

OPTIMAL MODEL ON CANAL WATER DISTRIBUTION BASED ON DYNAMIC PENALTY FUNCTION AND GENETIC ALGORITHM

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Abstract: The present optimal water delivery scheduling models are based on the assumed equal design discharges of lateral canals, which are not in accordance with practical water delivery scheduling demand in most irrigation systems. In order to solve this problem, a model of lateral canals with unequal discharges and a solution method were proposed; At present, traditional fixed penalty factor have some problem, such as it is difficulty to use unified dimension and to get a higher searching precision, besides, it prematurely converge to local optimal solution. Therefore, the thought of simulated annealing was referred to design a dynamic penalty function. In the progress of genetic operation, the SGA (Simple Genetic Algorithm) adopted adaptive crossover mutation method, and compared distinct solutions of model which based on the method in this paper, Adaptive genetic algorithm (AGA) and traditional methods used in irrigation district widely respectively. Comparing with water delivery plan compiled using traditional methods, the results illustrate that using this method can get much more reasonable lateral canals water delivery time and homogeneous discharges of upper canal. AGA can adjust the genetic controlling parameters automatically on the basis of values of individual fitness and degree of population dispersion, and get a high precision solution. So it has a higher practical value in irrigation system management.

Keywords: delivery model; penalty function; AGA; irrigation canal system; optimization delivery

1. INTRODUCTION

Optimum canal water distribution based on scientific decision-making can reduce the seepage loss and invalid disposable water in the course of water transfer, and improve the utilization of irrigation water. Optimization of water distribution in irrigation canal system means that certain method and technology are used to optimize the rotation irrigation combination of distribution canal, under the condition that the capacity of water transfer in distribution canals and lateral canals is limited to meet the particular irrigation needs of crops in irrigation district (Kang et al., 1996). There are ample of literature on the method which can optimize the decision-making for canal water distribution. Under the presupposition that the upper canals are made up of stream tubes with same water discharge and the amount of water discharge in stream tubes is equal to the amount of the water discharge in lateral canals. Suryavanshi et al.(1986) established linear programming water distribution model to fix the numbers of stream tubes and its algorithm with the purpose of reducing the investment in canal project. Wang et al. (1995) applied it into his research on optimal distribution of canal water. Having supposed the lateral canals had the same water discharge, he put forward the optimal canal water distribution model based on 0-1 programming method. However, the earlier models were expanded by Reddy et al. (1999) to allow for different users to request different discharges. Reddy et al. (1999) also introduced the concept of a time window, a window within which a user must be scheduled to receive water. Anwar and Clarke (2001) took this concept further to suggest that every user could have a requested start time and developed an integer programming model that developed a schedule where the difference between scheduled and target start time was minimized. Lv et al. (2000) improved the objective function in document (Wang et al. 1995), and pointed out the method to equally process water distribution time for all the lateral canals, and made the upper canal closed at the same time in order to decrease the times to regulate the gate. Wardlaw and Bhaktikul (2004) discussed a GA developed for solution of two lateral canal scheduling problems, results that the GA approach has been demonstrated to be robust and efficient in application to lateral canals scheduling problems. Song et al. (2004) analyzed the GA used in document (Lv et al.,2000) and did some contribution to significantly increasing the speed of getting solutions to the model and solving the problem that it difficult to get the results when there are many lateral canals. However, application of this model is limited to irrigation systems where the distribution outlets along the canal have the same discharge capacity and such systems are hypothetical. On the other hand, these researches fall short of the ability to solve the problem that majority of the lateral canals in irrigation system have unequal discharge amount of water distribution; Meanwhile, traditional fixed penalty factor have some problems, such as it is

difficulty to use unified dimension, can not get a higher searching precision, prematurely converge to local optimal solution. Therefore, this paper study the canal water distribution model based on dynamic penalty function and Genetic Algorithm, it is firmly believed that the model and method will provide powerful technical support for making decision on optimal water distribution of irrigation canal system.

