Group Decision Making with Triangular Fuzzy Linguistic Variables

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Abstract. In group decision making with linguistic information, the decision makers (DMs) usually provide their assessment information by means of linguistic variables. In some situations, however, the DMs may provide fuzzy linguistic information because of time pressure, lack of knowledge, and their limited attention and information processing capabilities. In this paper, we introduce the concepts of triangular fuzzy linguistic variable and its member function, and introduce some operational laws of triangular fuzzy linguistic variables. We propose a formula for comparing triangular fuzzy linguistic variables, and develop some operators for aggregating triangular fuzzy linguistic variables, such as the fuzzy linguistic averaging (FLA) operator, fuzzy linguistic weighted averaging (FLWA) operator, fuzzy linguistic ordered weighted averaging (FLOWA) operator, and induced FLOWA (IFLOWA) operator, etc. Based on the FLWA and IFLOWA operators, we develop a practical method for group decision making with triangular fuzzy linguistic variables, and finally, an illustrative example is given to verify the feasibility and effectiveness of the developed method.

1 Introduction

Group decision making with linguistic information involves aggregating different individual linguistic decision information into the collective linguistic decision information, which is a hot research topic having received more and more attention from researchers [1-14]. Herrera et al. [1-6], Bordogna et al. [7], Marimin et al. [8], and Xu [9-14] have investigated the group decision making problem, in which the decision information is expressed as linguistic variables. In some situations, however, the DMs may provide fuzzy linguistic information because of time pressure, lack of knowledge, and their limited attention and information processing capabilities. In this paper, we shall investigate another kind of group decision making problem, in which the DMs can only provide their preferences in the form of triangular fuzzy linguistic variables. To do so, the remainder of this paper is structured as follows. In Section 2 we define the concept of triangular fuzzy linguistic variable and some operational laws of triangular fuzzy linguistic variables, and propose a formula for comparing triangular fuzzy linguistic variables. In Section 3 we propose some operators for aggregating triangular fuzzy linguistic variables. Section 4 develops a practical method for group decision making with triangular fuzzy linguistic variables. Section 5 gives an illustrative example, and Section 6 concludes this paper.

2 Triangular Fuzzy Linguistic Variables

In decision making with linguistic information, the DM generally provides his/her assessment information by using linguistic scale [1-14]. Suppose that $S = \{s_i \mid i = -t,...,t\}$ is a finite and totally ordered discrete label set. Any label, s_i, represents a possible value for a linguistic variable, and it requires that $s_i < s_j$ iff $i < j$. For example, a label set S could be [12]:

$$S = \{s_{-4} = extremely\ \ poor,\ s_{-3} = very\ \ poor,\ s_{-2} = poor,$$
$$s_{-1} = slightly\ \ poor,\ s_0 = fair,\ s_1 = slightly\ good,$$
$$s_2 = good,\ s_3 = very\ good,\ s_4 = extremely\ good\}$$

In the process of aggregating information, some results may do not exactly match any linguistic labels in S. To preserve all the given information, Xu [12] extended the discrete label set S to a continuous label set $\overline{S} = \{s_\alpha \mid \alpha \in [-q, q]\}$, where $q\ (q > t)$ is a sufficiently large positive integer. If $s_\alpha \in S$, then s_α is called an original linguistic label, otherwise, s_α is called a virtual linguistic label. In general, the DMs use the original linguistic labels to evaluate alternatives, and the virtual linguistic labels can only appear in operation.

Definition 2.1. *Let* $s_\alpha, s_\beta \in \overline{S}$*, then we define the distance between* s_α *and* s_β *as:*

$$d(s_\alpha, s_\beta) = |\alpha - \beta| \tag{1}$$

In some situations, however, the DMs may provide fuzzy linguistic information because of time pressure, lack of knowledge, and their limited attention and information processing capabilities, in the following, we define the concept of triangular fuzzy linguistic variable.

