A Tabu Search Approach to Fuzzy Optimization of *Camellia Oleifera* Fertilization

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Abstract. Traditional optimization methods have been applied for years to high-yield fertilization models, which are usually well formulated by crisp coefficients and variables. Unfortunately, real-world crop growing environment and process are often not deterministic. In this paper we establish a fuzzy mathematical model between *Camellia oleifera* yield and fertilization application rates, in which variation coefficients of N, P, K are described with fuzzy numbers. In particular, we present a tabu search algorithm for finding a set of fertilization solutions in order to maximize *Camellia oleifera* yield based on fuzzy measures including expected value, optimistic value and pessimistic value. Our approach is more realistic and practical for real-world problems by taking vague and imprecise data into consideration, provides more comprehensive decision support by generating a set of high-quality alternatives, and can be applied to fertilizer decision for a variety of other crops.

Keywords: fuzzy optimization, tabu search, *Camellia oleifera*

1 Introduction

Crop yield response models have been playing an increasingly important role in variable-rate fertilization decision-making. There are several widely-used quantitative mathematical models including the linear-plus-plateau model, the quadratic-plus plateau model, the quadratic model, the exponential model, and the square root model, which often disagree when identifying the fertilizer application rates [1]. In many cases, it is hard for the farmers to place a high confidence level on the results of the models, the main reason of which is that the models always generate so exact and crisp solutions but the environment is uncertain and the measurement is imprecise in nature. For example, rather than say that “the crop yield would be 1000kg”, it is more reasonable to say that “the crop yield would be at least 800kg, at most 1150kg, and most likely to be 1000kg”.

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First initialized by Zadeh [2], the concept of fuzzy sets has been well developed and applied to many conventional mathematical methods to reflect the ambiguity and uncertainty in real world. However, until recent years, little attention has been given to fuzzy mathematic models on crop growth and yield. Kandala and Prajneshu [3] first proposed a fuzzy linear regression approach for crop yield forecasting using remotely sensed data which lies in an interval instead of a single number. In [4] Yu et al selected linear functions as membership functions in fuzzy synthesis evaluation of different irrigation and fertilization on growth of greenhouse tomato. Fuzzy decision concepts have also been used on the extension of some other optimization methods in agriculture fertilizer applications. For example, Li *et al* [5] presented a support vector machine algorithm to generate fertilization fuzzy rules to increase linguistic interpretability and acquisition capability of knowledge. Palaniswami *et al* [6] used a fuzzy neural network to predict the coconut yield in which fuzzy membership values of the independent variables were used as the input layer for the network.

In this paper we establish a fuzzy mathematical model between *Camellia oleifera* yield and fertilization application rates, in which variation coefficients of N, P, K are described with fuzzy numbers. Under the fuzzy environment, it is not desirable to find a single optimum for *Camellia oleifera* yield by applying one-step optimization techniques such as the Gauss-Newton method, and thus we present an extended tabu search algorithm for finding a set of Pareto optimal solutions based on fuzzy measures including expected value, optimistic value and pessimistic value. In comparison with crisp methods, our approach is more realistic and provides more comprehensive decision-making support by taking uncertain information and multiple criteria into consideration.

2 Preliminaries

First we present some basic concepts of fuzzy sets, sufficient to understand the paper. A fuzzy set is a pair (A,μ) where A is an ordinary set and μ is a function: $A\rightarrow[0,1]$. For each $x \in A$, $\mu(x)$ is called the grade of membership of x in (A,μ). x is said to be not included in (A,μ) if $\mu(x)=0$, x is said to be fully included if $\mu(x)=1$, and x is called a fuzzy member if $0<\mu(x)<1$.

Let X be the universe of discourse, A fuzzy set $\tilde{A}=(A,\mu)$ of X is said to be convex if and only if for all x_1 and x_2 in X and $\lambda\in[0,1]$ the following equation always holds:

$$\mu(\lambda x_1+(1-\lambda)x_2) \geq \min(\mu(x_1), \quad \mu(x_2)) \tag{1}$$

A fuzzy number is a convex, normalized fuzzy set (A,μ) whose membership function is at least segmentally continuous and has the functional value $\mu(x) = 1$ at precisely one element, where A is a subset of the real number R.