2. DESIGN AND APPLICATION IN THE GA OF PENALTY FUNCTION

At present, there are three commonly used methods in dealing with constraints in intelligent algorithms abandon infeasible solution (Wu et al.,1998), repair infeasible solution and penalty function method. Penalty function method, the most widely-used one, imposes certain penalty through infeasible solution and makes the resolution gradually close to feasible extreme point after iterations . The key point is to select proper penalty function for infeasible solutions: if the penalty value is too large, it is likely that the algorithm converge to non-extreme points; if the penalty value is too small, it's very likely that the convergence performance will be rather poor. In terms of constrained optimization question, the design of GA fitness function has some relations with the processing methods of constrained conditions. In the course of processing constrained conditions, having been inspired by the idea of simulated annealing, this paper takes dynamic penalty function to process constrained conditions. Annealing penalty factors σ_k is used to construct penalty function $P(x, \sigma_k)$. By using penalty function method, the equation (1) with the constrained optimization is changed into the unconstrained one shown as follows:

$$\begin{cases} \min & f(x) \\ s.t. & g_i(x) \geq 0, \quad i = 1, 2, m \\ & h_i(x) = 0, \quad i = 1, 2, n \\ & l_i(x) \leq 0, \quad i = 1, 2, p \end{cases}$$

$$F(x, \sigma_k) = 1/[f(x) + P(x, \sigma_k)] = 1/\{f(x) + \sigma_k[\sum_{i=1}^m |\min(0, \sigma_1 g_i(x))| + \sum_{i=1}^n |\sigma_2 h_i(x)| + \sum_{i=1}^p \max(0, \sigma_3 l_i(x))]\} \quad (1)$$

In this equation, supposing σ_k is penalty factor, σ_1 、 σ_2 、 σ_3 is penalty factor of different constrained conditions, according to actual engineering

dimension and the value of **feasible solution**. $\sigma_k = 1/T_p$, $T_{p+1} = \alpha T_p$, $\alpha \in [0,1]$ are fixed, then x is decision variable after decoding, F is fitness function, $f(x)$ is **objective function**, and m 、 n 、 P are numbers of constraints, standing for “ \geq 、 $>$ ”, “ $=$ ”, “ \leq 、 $<$ ” respectively.

3. OPTIMAL CANAL WATER DISTRIBUTION MODEL FOR THE UNEVEN WATER DISCHARGE IN LATERAL CANALS AND DESIGN OF ALGORITHM PARAMETERS

3.1 Foundation of the model

In the process of canal water distribution, the lateral canals are required to meet the following requirements: (1) the practical distributed water ought to be adequate to irrigate the farmlands; (2) the time for water distribution ought to change in the scheduled irrigation duration (“ T ” in short); (3) the total distributed water discharge of lateral canals should be equal to the one in upper canals at any time, and the distributed water discharge in lateral canals and upper canals should be limited to 0.6~1.0 times of the already designed discharge so that the requirement of the distributed water level can be satisfied and the canal burst resulted from over input can be avoided. When the distributed water discharge is same, the greater water discharge and the shorter time for water distribution, the smaller the lost water will be. Under the condition that the requirements for the decision scheme of optimal canal water distribution are satisfied, the distributed water discharge in upper and lateral canals at any time can be close to the designed discharge of each canal, the even discharge in upper canals can be realized and the times for regulating the gate can either be reduced.

3.2 The Establishment of Optimal Model

Supposing the upper canals’ designed discharge is Q_u ; and there are also several lateral canals(“ j ” in short), their designed discharge and distributed water discharge are q_{dj} and q_j ($j=1, 2, \dots, n$) respectively. Aiming at the minimal canal seepage loss of all the upper and lateral canals in all the distribution time during irrigation duration, the model is established as follows.

$$\text{Min } W = W_u + W_d \tag{2}$$

$$W_u = \sum_{i=1}^T f(A_u, m_u, q_u, l_u, t_u) \tag{3}$$

$$W_d = \sum_{i=1}^T \sum_{j=1}^N f(A_j, m_j, q_j, l_j, t_j) \tag{4}$$

Where, W_u and W_d is the total amount of water seepage loss while transferring in upper and lateral canals respectively during irrigation duration (m^3) ; q_u and q_j are distributed water discharge of upper and lateral canals respectively (m^3/s); A_u 、 A_j are permeability coefficients of upper and lateral canal beds, and m_u 、 m_j their permeable indexes respectively; l_u 、 l_j are the length of water transport in upper and lateral respectively (m), and t_u 、 t_j their corresponding distributed time (d). Therefore, the formula to measure the amount of canal seepage losses goes like this:

$$W = f(A, m, q, l, t) = [A \cdot l \cdot q^{(1-m)} \cdot t] / 100 \tag{5}$$

(parameters are same with the above-mentioned)

