MODELING AND DESIGNING OF A NONLINEAR TEMPERATURE-HUMIDITY CONTROLLER USING IN MUSHROOM-DRYING MACHINE

Xiuhua Wu*, Haiyan Luo, Minhui Shi

Information and Electrical Engineering Institute, Shenyang Agricultural University, Shenyang 110161, China

Abstract: Drying-process of many kinds of farm produce in a close room, such as mushroom-drying machine, is generally a complicated nonlinear and time-delay cause, in which the temperature and the humidity are the main controlled elements. The accurate controlling of the temperature and humidity is always an interesting problem. It's difficult and very important to make a more accurate mathematical model about the varying of the two. A math model was put forward after considering many aspects and analyzing the actual working circumstance in this paper. Form the model it can be seen that the changes of temperature and humidity in drying machine are not simple linear but an affine nonlinear process. Controlling the process exactly is the key that influences the quality of the dried mushroom. In this paper, the differential geometry theories and methods are used to analyze and solve the model of these small-environment elements. And at last a kind of nonlinear controller which satisfied the optimal quadratic performance index is designed. It can be proved more feasible and practical than the conventional controlling.

Key words: nonlinear controller, mathematical model, temperature-humidity control, mushroom-drying machine

1. INTRODUCTION

In recent years, the yield of mushroom has been increased year by year. It makes the demand of high quality drying-technology urgent. Drying mushroom needs to gain an ideal goal in a limited period or as quickly as it can. In order to reduce the cost of drying process, proper drying methods must be selected. The requirements can be satisfied by the drying equipments which used heat energy associated with solar energy. They not only make best use of the regenerative solar energy but also can ensure the drying time with heat energy when solar energy is not sufficient. It is a kind of combined and efficient drying method.

The environment of the drying machine likes a small green house. The main indexes are illumination, temperature, humidity, air-flowing speed and the quantity of heat. The temperature can't be too high in a long time during the process of mushroom-drying, otherwise the shape, color and the nutrition of the dried mushroom are damaged. The drying time can not be very long. The check of the products depends on the humidity of dried mushroom. In drying process, changes of temperature and humidity are not simple linear but complicated nonlinear. The traditional linear control can not content the needs to exactly control and get high quality products. A comparatively strict and exact control method is needed to get high quality and grades of the products. So, to make an ideal mathematical model about the varying of temperature and humidity in the drying machine is the important thing.

1. MODEL

Consideration of the cost, glass wall is used to make full use of solar in day. It's paved at night. The ventilation-holes is used to ventilate naturally. Heat energy associated with solar energy control the temperature in machine. So the temperature control dynamic model satisfies the following heat balance equation (Daskalov,1997):

$$V\rho C_p \frac{dT_i}{dt} = Q_{rad} + Q_{heat} + Q_{crad} - Q_{tran} - Q_{vent} - Q_{cac} \quad (1)$$

Where V is the volume of the machine (m^3) , ρ the air density $(1200g/m^3)$, C_p the specific heat of air $(1.006J/g \cdot K)$, T_i inner air temperature , Q_{rad} the solar radiant energy, Q_{heat} the heat energy by burning fuels, Q_{crad} the radiant energy of long waves, Q_{tran} the heat losses for transpiration , Q_{vent} the heat losses for air flowing , Q_{cac} the heat losses due to transmitting, and as the value very low compared with the others, it can be ignored. The right parts of the Eq. (1) as follows:

$$Q_{rad} = A_s \cdot I_a \cdot \tau_a \quad (2)$$

Where A_s is the areas of the sunshine（m2）, τ_a transmitance of the glass(0.89), I_a the solar radiant intensity（W/m^2）. I_a is a variable changed with the time. For simplification, the average radiant intensity of a day in summer or autumn is used. It's approximately 70 W/m2.

$$Q_{heat} = h_p(T_r - T_i), \tag{3}$$

Where T_r is the desired inside temperature (K), T_i the inside air temperature (K), h_p the heat of burning fuels per temperature unit (J/K).