One of the most commonly used fuzzy numbers is the triangular fuzzy number represented by (a,b,c), whose membership function is defined as follows:

$$\mu(x)=\begin{cases}(x-a)/(b-a), & \text{if } a\le x<b\\ (c-x)/(c-b), & \text{if } b\le x<c\\ 0, & \text{otherwise}\end{cases} \tag{2}$$

Let $\tilde{A}_1=(a_1,b_1,c_1)$ and $\tilde{A}_2=(a_2,b_2,c_2)$ be two triangular fuzzy numbers, the basic fuzzy arithmetic operations on them are defined as follows [7,8]:

$$\tilde{A}_1+\tilde{A}_2 = (a_1+a_2,b_1+b_2,c_1+c_2) \tag{3}$$

$$\tilde{A}_1-\tilde{A}_2 = (a_1-c_2,b_1-b_2,c_1-a_2) \tag{4}$$

$$\lambda\tilde{A}_1 = (\lambda a_1,\lambda b_1,\lambda c_1) \tag{5}$$

$$\tilde{A}_1\times\tilde{A}_2 = (a_1a_2,b_1b_2,c_1c_2) \tag{6}$$

$$\tilde{A}_1/\tilde{A}_2 = (a_1/c_2,b_1/b_2,c_1/a_2) \tag{7}$$

For fuzzy numbers, there is a variety of measures developed ranging from the trivial to the complex. Liu [9] proposed a credibility measure that satisfies normality, monotonicity, self-duality, and maximality. Given a triangular fuzzy number $\tilde{A}=(a,b,c)$ and a credibility level $\lambda\in[0,1]$, the expected value, α-optimistic value and α-pessimistic value of $\tilde{A}$ are respectively defined as follows:

$$E(\tilde{A}) = (a+2b+c)/4 \tag{8}$$

$$O_\lambda(x)=\begin{cases}2\lambda b+(1-2\lambda)c, & \text{if } \lambda\le 0.5\\ (2\lambda-1)a+(2-2\lambda)b, & \text{else}\end{cases} \tag{9}$$

$$P_\lambda(x)=\begin{cases}(1-2\lambda)a+2\lambda b, & \text{if } \lambda\le 0.5\\ (2-2\lambda)b+(2\lambda-1)c, & \text{else}\end{cases} \tag{10}$$

3 A Fuzzy Fertilizer Response Model for *Camellia Oleifera*

A field experiment was conducted in Yichun area, Jiangxi province, China. The selected *Camellia oleifera* trees were in full bearing period. The experimental area was 360 ha and divided into 45 parts, and the plant density is about 68~75 trees/ha. The three-factor quadratic model was used for describing the relationship between the yield of the fruits (kg/ha) and the concentration of N, P, and K:

$$\tilde{Y} = \tilde{a}_1 N^2 + \tilde{a}_2 P^2 + \tilde{a}_3 K^2 + \tilde{b}_1 NP + \tilde{b}_2 PK + \tilde{b}_3 NK + \tilde{c}_1 N + \tilde{c}_2 P + \tilde{c}_3 K + \tilde{d} \tag{21}$$

where coefficients $\tilde{a}_1, \tilde{a}_2$, $\tilde{a}_3$, $\tilde{b}_1$, $\tilde{b}_2$, $\tilde{b}_3$, $\tilde{c}_1$, $\tilde{c}_2$, $\tilde{c}_3$ and constant $\tilde{d}$ are all triangular fuzzy numbers. Based on the historical data, the value ranges are set as $6.5 \leq N \leq 8.4$, $11.5 \leq P \leq 15.8$, $25.0 \leq K \leq 40.2$. By performing fuzzy regression analysis based on the quadratic programming formulation [10], we obtained the regression coefficients in Equation (11) as shown in Table 1.

Table 1. The fuzzy regression coefficients

	$\tilde{a}_1$	$\tilde{a}_2$	$\tilde{a}_3$	$\tilde{b}_1$	$\tilde{b}_2$	$\tilde{b}_3$	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{c}_3$	$\tilde{d}$
a	-0.66	-4.55	-0.39	6.45	1.84	-3.27	9.17	6.05	14.67	832.1
b	-0.60	-4.10	-0.35	6.70	2.00	-3.10	9.30	6.10	14.90	850.2
c	-0.52	-3.96	-0.32	7.20	2.25	-2.93	9.65	6.52	15.40	874.5

Nevertheless, we should be careful here for negative coefficients: the addition of two negative fuzzy numbers should use the Equation (3), while the addition of a positive fuzzy number and a negative fuzzy number should use the Equation (4); if a fuzzy number contains both positive and negative components, e.g., $\tilde{a}_1$ = (-0.5, 0.5, 1.0), we should perform a normalization on the equation to make it contain only pure positive numbers and pure negative numbers.

4 A Tabu Search Algorithm for Finding Optimal Fertilization Solutions

Since fuzzy numbers are represented in nature by possibility distributions, it is difficult to determine clearly whether one fuzzy number is larger or smaller than the other [11]. In consequence, for quadratic regression models with fuzzy coefficients, it is not desirable to find a single optimum for by applying one-step optimization techniques such as the Gauss-Newton method. Instead, we should use a comprehensive approach that takes more than one ranking criteria into consideration.