Definition 2.2. *Let* $\hat{s} = (s_\alpha, s_\beta, s_\gamma) \in \hat{S}$*, where* s_α, s_β *and* s_γ *are the lower, modal and upper values of* $\hat{s}$*, respectively, then we call* $\hat{s}$ *a triangular fuzzy linguistic variable, which is characterized by the following member function:*

$$\mu_{\hat{s}}(\theta) = \begin{cases} 0, & s_{-q} \le s_\theta \le s_\alpha \\ \dfrac{d(s_\theta, s_\alpha)}{d(s_\beta, s_\alpha)}, & s_\alpha \le s_\theta \le s_\beta \\ \dfrac{d(s_\theta, s_\gamma)}{d(s_\beta, s_\gamma)}, & s_\beta \le s_\theta \le s_\gamma \\ 0, & s_\gamma \le s_\theta \le s_q \end{cases} \tag{2}$$

Clearly, s_β gives the maximal grade of $\mu_{\hat{s}}(\theta)$ ($\mu_{\hat{s}}(\theta)=1$), s_α and s_γ are the lower and upper bounds which limit the field of the possible evaluation. Especially, if $s_\alpha = s_\beta = s_\gamma$, then $\hat{s}$ is reduced to a linguistic variable.

Let $\hat{S}$ be the set of all triangular fuzzy linguistic variables. Consider any three triangular fuzzy linguistic variables $\hat{s} = (s_\alpha, s_\beta, s_\gamma)$, $\hat{s}_1 = (s_{\alpha_1}, s_{\beta_1}, s_{\gamma_1})$, $\hat{s}_2 = (s_{\alpha_2}, s_{\beta_2}, s_{\gamma_2})$, and $\lambda \in [0,1]$, then we define their operational laws as follows:

1) $\lambda\hat{s} = \lambda(s_\alpha, s_\beta, s_\gamma) = (\lambda s_\alpha, \lambda s_\beta, \lambda s_\gamma) = (s_{\lambda\alpha}, s_{\lambda\beta}, s_{\lambda\gamma})$;

2) $\hat{s}_1 \oplus \hat{s}_2 = (s_{\alpha_1}, s_{\beta_1}, s_{\gamma_1}) \oplus (s_{\alpha_2}, s_{\beta_2}, s_{\gamma_2}) = (s_{\alpha_1} \oplus s_{\alpha_2}, s_{\beta_1} \oplus s_{\beta_2}, s_{\gamma_1} \oplus s_{\gamma_2})$
$= (s_{\alpha_1+\alpha_2}, s_{\beta_1+\beta_2}, s_{\gamma_1+\gamma_2})$;

3) $\lambda(\hat{s}_1 \oplus \hat{s}_2) = \lambda\hat{s}_1 \oplus \lambda\hat{s}_2$;

4) $\hat{s}_1 \oplus \hat{s}_2 = \hat{s}_2 \oplus \hat{s}_1$.

In the following, we introduce a formula for comparing triangular fuzzy linguistic variables:

Definition 2.3. *Let $\hat{s}_1 = (s_{\alpha_1}, s_{\beta_1}, s_{\gamma_1})$, $\hat{s}_2 = (s_{\alpha_2}, s_{\beta_2}, s_{\gamma_2}) \in \hat{S}$, then the degree of possibility of $\hat{s}_1 \geq \hat{s}_2$ is defined as:*

$$p(\hat{s}_1 \geq \hat{s}_2) = \rho \max\left\{1 - \max\left(\frac{d(s_{\beta_2}, s_{\alpha_1})}{d(s_{\beta_1}, s_{\alpha_1}) + d(s_{\beta_2}, s_{\alpha_2})}, 0\right), 0\right\}$$
$$+ (1-\rho)\max\left\{1 - \max\left(\frac{d(s_{\gamma_2}, s_{\beta_1})}{d(s_{\gamma_1}, s_{\beta_1}) + d(s_{\gamma_2}, s_{\beta_2})}, 0\right), 0\right\} \quad (3)$$