According to Stefan-Boltzmann law, the radiant energy of long waves as follow:

$$Q_{crad} = \varepsilon_{12} A \sigma T_i^4 \tag{4}$$

Where $\varepsilon_{12} = (\varepsilon_1 + \varepsilon_2 - 1)^{-1}$ is emissivity between interfaces, $\varepsilon 1, \varepsilon 2$ are the emissivities of glass and air respectively, $\varepsilon 1 = 0.9, \varepsilon 2 = 0.3$. σ coefficient of Stefan-Boltzmann, $\sigma = 5.67 \times 10^{-8}\,\mathrm{W}/m^2 \cdot K^4$, $K = -273^\circ C$.

$$Q_{tran} = \lambda \cdot E_m A \tag{5}$$

Where Q_{tran} is the absorbing energy of transpiration of the products(J/h), A the area of the products(m^2), $\lambda = 4.1868(597 - 0.57t)$ the parameter of the transpiration heat(J/g), t the temperature of the products in transpiration, in this case it used the T_r , E_m the intensity of transpiration($g/m^2 \cdot h$), E_m can be denoted as:

$$E_m = 1.2k_e[3.9 \times 10^{-3} \exp(\frac{T_i}{15.2}) - q_i] \tag{6}$$

Where k_e is the coefficient of the evaporation which has relation to the air speed, $k_e = 0.223 + 0.149 \times f(v_{air})$ (g/h) (Daskalov,2006) ,for comprehensive consideration, it uses 0.5(Govriachev, 1968).

Substitute Eq.(6) into Eq.(5), it gains:

$$\begin{aligned} Q_{tran} &= 4.1868(597 - 0.57T_r) \times 1.2 \times 0.5 \times (3.9 \times 10^{-3} \exp(T_i/15.2) - q_i)A \\ &= 5.85A\exp(T_i/15.2) - 1500Aq_i - 0.005T_rA\exp(T_i/15.2) + 1.432AT_rq_i \\ &= (5.85A - 0.005T_rA)\exp(T_i/15.2) + (1.432AT_r - 1500A)q_i \end{aligned} \tag{7}$$

$$Q_{\mathrm{vent}} = \rho C_p v_{na}(T_r - T_i) \tag{8}$$

Where Q_{vent} is the heat losses of the air flowing, v_{na} the natural ventilation ratio ($m^3/\min$), it uses the average value 2.5 $m^3/\min$.

Substitute Eqs.(2)-(4)and (7)-(8) into Eq.(1), it can get:

$$\begin{aligned} V\rho C_p \frac{dT_i}{dt} &= A_s I_a \tau_a + h_p(T_r - T_i) + (\varepsilon_1 + \varepsilon_2 - 1)^{-1} A \sigma T_i^4 - (5.85A \\ &- 0.005T_rA)\exp(T_i/15.2) - (1.432AT_r - 1500A)q_i - \rho C_p v_{na}(T_r - T_i) \end{aligned} \tag{9}$$

The humidity dynamic model of the drying machine is as follow:

$$\frac{dq_i(t)}{dt} = \frac{AE_m}{V} - \frac{v_{na}}{V}[q_o - q_i] \tag{10}$$

Where q_o the outside absolute humidity (g/m^3), q_i the inside controlled absolute humidity(g/m^3).

Substitute the Eq.(6) into Eq.(10),it can get ：

$$\begin{aligned}\frac{dq_i}{dt} &= \frac{A}{V} \times 0.6 \times [3.9 \times 10^{-3} \exp(T_i / 15.2) - q_i] + \frac{v_{na}}{V} q_i - \frac{v_{na}}{V} q_0 \\ &= 0.002 \frac{A}{V} \exp(T_i / 15.2) + \frac{v_{na} - 0.6A}{V} q_i - \frac{v_{na}}{V} q_o\end{aligned} \tag{11}$$

In addition，as the temperature and humidity inside are higher than those outside, the temperature and humidity outside have little effect on those of inside. So T_r, q_o, v_{na} can be regarded as constants.

