshold is chosen too big, it will cause distortion. So it is a key problem to select threshold while de-noising. Donoho has given the threshold value based on orthogonal Wavelet transform $\lambda = \sigma\sqrt{2\ln^{\mathrm{N}}}$, where n is the length of the signal, σ is the noise standard deviation, which can be evaluated by the following equation.

$$\sigma = \frac{\mathrm{median}\ (|\mathrm{d}_{\mathrm{j}}(\mathrm{k})|)}{0.6745} \tag{3}$$

Where dj(k) is the high frequency coefficient after the Wavelet Decomposition, median is calculating intermediate values of operation command in MATLAB. The threshold selection method results are unsatisfactory in the practical application, the following guidelines can be chosen:

(1) Fixed threshold

The selected algorithm is:

$$\lambda = \sqrt{2\lg N} \tag{4}$$

Where N is the signal length.

(2) Stein’s unbiased likelihood estimation for threshold values (rigrsure threshold)

This calculated SURE threshold is based on Stein's unbiased maximum likelihood estimator, for each threshold value, find out its corresponding value-at-risk, and then choose a threshold to make the risk threshold values to be smallest. Specific algorithms are:

$$P = [P_0, P_1, ...P_{\mathrm{n}-1}], P_0 \prec P_1 \prec ... \prec P_{n-1} \tag{5}$$

The elements of P are square of the wavelet coefficients, which is from small to large order. Risk algorithm is:

$$R(K) = [N - 2K - (N-K)P_K + \sum_{I=1}^{K} P_{\mathrm{i}}] / \mathrm{N} \tag{6}$$

Where k=0,1,⋯, N-1. According to the resulting risk curve R(k), Mark their minimum corresponding value to be kmin,Then threshold is defined as:

$$\lambda = \sigma\sqrt{\mathrm{pk}_{\min}} \quad (7)$$

(3) Heuristic threshold

It is synthesized of the former two thresholds, what is chosen is optimal prediction variable threshold. When the ratio of the signal and noise is small, a fixed threshold is adopted; otherwise, using rigrsure norm.

(4) Max and minimum threshold

Its principle is to make the estimated maximum risk minimization, the threshold selection algorithm is:

$$\lambda = \begin{cases} \sigma[0.3936 + 0.1829(\ln N / \ln 2)] N \succ 32 \\ 0 N \leq 32 \end{cases} \quad (8)$$

(5) Penalty threshold based on wavelet packet transform

Wavelet packet analysis on low-frequency signal decomposition, on high-frequency signal decomposition, it has a more accurate analysis capability. Principle of de-noising based on wavelet packet analysis more or less agrees with Wavelet analysis. The key problem is to select threshold, the following are penalty threshold value method.

Wavelet packets coefficients sort in the order from small to large, $C = [C_1, C_2, \ldots.. C_n]$ .Supposed function: $\mathrm{crit}(t) = -\sum_{k \prec t} {c_k}^2 + 2\sigma^2 + (\alpha + \log(n/t))$,which t = 1 ,2 , ⋯, n, n is the number of wavelet cofficents, α is experience coefficient whose value must be greater than 1,and the typical value is 2.Take t as variable, calculate the minimal value of crit (t). Set the crit(t) for the minimum value of t for t 0,the function λ = | ct0 |.

3. The application of wavelet analysis in GPS signal noise

Each GPS satellite launched signal which can be distinguished, including carrier signals, P-code (Y-codes), C/A code, data code (D-code) and other signal component, that all these signal components are produced on the basis of the same fundamental frequency f0 which is 10.23MHz. P-code and the C/A code collectively are known as the ranging codes, whose carrier signals include L1 and L2 carrier, frequency 1575.42MHz and 1227.60MHz respectively. On the carrier frequency L1, the modulation includes C/A code, P-code and D-code, however on the frequency L2 the modulation P-code and D-code only. Both L1 and L2 carrier is a sine wave, and the frequency of P-code is 10.23MHz, the sequence of which the length of an element is 100ns, corresponding to 30m distance. When there is no other error, the measurement accuracy on P-code can achieve over a meter, and whose cycle is 267d. C/A code is much simpler than the P-code, which frequency is 1.023MHz, corresponding to the wavelength of 300m. Due to the C/A code cycle of 1ms, it could be locked by receiver soon. The work procedures of P-code also firstly lock the C/A code by receiver, worked out the system information, then switched to P-code precision pseudo observations [1-2].

Ranging codes and D-code of GPS satellites, are modulated to a carrier wave with the technology of phase modulation, and the amplitude of modulation code only is 0 or 1. If the value of code is 0, take the code corresponding to the State as +1. And the value of code is 1, the corresponding status codes get -1. Then, after the carrier multiplied by the appropriate status code, it enables carrier modulation, that is, the code signal is onto a carrier wave.