And the model mainly has the flowing constrained conditions:

Constrained conditions for distributed water discharge in lateral canals:

Any lateral canal's distributed water discharge “ q_j ” is α_j times of its designed discharge.

$$q_j = \alpha q_{dj} (j=1, 2, \dots, N) \tag{6}$$

$$0.6 \leq \alpha_j \leq 1.0 \tag{7}$$

Constrained conditions for irrigation duration:

The distributed time is t_j , the starting time is “ t_j' ” , and ending time is “ t_j'' ” . The t_j' and t_j'' should be in the irrigation duration “ T ” .

$$0 \leq t_j' \tag{8}$$

$$t_j' = t_j'' - t_j \tag{9}$$

$$t_j'' \leq T \tag{10}$$

Constrained conditions for water volume:

The product of any lateral canals' distributed water discharge and delivery time “ t_j ” should be equal to the required distributed water discharge “ W_j ” in the area.

$$W_j = q_j \times t_j \quad (11)$$

Constrained conditions for water balance:

The practical distributed water discharge of upper canals at any time (“ q_u ” in short) should be equal to the total amount of the discharge of lateral canals at the same period of time.

$$q_u = \sum_{j=1}^n q_j \times x_{ij} \quad (i=1, 2, \dots, T) \quad (12)$$

$$\text{if } t_j' \leq i \leq t_j'', x_{ij} = 1; \text{ otherwise } x_{ij} = 0 \quad (13)$$

Constrained conditions for distributed water discharge in upper canals:

The practical distributed water discharge of upper canals at any time (“ q_u ” in short) gets close to designed discharge and smaller than the maximally allowed discharge.

$$q_u \approx Q_u \quad (14)$$

$$0.6Q_u \leq q_u \leq 1.2Q_u \quad (15)$$

3.3 Genetic Operations and the Setting of Algorithm Parameters

It's extremely difficult to get the solutions by the above-mentioned model. Using the method in this paper, it is easy get solutions to the global optimal values of this mathematical model, because the ordinary irrigation canal system has lots of lateral canals and irrigation duration lasts 10 to 20 days, there are lots of decision-making periods based on the criterion that a period lasts 12 hours. The constrained conditions which can satisfy the requirements of formulas (6) ~ (13) and the balanced distributed water discharge in upper and lateral canals are accounted to several hundreds. The obvious problems of genetic algorithm are that it's hard to get feasible solutions to the questions at the beginning of evolution. Therefore, properly cope with the problems of constrained conditions, scientific selection of decision factors, coding design and determination of genetic operators are all included in this paper.

3.3.1 Coding design

After the construction of irrigation district, the length of water delivery (l), permeability parameter (A and m) and design discharge (q_{dj}), the maximum and minimum of the allowed discharge through canals are fixed. The decision of canal water distribution in irrigation district is optimize the

decision variables like the starting time, the duration, and the ending time of the distributed water and the distributed water discharge under the condition that the water distribution quantity is fixed in all the canals. Having considered the variables are closely-linked and thoroughly analyzed the model's characteristics, this paper regards the ratio of distributed water discharge in lateral canals and their designed discharge (" α_j " in short) and the ending time of water distribution (" t_j " in short) as decision variables to encode. Therefore, the variable α_j can make the algorithm automatically satisfy the constrained conditions in formulas (6)~(7); the ratio of declared water quantity and distributed water discharge can satisfy the constrained conditions in formula(11); the encoding of ending time of the distributed water t_j can satisfy the constrained conditions in formula(10); the starting time, which is got from the margin between the ending time and the irrigation duration time, can satisfy the constrained conditions in formula(9). This method of dealing with constrained conditions remarkably facilitates the algorithm to get feasible solutions.

3.3.2 Selection of operators

Based on the evaluation of population's individual fitness degree, some individuals are selected according to certain proportion and regarded as parents for propagation. Roulette bet method is adopted in this paper to select the individuals.