where the value ρ is an index of rating attitude. It reflects the DM's risk-bearing attitude. If $\rho < 0.5$, then the DM is a risk lover. If $\rho = 0.5$, then the DM's attitude is neutral to the risk. If $\rho > 0.5$, the DM is a risk avertor. Especially, if both the linguistic variables $\hat{s}_1$ and $\hat{s}_2$ express precise linguistic information (i.e. if

$$d(s_{\gamma_1}, s_{\alpha_1}) + d(s_{\gamma_2}, s_{\alpha_2}) = 0 \quad (4)$$

then we define the degree of possibility of $\hat{s}_1 > \hat{s}_2$ as:

$$p(\hat{s}_1 > \hat{s}_2) = \begin{cases} 1, & if \quad \hat{s}_1 > \hat{s}_2 \\ 1/2, & if \quad \hat{s}_1 = \hat{s}_2 \\ 0, & if \quad \hat{s}_1 < \hat{s}_2 \end{cases} \quad (5)$$

Obviously, the possibility degree $p(\hat{s}_1 \geq \hat{s}_2)$ satisfies the following properties:

1) $0 \leq p(\hat{s}_1 \geq \hat{s}_2) \leq 1$;

2) $p(\hat{s}_1 \geq \hat{s}_2) + p(\hat{s}_2 \geq \hat{s}_1) = 1$. Especially, $p(\hat{s}_1 \geq \hat{s}_1) = p(\hat{s}_2 \geq \hat{s}_2) = \frac{1}{2}$.

3 Some Aggregation Operators

In the following, we develop some operators for aggregating triangular fuzzy linguistic variables.

Definition 3.1. *Let* $FLA : \hat{S}^n \to \hat{S}$ *, if*

$$FLA(\hat{s}_1, \hat{s}_2, ..., \hat{s}_n) = \frac{1}{n}(\hat{s}_1 \oplus \hat{s}_2 \oplus \cdots \oplus \hat{s}_n) \qquad (6)$$

where $\hat{s}_i \in \hat{S}$ *,* $i = 1,2,..., n$ *, then* FLA *is called a fuzzy linguistic averaging (FLA) operator.*

Definition 3.2. *Let* $FLWA : \hat{S}^n \to \hat{S}$ *, if*

$$FLWA_w(\hat{s}_1, \hat{s}_2, ..., \hat{s}_n) = w_1 \hat{s}_1 \oplus w_2 \hat{s}_2 \oplus \cdots \oplus w_n \hat{s}_n \qquad (7)$$

where $w = (w_1, w_2, ..., w_n)^T$ *is the weighting vector of the* $\hat{s}_i$ *,* $\hat{s}_i \in \hat{S}$ *,* $w_i \geq 0,$ $i = 1,2,..., n,$ $\sum_{i=1}^{n} w_i = 1$ *then* $FLWA$ *is called a triangular fuzzy linguistic weighted averaging (FLWA) operator.*

In [15], Yager introduced the ordered weighted averaging (OWA) operator, whose fundamental feature is the reordering step. The OWA operator combines the arguments by giving weights to the values in relation to their ordering position, and diminishes the importance of extreme values by increasing the importance of central ones. In the following, we develop a fuzzy linguistic OWA operator to accommodate the situations where the input arguments are triangular fuzzy linguistic variables.

Definition 3.3. *A fuzzy linguistic ordered weighted geometric (FLOWA) operator of dimension* n *is a mapping* $FLOWA : \hat{S}^n \to \hat{S}$ *that has associated with* it a weighting *vector* $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ *such that* $\omega_j \geq 0,$ $j = 1,2,..., n,$ $\sum_{j=1}^{n} \omega_j = 1$*. Furthermore*

$$FLOWA_\omega(\hat{s}_1, \hat{s}_2, ..., \hat{s}_n) = \omega_1 \hat{r}_1 \oplus \omega_2 \hat{r}_2 \oplus \cdots \oplus \omega_n \hat{r}_n \qquad (8)$$

where $\hat{r}_j$ *is the* j *th largest of* $\hat{s}_i$ $(i = 1, 2, ..., n)$ *,* $\hat{s}_i \in \hat{S}$ *. Especially, if* $\omega = (1/n, 1/n, ..., 1/n)^T$*, then the FLOWA operator is reduced to the FLA operator.*