GPS technology is inseparable with the pseudo code. GPS receivers use pseudo code received signal, to identify the GPS satellites. In precision GPS phase measurements, solve phase ambiguity with the use of pseudo code. There are many errors source in pseudo measurement, such as ionosphere refraction, troposphere refraction, multipath effect, time difference effect, receiver noise and so on. On the ionosphere refraction and troposphere refraction, there already are many technologies or models to eliminate and correct. Time difference effect also can eliminate well with certain model. But it is different to multiple paths effect, because it gets different multiple paths effect due to receivers or locals. Multipath effects in GPS data processing cannot be ignored, which is also an important error source in static GPS precise positioning besides receiver noise.

Because measurement noises of GPS receiver and multipath delay errors are noise signals of GPS phase observations, from wavelet detection signal analysis, they can consider on the amplitude of the mutation, are stationary signals. In GPS signals, the useful frequency of signal could include signal launch frequency, data sampling frequency, data solutions frequency and so on. As which kind of signal is the useful signal what we need of, it depends on we prepared to extract the information from data, so the frequency range of the information is just the interested and useful frequency. The information which go beyond the above frequency range, can be considered useless information or noise. From the perspective of pure frequency, various measurement noise and multipath effect signals are in a certain frequency range. The time-frequency characteristics between useful signal in the GPS observation sequence and noise are usually not same. In time domain and frequency domain, the useful signals are localized, characterized in low frequency or stationary signal. But, observation noise and multipath effects distribute globally in time-frequency spaces, existing everywhere in the whole observation time domain, and in frequency domain performance as high frequency signal. In observation of GPS relative positioning, access to the useful signal a steady signal, appeared in the low-pass filtering of results, and various noises are reflected in the high-pass filtering results.

GPS signal noise processing with Wavelet analysis, firstly should get one dimensional sequence carrier signal. Due to the different data formats designed by GPS receiver manufacturers, GPS raw data should be transferred from the receiver to the computer, and then take original observation data file to form GPS RINEX observation files in standard format with the random software. This study exploits a GPS signal processing software with MATLAB and VC++ language. The software firstly extracts the original carrier signal from observation data in the format of the RINEX, and makes the difference process on one dimension original carrier sequence signal to form one-dimensional single and double observation sequence signal, as the input signal of Wavelet decomposition. This article takes Wavelet Decomposition for GPS signals by the Wavedec function. The decomposition should reconstruct high-frequency signal by the specified wavelet functions, that is to say that it could de-compound the GPS signal into low frequency and high frequency. On account of the high frequency characteristic of the noise, analysis the part of high frequency decomposition to research the characteristics of GPS signal noise.

Figure 1 has shown GPS signal of the monitoring ground surface in Datong mining area, apparently the signal affected by noises. Using quadratic spline Wavelet, make Wavelet transform on the four-level of binary scale, and the details reflected GPS signals changes at all levels.

Figure 2 is the curves of modulus maxima resulting from wavelet transform of GPS signal in all scales, which you can distinguish the modulus maxima formed by the effective GPS signal and noises. From Figure 2, as the scale increases, the modulus maxima of amplitude and thickness of noise wavelet transform rapidly decrease. When in large scale, the singularity of effective signal is gradually emerging, and the modulus maxima of amplitude about wavelet transform increases as the scale increases.

Figure 3 is the contrastive curve between the original GPS signal and the reconstructed GPS signal after de-noising which following the above process. The de-noising GPS signal is used in line adjustment of controller parameters, which ensured data accuracy finally.

4 Conclusion

In the occasion of small signal-to-noise, this paper gives an de-noising algorithm, which provides the precise position of singular point of the original signal in strong noise background, this is the traditional spectrum analysis method cannot match. Besides, the impromptu algorithm of searching the great value saves calculation time but sometimes it is not too accurate. If certain part of original signal has min shaped structure, what means that there are many singular points in subsection of signal, extreme point in j level become j-1 level, and the candidate extreme points are more than one. So in the case, this method is not fit. Summary out some experiential criterion by specific debugging. Based on wavelet transform, the algorithm of alternating projection of modulus Maxima is an approximate recovery to the original signal. Most of the signal information is available and the recovery and reconstruction is in very high precision. In generally GPS signal processing, refactoring error is tolerable. The computing time of alternating projection algorithm is a little longer, so it could not meet real-time requirements. If a more efficient algorithm is established, it is of great significance.