3.3.3 Crossover operator

After the match and single crossover of the parental individuals, the **offspring individuals** are produced. In this process, the **adaptive crossover rate** is adopted in order to reduce the possibility of damaging some individuals who have high fitness(Srinivas et al.,1994; Wang et al.,2002). In

AGA, P_c according to formulas (16).

$$P_c = \begin{cases} P_{c1} - \frac{(P_{c1} - P_{c2})(f' - f_{avg})}{f_{max} - f_{avg}}, & f' \geq f_{avg} \\ P_{c1}, & f < f_{avg} \end{cases} \quad (16)$$

3.3.4 Mutation operators

The individuals and the location of their genetic codes are randomly selected, and the individual can freely mutated in the gene's allowed range. The adaptive mutation probability is also used for mutation probability. In AGA, P_m according to formulas (17).

$$P_m = \begin{cases} P_{m1} - \frac{(P_{m1} - P_{m2})(f - f_{avg})}{f_{max} - f_{avg}}, & f \geq f_{avg} \\ P_{m1}, & f < f_{avg} \end{cases} \quad (17)$$

In the formulas, f_{max} is the maximal individual fitness value of population; f_{avg} is the average individual fitness value of population; f' is the larger fitness value of the two in the crossover; f is the fitness value of the individual that is involved in variation. $P_{c1} = 0.9$, $P_{c2} = 0.6$, $P_{m1} = 0.1$, $P_{m2} = 0.001$.

4. APPLICATION EXAMPLE

11 branch canals have 24 lateral canals in northern main branch, Feng Jiashan irrigation district, Shaanxi Province are chosen as application example. During the practical process of water distribution in certain irrigation duration in the spring of 2005. The designed discharge of the upper canals Q_u is $1.2\text{m}^3/\text{s}$, the designed discharge of lateral canals varies among $0.03 \sim 0.18\text{m}^3/\text{s}$, the soil in canal bed is medium loam and the permeability parameters of soil "A" and "m" are 1.9 and 0.4 individually (Wang, 2000). Besides the present experience, the AGA and the method based on dynamic penalty function and AGA were adopted to make contrast verification for the marshalling scheme of canal water distribution model, the results are shown as follows in Fig.1、Fig.2 and Fig.3.

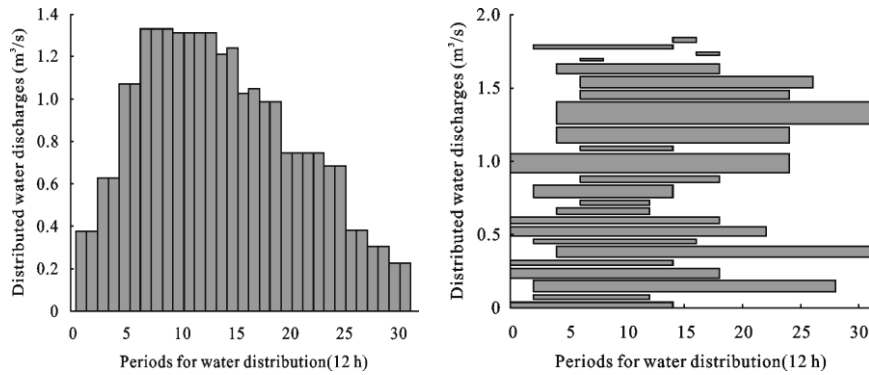


Fig. 1 Procedures of water distribution of a) upper and b) lateral canals by experiential scheme

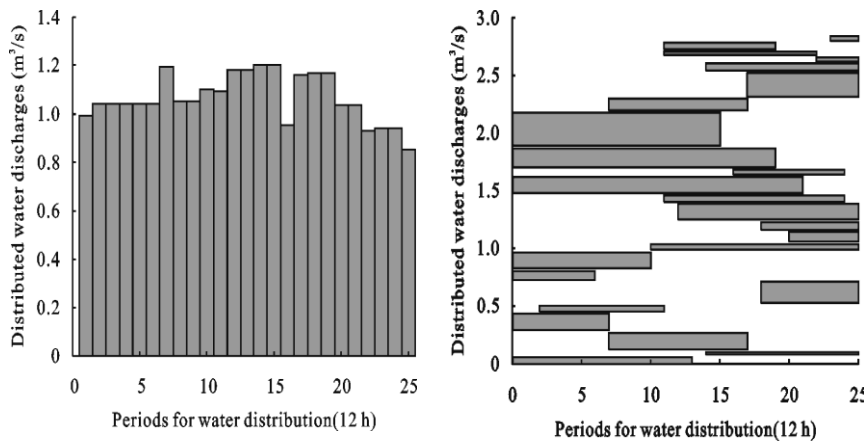


Fig.2 Procedures of water distribution of a) upper and b) lateral canals by AGA optimal scheme