To rank these triangular fuzzy linguistic variables $\hat{s}_j$ $(j = 1,2,..., n)$, we first compare each argument $\hat{s}_i$ with all triangular fuzzy linguistic variables $\hat{s}_j$

$(j=1,2,...,n)$ by using (3), and let $p_{ij}=p(\hat{s}_i \geq \hat{s}_j)$. Then we construct a complementary matrix [16-22] $P=(p_{ij})_{n\times n}$, where

$$p_{ij}\geq 0,\ p_{ij}+p_{ji}=1,\ p_{ii}=0.5,\ i,j=1,2,...,n \tag{9}$$

Summing all the elements in each line of matrix P, we have

$$p_i=\sum_{j=1}^{n} p_{ij},\ \ i=1,2,...,n \tag{10}$$

Then we can rank the arguments $\hat{s}_i\ (i=1,2,...,n)$ in descending order in accordance with the values of $p_i\ (i=1,2,...,n)$.

Yager and Filev [23] introduced an induced ordered weighted averaging (IOWA) operator. The IOWA operator allows the introduction of semantics or meaning in the aggregation of arguments, and therefore allows for better control over the aggregation stage developed in the resolution process. Below we develop an induced FLOWA (IFLOWA) operator to accommodate the situations where the input arguments are triangular fuzzy linguistic variables.

Definition 3.4. *An IFLOWA operator is defined as:*

$$IFLOWA_{\omega}\left(\langle\delta_1,\hat{s}_1\rangle,\langle\delta_2,\hat{s}_2\rangle...,\langle\delta_n,\hat{s}_n\rangle\right)=\omega_1\hat{s}_{\gamma_1}\oplus\omega_2\hat{s}_{\gamma_2}\oplus\cdots\oplus\omega_n\hat{s}_{\gamma_n} \tag{11}$$

where $\omega=(\omega_1,\omega_2,...,\omega_n)^T$ is a weighting vector, $\omega_j\geq 0$, $j=1,2,...,n$, $\sum_{j=1}^{n}\omega_j=1$, $\hat{s}_{\gamma_j}$ is the $\hat{s}_i$ value of the FLOWA pair $\langle\delta_i,\hat{s}_i\rangle$ having the j th largest δ_i ($i=1,2,...,n$), and δ_i in $\langle\delta_i,\hat{s}_i\rangle$ is referred to as the order inducing variable and $\hat{s}_i$ as the triangular fuzzy linguistic argument variable. Especially, if $\omega=(1/n,1/n,...,1/n)^T$, then IFLOWA is reduced to the FLA operator; if $\delta_i=\hat{s}_i$, for all i, then IFLOWA is reduced to the FLOWA operator; if $\delta_i=No.\,i$, for all i, where $No.\,i$ is the ordered position of $\hat{s}_i$ ($i=1,2,...,n$), then IFLOWA is reduced to the FLWA operator.

However, if there is a tie between $\langle\delta_i,\hat{s}_i\rangle$ and $\langle\delta_j,\hat{s}_j\rangle$ with respect to order inducing variables. In this case, we can follow the policy presented by Yager and Filev [23], that is, to replace the arguments of the tied objects by the average of the arguments of the tied objects, i.e., we replace the argument component of each of $\langle\delta_i,\hat{s}_i\rangle$ and $\langle\delta_j,\hat{s}_j\rangle$ by their average $(\hat{s}_i\oplus\hat{s}_j)/2$. If k items are tied, then we replace these by k replicas of their average.

The IFLOWA operator reflects the fuzzy majority by utilizing a fuzzy linguistic quantifier [14] to calculate its weighting vector. In the case of a non-decreasing proportional quantifier Q, the weighting vector can be obtained by the following expression:

$$\omega_k = Q\left(\frac{k}{n}\right) - Q\left(\frac{k-1}{n}\right), \quad k = 1,2,...,n \tag{12}$$

where

$$Q(r) = \begin{cases} 0 & if \ \ r < a, \\ \dfrac{r-a}{b-a} & if \ \ a \le r \le b, \\ 1 & if \ \ r > b \end{cases} \tag{13}$$

with $a, b, r \in [0, 1]$.