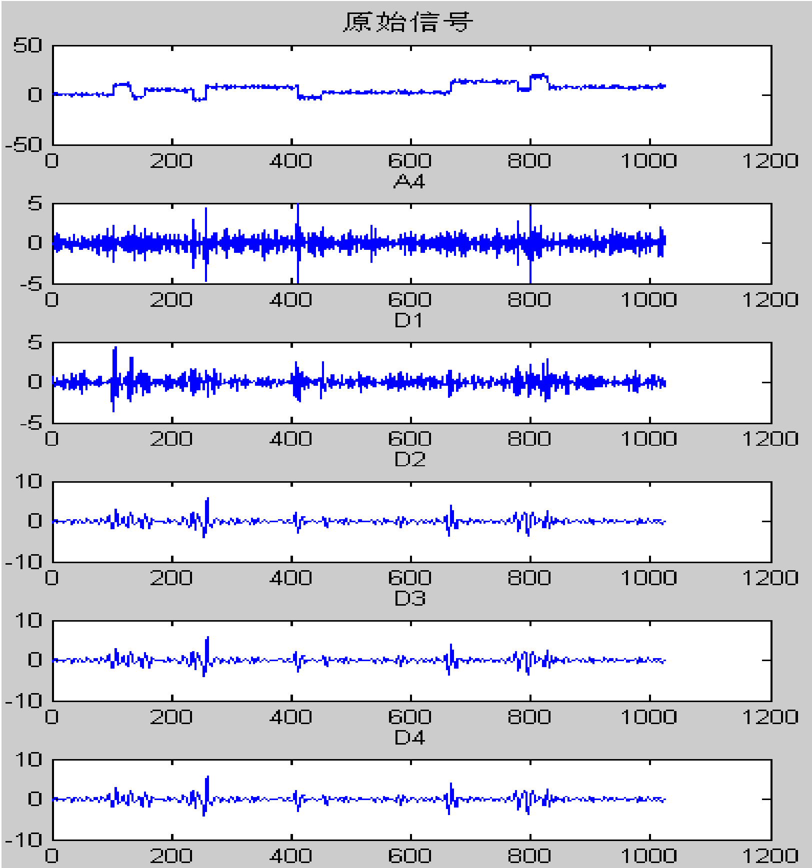


Fig 1 GPS signal including chirp and wavelet transformation under each criterion

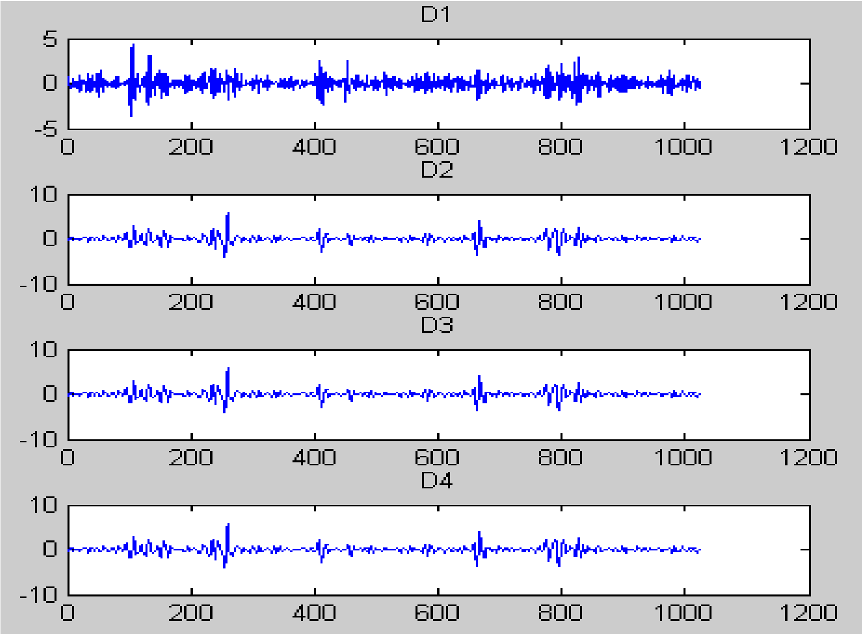


Fig2 GPS signal and mold maximum value curve under each criterion wavelet transformation

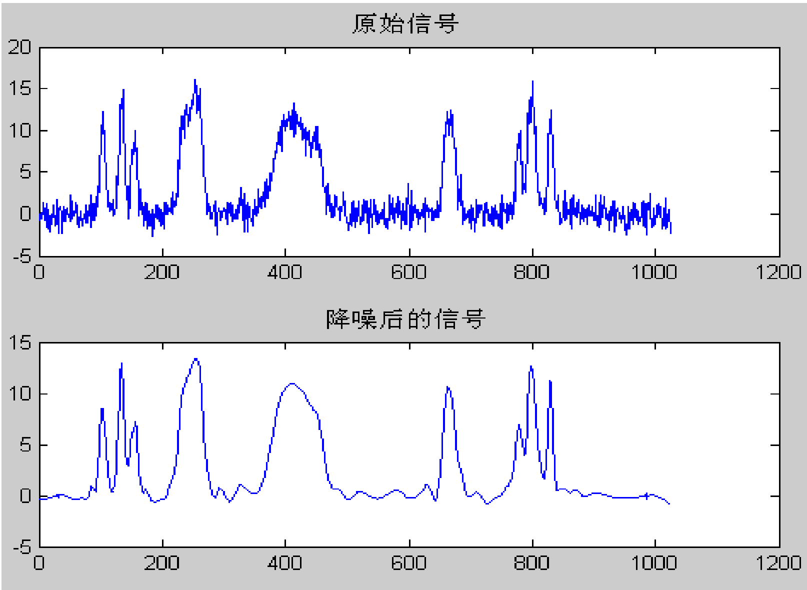


Fig3　Original signal and denoising signal of GPS

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