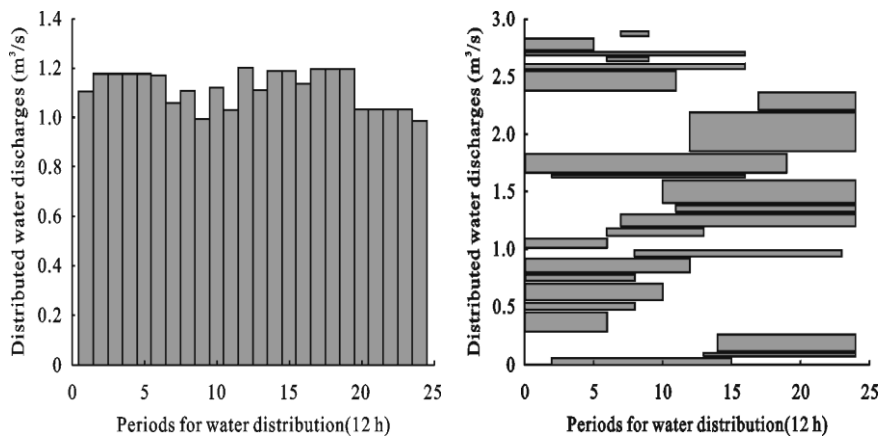


Fig.3. Procedures of water distribution of a) upper and b) lateral canals by this paper optimal scheme

In Fig.1, a、 b are the results of the experienced methods used in irrigation district widely respectively, from which the practical distributed water discharge in branch canals verifies remarkably in the process of experiential water distribution and it's quite common that great fluctuation happened in the upper canal distributed water discharge, which make it difficult to manage the water distribution, as the Fig.1-a displayed. The distribution water time in the practical distribution water process is comparatively concentrated, the total discharge is rather small, the practical distributed water discharge is larger than the designed discharge or smaller than the allowed minimum, as the Fig.1-b shown.

By AGA optimal scheme, the distributed water discharge that are made according to the typical optimal scheme is comparatively even, which facilitates the water distribution and shortens 6 periods (from 31 to 25), as the Fig.2-a displayed. If the distributed water discharge in lateral canals by AGA optimal scheme is equally distributed based on the allowed discharge range, the distribution will be reasonable and practical, which is more optimal than the result of the experienced methods, indicated in Fig.2-b.

Compared with the AGA optimal scheme, the optimal method in this paper has a excellent accuracy in search, water distribution period is shortened(from 25 to 24), the water discharge in upper canals becomes more even, water distribution is more reasonable, and the quality of distributed water is obviously improved, as the Fig.3 shown.

5. CONCLUSIONS

This paper proposed a model of lateral canals with unequal discharges and a solution method, and cope with constraint condition adopted the thought of simulated annealing and studied the way how to deal with the issue of constraints by dynamic penalty function. This method can save feasible solution, use the part of the useful gene of infeasible solution, and can avoid optimal solutions which is premature convergence to local in the process of genetic process effectively, thereby obtaining global optimization solutions and overcoming traditional problem of fixed penalty factor ,such as it's difficulty to use unified dimension, can not get a higher searching precision ,prematurely converge to local optimal solution. In the genetic operation, adaptive genetic algorithm can adjust the genetic controlling parameters automatically on the basis of values of individual fitness and degree of population dispersion, thereby improving the precision and stability of the AGA. The example illustrates that using the method in this paper can significantly decrease the water distribution time ,get much more reasonable lateral canals water delivery time , homogeneous discharges of upper canal and reduce the water seepage loss while transferring. So this

theory can provide powerful technical support for optimal water allocation of irrigation canal system.

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