4 A Method for Group Decision Making with Triangular Fuzzy Linguistic Variables

Based on the above operators, we develop a practical method for ranking alternatives in group decision making with triangular fuzzy linguistic variables as follows.

Step 1. For a group decision making problem with fuzzy linguistic information. Let $X = \{x_1, x_2, ..., x_n\}$ be the set of alternatives, and $G = \{G_1, G_2, ..., G_m\}$ be the set of attributes. Let $w = (w_1, w_2, ..., w_m)^T$ be the weight vector of attributes, where $w_i \ge 0,\ i = 1,2,...,m, \sum_{i=1}^{m} w_i = 1$. Let $D = \{d_1, d_2, ..., d_m\}$ be the set of DMs, and $v = (v_1, v_2, ..., v_m)^T$ be the weight vector of DMs, where $v_l \ge 0, l = 1,2,..., m$, $\sum_{l=1}^{m} v_l = 1$. Suppose that $\hat{A}^{(k)} = (\hat{a}_{ij}^{(k)})_{m \times n}$ is the fuzzy linguistic decision matrix, where $\hat{a}_{ij}^{(k)} \in \hat{S}$ is a triangular fuzzy linguistic variable, given by the DM $d_k \in D$, for the alternative $x_j \in X$ with respect to the attribute $G_i \in G$.

Step 2. Utilize the IFLOWA operator

$$\hat{a}_{ij} = IFLOWA_{\omega}\left(\left\langle v_1, \hat{a}_{ij}^{(1)} \right\rangle, \left\langle v_2, \hat{a}_{ij}^{(2)} \right\rangle, \cdots, \left\langle v_l, \hat{a}_{ij}^{(l)} \right\rangle\right),$$
$$i = 1,2,...,m;\ j = 1,2,...,n \tag{14}$$

to aggregate all the decision matrices $\hat{A}^{(k)}\ (k = 1,2,...,l)$ into a collective decision matrix $\hat{A} = (\hat{a}_{ij})_{m \times n}$, where $v = (v_1, v_2, ..., v_l)^T$ is the weight vector of DMs, $\omega = (\omega_1, \omega_2, ..., \omega_l)^T$ is the weighting vector of the IFLOWA operator, $\omega_k \ge 0,\ k = 1,2,..., l, \sum_{k=1}^{l} \omega_k = 1$.

Step 3. Utilize the FLWA operator

$$\hat{a}_j = FLWA_w(\hat{a}_{1j}, \hat{a}_{2j}, ..., \hat{a}_{mj}) = w_1\hat{a}_{1j} \oplus w_2\hat{a}_{2j} \oplus \cdots \oplus w_m\hat{a}_{mj},\ j = 1,2,...,n \quad (15)$$

to derive the collective overall preference value $\hat{a}_j$ of the alternative x_j, where $w = (w_1, w_2, ..., w_m)^T$ is the weight vector of attributes.

Step 4. Compare each $\hat{a}_j$ with all $\hat{a}_i\ (i = 1,2,...,n)$ by using (3), and develop a complementary matrix $P = (p_{ij})_{n\times n}$, where $p_{ij} = p(\hat{a}_i \geq \hat{a}_j)$.

Step 5. Rank $\hat{a}_j\ (j = 1,2,...,n)$ in descending order in accordance with the values of $p_i\ (i = 1,2,...,n)$ obtained by using (11).

Step 6. Rank all the alternatives $x_j\ (j = 1,2,...,n)$ and select the best one(s) in accordance with $\hat{a}_j\ (j = 1,2,...,n)$.

Step 7. End.

5. Illustrative Example

In this section, a group decision making problem of evaluating university faculty for tenure and promotion (adapted from [24]) is used to illustrate the proposed procedure.