To measure and compare the results of the fuzzy yield response model (11), we employed here three important fuzzy ranking criteria including expected value, optimistic value and pessimistic value. Given two fertilizer solutions which result in yields $\tilde{Y}_1$ and $\tilde{Y}_2$ respectively, we say $\tilde{Y}_1$ dominates $\tilde{Y}_2$ if they satisfies $E(\tilde{Y}_1) \geq E(\tilde{Y}_2)$, $O(\tilde{Y}_1) \geq O(\tilde{Y}_2)$ and $P(\tilde{Y}_1) \geq P(\tilde{Y}_2)$. Under the circumstance of multiple criteria decision, it is desired to find a set of non-dominated solutions instead of a single one.

Tabu search, first proposed by Glover [12,13], is a meta-heuristic search that repeatedly moves from a current solution to the best of neighboring solutions while avoiding being trapped in local optima by keeping a tabu list of forbidden moves. It was extended to continuous problems by Cvijovic and Klinowski [14] and to multiobjective optimization problems by Hansen [15]. The follows present an innovative tabu search algorithm for working out the non-dominated fertilizer solutions for the *Camellia oleifera* yield response model:

Step 1. Find an arbitrary initial solution x_0, initializes the empty tabu list T, let $x = x_0$, $E^* = E(x_0)$, $O^* = O(x_0)$, $P^* = P(x_0)$, $Q = \{x_0\}$.

Step 2. In the current solution's neighborhood $N(x)$, select all the solution x' that satisfies any of the following criteria: (1) $E(x') > E^*$, (2) $O(x') > O^*$, (3) $P(x') > P^*$, add x' to the solution set Q and delete those solutions being dominated by x' from Q, and update E^*, O^*, or P^*. If $E(x') > E^*$, update the tabu list T and let $x = x'$, go *Step 2*.

Step 3. If no such a solution satisfying $E(x') > E^*$ and all the neighborhood solutions are tabu, or some of the termination conditions are reached, the algorithm stops and returns Q.

Step 4. Otherwise, select an x' in $N(x)\backslash T$ with the best expected value in $E(x')$, update the tabu list T and let $x = x'$, go *Step 2*.

In the above algorithmic framework, the neighborhood of a fertilizer solution $[x_1, x_2, x_3]$ is defined as the set of six solutions including $[x_1 \pm 0.1, x_2, x_3]$, $[x_1, x_2 \pm 0.1, x_3]$ and $[x_1, x_2, x_3 \pm 0.1]$, where 0.1 is a preset increment value (for other yield response models, the increment value can be adjusted according to the units of measurement used and the fertilizer application quantities estimated).

We run the tabu search algorithm for solving the *Camellia oleifera* yield response model. The credibility level λ was set to 0.25; the algorithm stopped after 528 iterations, and the result non-dominated solution set contained 6 solutions as shown in Table 2. As we can see, solution #5 and #6 reached the maximum expected yield value 1162.5, in which #6 reached the maximum optimistic value 1295, and #1 reached the maximum pessimistic value 1036.

Table 2. Non-dominated solution set for the *Camellia oleifera* yield response model

	N	P	K	Y	$E(Y)$	$O(Y)$	$P(Y)$
#1	6.6	11.7	25.0	(931, 1140, 1365)	1144	1252	1036
#2	6.8	11.8	25.0	(929, 1141, 1369)	1145	1255	1035
#3	7.1	12.1	25.0	(924, 1143, 1380)	1147.5	1262	1034
#4	7.3	12.3	25.0	(921, 1145, 1386)	1149.25	1266	1033
#5	8.4	13.4	25.0	(905, 1158, 1429)	1162.5	1294	1032
#6	8.4	13.6	25.0	(902, 1158, 1432)	1162.5	1295	1030

5 Conclusion

The paper establishes a fuzzy mathematical model between *Camellia oleifera* yield and fertilization application rates, in which variation coefficients of N, P, K are described with fuzzy numbers. In particular, we present a tabu search algorithm for finding the non-dominated fertilization solution set on three fuzzy measures including expected value, optimistic value and pessimistic value of the *Camellia oleifera* yield. Our approach is more realistic and practical by taking vague and imprecise data into consideration, and supports more comprehensive decision-making by generating a set of high-quality alternatives.

The fuzzy yield response model can be applied to a wide variety of crops more reasonably and effectively, and the algorithmic framework can be applied/extended for solving the quadratic and other kinds of models. Moreover, more fuzzy ranking criteria can be included in order to providing more comprehensive and complicated decision support. Our ongoing work also includes developing an integrated software tool to support fuzzy data analysis, regression modeling, problem solving, and visualized fertilizer decision-making.

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