We take place all the constants in Eqs.(9) and (11) with $a_0,..., a_5, b_0,..., b_2$, then we can observe：

$$\begin{cases}\dfrac{dT_i}{dt} = a_0 T_i^4 + a_1 T_i + a_2 e^{kT_i} + a_5 + a_3 q_i + (a_4 - T_i) h_p \\ \dfrac{dq_i}{dt} = b_0 e^{kT_i} + b_1 q_i + b_2\end{cases} \tag{12}$$

Where $X = [T_i q_i]^T, X \in R^n$ is the state vector, $\dot{X} = [\dot{T}_i \dot{q}_i]^T$ the differential of the state variables, $u = h_p$, $u \in R$ the control variable, $y(t) = q_i, y \in R$ the output variable.

Apparently, the differential Equation set contents the normal form:

$$\begin{cases}\dot{X}(t) = F(X) + g(X)u \\ y(t) = h(X)\end{cases} \tag{13}$$

So it is a SISO multivariable affine nonlinear control system. Where

$$F(X) = \begin{bmatrix} a_0 T_i^4 + a_1 T_i + a_2 e^{kT_i} + a_3 q_i + a_5 \\ b_0 e^{kT_i} + b_1 q_i + b_2 \end{bmatrix} \tag{14}$$

$$g\ (X) = \begin{bmatrix} a_4 - T_i \\ 0 \end{bmatrix} \tag{15}$$

$$y(X) = h(X) = q_i \tag{16}$$

The control purpose is to find an optimal control law u which can make the state variables reach the idealist values and the energy consumed is the least. At the same time, it's to find a coordination transform $\varphi(\mathrm{Z})$, changing the nonlinear system into a linear system as equation set (17) , namely exactly linearization.

$$\begin{cases} \dot{\mathrm{X}} = \mathrm{AX} + \mathrm{Bu} \\ \mathrm{Y} = \mathrm{CX} + \mathrm{Du} \end{cases} \tag{17}$$

2. METHODS AND RESULTS

According to the theory of differential geometry about exact linearization, the affine nonlinear system must have the relative degree r equal the order of the state variables n. so we first calculate the following Lie derivative (Lu Q.,1996).

$$L_g L_f{}^0 h(X) = L_g h(X) = \frac{\partial h(X)}{\partial X} g(X) = [0\,1]\begin{bmatrix} a_4 - T_i \\ 0 \end{bmatrix} = 0 \tag{18}$$

$$L_f h(X) = \frac{\partial h(X)}{\partial X} F(X) = [01]\begin{bmatrix} a_0 T_i^4 + a_1 T_i + a_2 e^{kT_i} + a_3 q_i + a_5 \\ b_0 e^{kT_i} + b_1 q_i + b_2 \end{bmatrix} \tag{19}$$

$$= b_0 e^{kT_i} + b_1 q_i + b_2$$

$$L_g L_f h(X) = L_g (b_0 e^{kT_i} + b_1 q_i + b_2) = [b_0 k e^{kT_i} b_1]\begin{bmatrix} a_4 - T_i \\ 0 \end{bmatrix} \tag{20}$$

$$= (a_4 - T_i) b_0 k e^{kT_i} \neq 0 (a_4 \neq T_i, alsowhen T_r \neq T_i)$$

Where $L_f{}^n h(X)$ denotes the rth order Lie derivative of $h(X)$ along f. $L_g L_f\ h(X)$ the mix Lie derivative of $h(X)$ along f and g.

So the relative degree of this nonlinear system r-1=1, r=2=n .

Now looking for a coordination mapping $\varphi(\mathrm{Z})$:

$$\mathrm{Z}_1 = h(X) = q_i \tag{21}$$

$$\dot{\mathrm{Z}}_1 = \mathrm{Z}_2 = \frac{\partial h(X)}{\partial X}\dot{X} = \frac{\partial h(X)}{\partial X}(F(X) + g(X)u) \tag{22}$$

$$= L_f h(X) + L_g L_f{}^0 h(X) u = L_f h(X) = b_0 e^{kT_i} + b_1 q_i + b_2 (T_r \neq T_i)$$