A practical use of the proposed approach involves the evaluation of university faculty for tenure and promotion. The criteria (attributes) used at some universities are G_1 : teaching, G_2 : research, and G_3 : service (whose weight vector $w = (0.36, 0.31, 0.33)^T$). Five faculty candidates (alternatives) $x_j\ (j = 1, 2, 3, 4, 5)$ are evaluated using the label set (listed in Section 2) by three DMs $d_k\ (k = 1,2,3)$ (whose weight vector $v = (0.4, 0.5, 0.1)^T$) under these three attributes, as listed in Tables 1-3.

Table 1. Fuzzy linguistic decision matrix $A^{(1)}$

G_i	x_1	x_2	x_3	x_4	x_5
G_1	(s_{-2}, s_{-1}, s_0)	(s_{-1}, s_0, s_1)	(s_{-2}, s_{-1}, s_1)	(s_1, s_3, s_4)	(s_0, s_1, s_3)
G_2	(s_0, s_2, s_3)	(s_1, s_3, s_4)	(s_{-3}, s_{-1}, s_0)	(s_{-1}, s_1, s_2)	(s_1, s_2, s_3)
G_3	(s_1, s_2, s_4)	(s_2, s_3, s_4)	(s_1, s_3, s_4)	(s_{-2}, s_{-1}, s_1)	(s_0, s_1, s_2)

Table 2. Fuzzy linguistic decision matrix $A^{(2)}$

G_i	x_1	x_2	x_3	x_4	x_5
G_1	(s_{-1}, s_0, s_1)	(s_1, s_2, s_4)	(s_1, s_2, s_3)	(s_1, s_2, s_4)	(s_{-1}, s_1, s_2)
G_2	(s_{-2}, s_{-1}, s_0)	(s_{-1}, s_0, s_2)	(s_1, s_2, s_3)	(s_0, s_1, s_2)	(s_2, s_3, s_4)
G_3	(s_1, s_2, s_3)	(s_2, s_3, s_4)	(s_0, s_1, s_3)	(s_0, s_1, s_2)	(s_{-1}, s_0, s_1)

Table 3. Fuzzy linguistic decision matrix $A^{(3)}$

G_i	x_1	x_2	x_3	x_4	x_5
G_1	(s_{-1}, s_0, s_2)	(s_{-1}, s_0, s_1)	(s_{-1}, s_1, s_2)	(s_2, s_3, s_4)	(s_0, s_1, s_3)
G_2	(s_1, s_3, s_4)	(s_{-2}, s_{-1}, s_0)	(s_0, s_2, s_3)	(s_{-1}, s_1, s_3)	(s_1, s_2, s_4)
G_3	(s_2, s_3, s_4)	(s_1, s_2, s_3)	(s_{-2}, s_0, s_1)	(s_2, s_3, s_4)	(s_{-1}, s_0, s_1)

In what follows, we utilize the method developed in this paper to get the most desirable alternative(s):

Step 1. Utilize (11) (by using the linguist quantifier "*most*", with the corresponding weighting vector $\omega = (1/15, 10/15, 4/15)^T$):

$$\hat{a}_{ij} = IFLOWA_{\omega}\left(\left\langle v_1, \hat{a}_{ij}^{(1)} \right\rangle, \left\langle v_2, \hat{a}_{ij}^{(2)} \right\rangle, \left\langle v_3, \hat{a}_{ij}^{(3)} \right\rangle\right),\ i = 1,2,3;\ j = 1,2,3,4,5$$

to aggregate all the decision matrices $\hat{A}^{(k)}$ $(k = 1,2,3)$ into a collective decision matrix $\hat{A} = (\hat{a}_{ij})_{3\times 4}$ (see Table 4).

Table 4. The collective Fuzzy linguistic decision matrix $\hat{A}$

G_i	x_1	x_2	x_3	x_4	x_5
G_1	$(s_{-1.67}, s_{-0.67}, s_{0.60})$	$(s_{-0.87}, s_{0.13}, s_{1.20})$	$(s_{-1.53}, s_{-0.27}, s_{1.40})$	$(s_{1.27}, s_{2.93}, s_{4.00})$	$(s_{-0.07}, s_{1.00}, s_{2.93})$
G_2	$(s_{0.13}, s_{2.07}, s_{3.07})$	$(s_{0.07}, s_{1.73}, s_{2.80})$	$(s_{-1.93}, s_{0.00}, s_{1.00})$	$(s_{-0.93}, s_{1.00}, s_{2.27})$	$(s_{1.07}, s_{2.07}, s_{3.33})$
G_3	$(s_{1.27}, s_{2.27}, s_{3.93})$	$(s_{1.73}, s_{2.73}, s_{3.73})$	$(s_{0.13}, s_{2.07}, s_{3.13})$	$(s_{-0.80}, s_{0.20}, s_{1.87})$	$(s_{-0.38}, s_{0.67}, s_{1.67})$

Step 2. Utilize the FLWA operator

$$\hat{a}_j = FLWA_w(\hat{a}_{1j}, \hat{a}_{2j}, \hat{a}_{3j}),\ j = 1,2,3,4,5$$

to aggregate $\hat{a}_{ij}$ $(i = 1,2,3)$ corresponding to the alternative x_j, and then get the collective overall preference value $\hat{a}_j$ of the alternative x_j:

$$\hat{a}_1 = (s_{-0.142}, s_{1.150}, s_{2.465}),\ \hat{a}_2 = (s_{0.279}, s_{1.484}, s_{2.531}),\ \hat{a}_3 = (s_{-1.106}, s_{0.586}, s_{1.847})$$

$$\hat{a}_4 = (s_{-0.095}, s_{1.431}, s_{2.761}),\ \hat{a}_5 = (s_{0.181}, s_{1.223}, s_{2.638})$$

Step 3. Suppose that the DM's attitude is neutral to the risk, i.e., $\rho = 0.5$, then compare each $\hat{a}_i$ with all the $\hat{a}_j$ $(j = 1,2,3,4,5)$ by using (3), and develop a complementary matrix:

$$P = \begin{bmatrix} 0.5 & 0.3821 & 0.7427 & 0.4163 & 0.4351 \\ 0.6179 & 0.5 & 0.8683 & 0.5205 & 0.5556 \\ 0.2573 & 0.1317 & 0.5 & 0.1861 & 0.1906 \\ 0.5837 & 0.4795 & 0.8139 & 0.5 & 0.5235 \\ 0.5649 & 0.4444 & 0.8094 & 0.4765 & 0.5 \end{bmatrix}$$

Summing all elements in each line of matrix P, we have

$$p_1 = 2.4762\ ,\ p_2 = 3.0623\ ,\ p_3 = 1.2657\ ,\ p_4 = 2.9006\ ,\ p_5 = 2.7952$$

Then we rank $\hat{a}_j\ (j = 1,2,3,4,5)$ in descending order in accordance with the values of p_j ($j = 1,2,3,4,5$): $\hat{a}_2 > \hat{a}_4 > \hat{a}_5 > \hat{a}_1 > \hat{a}_3$.

Step 4. Rank all the alternatives $x_j\ (j = 1,2,3,4,5)$ and select the best one(s) in accordance with $\hat{a}_j\ (j = 1,2,3,4,5)$, then we get $x_2 \succ x_4 \succ x_5 \succ x_1 \succ x_3$, and thus, the most desirable alternative is x_2.

6. Concluding Remarks

In group decision making with linguistic information, the DMs sometimes provide only fuzzy linguistic information because of time pressure, lack of knowledge, and their limited attention and information processing capabilities. In this paper, we have investigated the group decision making problem with triangular fuzzy linguistic variables. We have defined the concepts of triangular fuzzy linguistic variable and some of its operational laws, and then proposed some operators for aggregating triangular fuzzy linguistic variables. Based on these aggregation operators, we have developed a practical method for group decision making with triangular fuzzy linguistic variables. In future research, our work will focus on the application of triangular fuzzy linguistic variables in the field of computing with words.

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