$$\dot{\mathrm{Z}}_2 = \mathrm{Z}_3 = L_f{}^2 h(X) + L_g L_f{}^1 h(X) = \frac{\partial (L_f h(X))}{\partial X} F(X) + L_g L_f h(X) u \tag{23}$$

$$L_g L_f h(X)u = (a_4 - T_i) b_0 k e^{kT_i} u \tag{24}$$

$$L_f^{\,2} h(X) = [b_0 k e^{kT_i} b_1] \begin{bmatrix} a_0 T_i^{\,4} + a_1 T_i + a_2 e^{kT_i} + a_3 q_i + a_5 \\ b_0 e^{kT_i} + b_1 q_i + b_2 \end{bmatrix}$$

$$= b_0 k e^{kT_i} (a_0 T_i^{\,4} + a_1 T_i + a_2 e^{kT_i} + a_3 q_i + a_5) + b_1 (b_0 e^{kT_i} + b_1 q_i + b_2) \tag{25}$$

$$= a_0 b_0 k e^{kT_i} T_i^4 + a_1 b_0 k e^{kT_i} T_i + a_2 b_0 k e^{2kT_i} + (a_5 b_0 k + b_1 b_0) e^{kT_i} +$$

$$(a_3 b_0 k e^{kT_i} + b_1^2) q_i + b_1 b_2$$

$$\dot{Z}_2 = L_f^{\,2} h(X) + L_g L_f^{\,1} h(X) u \tag{26}$$

$$= \alpha(X) + \beta(X) u = V$$

So, we get the Brunovsky normal form:

$$\begin{cases} \dot{Z}_1 = A Z_2 \\ \dot{Z}_2 = B V \\ y = h(X) = q_i \end{cases} \tag{27}$$

Compared with the Eq. set (17), we can get:

$$\mathrm{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, D = 0, y = h(X) = q_i \,.$$

While $\begin{cases} V^* = \alpha(X) + \beta(X) u^*, \\ u^* = -\dfrac{\alpha(X)}{\beta(X)} + \dfrac{V^*}{\beta(X)}, (\beta(X) \neq 0) \end{cases}$ and

$\begin{cases} \alpha(\mathrm{X}) = L_f^{\,2} h(X), \beta(X) = L_g L_f h(X) \\ u^* = \dfrac{-L_f^{\,2} h(X) + V^*}{L_g L_f h(X)} \end{cases}$ is the control law.

Then we will determine the input variable V which is the control input of the linear system of the Brunovsky normal form. To the most reasonable way is using the linear optimal control design method with the quadratic performance index (LQR method) to produce the V*.

It can be proved that the system is a controllable system as the rank of the matrix $[B \mid AB \mid A^2 B \mid ....]$ is n.

Selected the linear quadratic performance index (LQR) as follow:

$$J = \frac{1}{2} \int_0^{\infty} (Z^T Q Z + V^T R V) dt \tag{28}$$

Then we can get the control vector V which makes the performance index functional J reach its extremum.

$$V^* = -R^{-1} B^T P^* Z(t) = -K^* z(t) \tag{29}$$

Where V^* denotes the optimal control vector, K^* is the optimal feedback gain matrix:

$$K^* = R^{-1}B^T P^* \tag{30}$$

P^* is the solution of the Riccati algebraic equation, Q, R are semi-positive and positive definite weighting matrices, respectively.

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{31}$$

Generally, Q is a diagonal matrix. In this question, $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, R = 1,$ we can deduce $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \mathrm{P} + \mathrm{P} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \mathrm{P} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} \mathrm{P} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$.

Here we suppose $\mathrm{P} = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$, substitute it into the upper equation. We get:

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} + \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0 \tag{32}$$

Then,

$$\begin{bmatrix} 0 & 0 \\ x_1 & x_2 \end{bmatrix} + \begin{bmatrix} 0 & x_1 \\ 0 & x_3 \end{bmatrix} - \begin{bmatrix} x_2 x_3 & x_2 x_4 \\ x_3 x_4 & x_4^2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0 \tag{33}$$

Solving the equation, we gain: $\mathrm{P}^* = \begin{bmatrix} \sqrt{3} & 1 \\ 1 & \sqrt{3} \end{bmatrix}$,which is a positive definite matrix. Substitute it into the expression (30), we can get

$$K^* = R^{-1}B^T P^* = \begin{bmatrix} 1 & \sqrt{3} \end{bmatrix} \tag{34}$$

Then,

$$\begin{aligned} V^* &= -K^* Z(t) = -Z_1(t) - \sqrt{3} Z_2(t) = -h(X) - \sqrt{3} L_f h(X) \\ &= -q_i - \sqrt{3}(b_0 e^{KTi} + b_1 q_i + b_2) \end{aligned} \tag{35}$$

$$\begin{aligned} u^* &= \frac{-\alpha(X) + V^*}{\beta(X)} = \frac{-L_f{}^2 h(X) + V^*}{L_g L_f h(X)} \\ &= -\frac{a_0 b_0 k e^{kT_i} T_i^4 + a_1 b_0 k e^{kT_i} T_i + a_2 b_0 k e^{2kT_i} + (a_5 b_0 k + b_1 b_0) e^{kT_i}}{a_4 b_0 K e^{KT_i} - b_0 K T_i e^{KT_i}} \\ &+ \frac{(a_3 b_0 k e^{kT_i} + b_1^2) q_i + b_1 b_2}{a_4 b_0 K e^{KT_i} - b_0 K T_i e^{KT_i}} - \frac{q_i + \sqrt{3}(b_0 e^{KT_i} + b_1 q_i + b_2)}{a_4 b_0 K e^{KT_i} - b_0 K T_i e^{KT_i}} \end{aligned} \tag{36}$$

It is the optimal control law of the original affine nonlinear control system.

3. CONCLUSIONS

By analyzing the real working environment of the drying-machine and all kinds of effects, dynamic mathematical model of the complicated drying-process was made. It is an affine nonlinear control model. An optimal control law was obtained by exactly linearizing the model with differential geometry theories and solving the Riccati equation. The final control law is satisfied the optimal quadratic performance index. It can make loss of the energy least and the control object ideal.

REFERENCES

Cai Jiabing. Estimatingreference evapotranspiration with the FAO Penman–Monteith equation using daily weather forecast messages. Agricultural and Forest Meteorology . 2007. 145, pp.22–35

Chua K.J, etc. A comparative study of different control strategies for indoor air humidity. Energy and Buildings,2007.39, pp.:537-545

Daskalov P.I,etc.Non-linear adaptive temperature and humidity control in animal buildings .Biosystems Engineering ,2006.93(1),pp.1-24

Garcia, Magali el al. Dynamics of reference evapotranspiration in the Bolivian highlands (Altiplano). Agricultural and Forest Meteorology. 2004.125, pp.67-82

Lu, Q etc. Decentralized nonlinear optimal excitation control. IEEE Transactions on power systems, 1996.11(4) ,pp.1957-1962

Lu, Qiang el al. Nonlinear control systems and power system dynamics. Kluwer Academic Publishers.Boston.2001

Mesquita L.C.S.,etc. Modeling of heat and mass transfer in parallel plate liquid-desiccant dehumidifiers. Solar Energy .2006. 80,pp. 1475–1482

Ríos-Moreno G.J.,etc. Modeling temperature in intelligent buildings by means of autoregressive models. Automation in Construction.2007. 16,pp.713–722

Soldatos A.G..etc. Nonlinear robust temperature humidity control in livestock buildings. Computers and Electronics in Agriculture.2005.49, pp.357-376

Stefan Heinrich,etc. Modeling of the batch treatment of wet granular solids with superheated steam in fluidized beds, Chemical Engineering and Processing, 1999. 38,pp.131–142

Yan Zhengyong,etc. Mathematical modeling of the kinetic of quality deterioration of intermediate moisture content banana during storage. Journal of Food Engineering.2008. 84,pp.359–367

Zhang yun,etc. Cold–humid ecological effects of the Sanjiang Plain. Ecology and environment,2004.13, pp.37-39