Analyzing the Influence of Overconfident Investors on Financial Markets through Agent-Based Modeling

Hiroshi Takahashi and Takao Terano

Graduate School of Humanities and Social Sciences, Okayama University, 3-1-1 Tsushima-naka, Okayama-city,700-8530,Japan,
Interdisciplinary Graduate School of Science and Engineering, Tokyo Institute of Technology, 4259 Nagatsuda-Cho, Midori-ku, Yokohama 226-8502,Japan
htaka@e.okayama-u.ac.jp, terano@dis.titech.ac.jp

Abstract. In this research, we employ Agent-Based Modeling to analyze how asset prices are affected by investors' Behavior. We construct a virtual financial markets that contains several types of investors: fundamentalists and non-fudamentalists. In this analysis, we place focus on the influence of overconfident investors on financial markets. As a result of intensive analysis, we find that overconfident investors are generated in a bottom-up fashion in the market. Furthermore, we also find that overconfident investors have the ability to contribute to market efficiency.

1 Introduction

In the area of computer science, Agent-Based Modeling is proposed as an effective method to analyze the relation between micro-rules and macro-behavior[2][3]. Agent-Based Modeling is an attempt to explain the macro-behavior of systems by local rules. As a result of applying this Model to social science, it is found that a variety of macro-behavior emerges bottom-up from local micro-rules[7]. An artificial market is one of the good applications of Agent-Based Modeling to financial markets[1][24][17][1]. Recently various kinds of financial services have been proposed. In order to improve them, it is necessary to analyze financial markets from the bottom up. Agent-Based Modeling provides an effective method for them.

In recent years, there has been rising interest in a field called behavioral finance which incorporates psychological methods in analyzing investor behavior. There are numerous arguments in behavioral finance that investors' decision making bias can explain phenomenon in the financial market which until now had gone unexplained by pointing out limit to arbitrage and existence of systematic biases in decision Making [21][13][15].

However, there is also criticism that most such arguments in behavioral finance are simply ad hoc, applying decision making bias exogenously, and only introducing decision making bias conveniently in order to explain certain phenomenon in the financial market.

With such underlying arguments, this analysis aims to show that decision making bias discussed in financial economics appears in a bottom-up fashion in the financial market. Above all, this research is undertaken with a focus on overconfident decision making which has been under the spotlight in recent years[4][10][22]. Furthermore, this research inquires into the conditions under which transaction prices reflect fundamental values.

The next section of this paper explains the model utilized for this analysis, before analysis results are looked at in section 3. Section 4 contains a summary.

[1] Hirshleifer describes how Agent-Based Modeling is effective to analyze financial markets[12].

2 Model

A computer simulation of the financial market involving 1000 investors was used as the model for this research, shares and risk-free assets being the 2 possible transaction methods [1][23]. Several types of investors exist in the market, each undertaking transactions based on their own stock valuations. This market is composed of 3 major steps, (1) generation of corporate earnings, (2) formation of investor forecasts, (3) setting transaction prices. The market advances through repetition of these steps[23].

2.1 Assets traded in the Market

This market consists of both risk-free and risky assets. There is a financial security (as risky assets) in which all profits gained during each term are distributed to the shareholders. The corporate earning (y_t) accrues according to the process of $y_t = y_{t-1} \cdot (1 + \varepsilon_t), where\, \varepsilon_t \sim N(0, \sigma_y^2)$[18], and the stock is traded after the corporate profit of current period is announced. Each investor is given common asset holdings at the start of the term and is able to borrow or lend the risk-free asset unlimitedly in principle. The initial asset amount of every investor is 1,000 in stock and 1,000 in risk-free asset.

2.2 Modeling Investor Behavior

Investors in the market evaluate transaction prices based on their own forecast for market tendency, taking into account both risk and return rates when making investment decisions. Each investor decides on the investment ratio (w_t) of stock for each term based on the maximum objective function of $f(w_t^i) = r_{t+1}^{int,i} \cdot w_t^i + r_f \cdot (1 - w_t^i) - \lambda \cdot (\sigma_{t-1}^{s,i})^2 (w_t^i)^2$. In this case, $r_{t+1}^{int,i}$ and $\sigma_{t-1}^{s,i}$ express the expected rate of return and risk for stock as estimated by each investor i. r_f represents the risk-free rate. w_t^i is the stock investment ratio of investor i for term t [6][23].

Expected rate of return for shares ($r_{t+1}^{int,i}$) is calculated as $r_{t+1}^{int,i} = (1 \cdot c^{-1} \cdot (\sigma_{t-1}^{s,i})^{-2})/(1 \cdot c^{-1} \cdot (\sigma_{t-1}^{s,i})^{-2} + 1 \cdot (\sigma_{t-1}^{s,i})^{-2}) \cdot r_{t+1}^{f,i} + (1 \cdot (\sigma_{t-1}^{s,i})^{-2})/(1 \cdot c^{-1} \cdot (\sigma_{t-1}^{s,i})^{-2} + 1 \cdot (\sigma_{t-1}^{s,i})^{-2}) \cdot r_t^{im}$. Here, $r_{t+1}^{f,i}$,r_t^{im} express the expected rate of return, calculated respectively from short-term expected rate of return, and risk and gross current price ratio of stock etc[6][23]. c is adjustment coefficient[6][2].

Short-term expected rate of return ($r_t^{f,i}$) is obtained by $r_{t+1}^{f,i} = ((P_{t+1}^{f,i} + y_{t+1}^{f,i})/P_t - 1) \cdot (1 + \eta_t^i)$, ($P_{t+1}^{f,i}, y_{t+1}^{f,i}$) being the equity price and profit forecast for term t+1 as estimated by the investor. Short-term expected rate of return includes the error term ($\eta_t^i \sim N(0, \sigma_n^2)$) reflecting that even investors of the same forecast model vary slightly in their detailed outlook.

Expected rate of return for stock (r_t^{im}) as obtained from stock risk etc. is calculated from stock risk ($\sigma_{t-1}^{s,i}$), benchmark equity stake (W_{t-1}), investors' degree of risk avoidance (λ), and risk-free rate (r_f) in the equation $r_t^{im} = 2 \cdot \lambda \cdot (\sigma_{t-1}^s)^2 \cdot W_{t-1} + r_f$[6][19].

At every simulation step, the investors determine its asset allocation based on its prediction described below. The excess returns of the investors are relatively measured based on the buy-and-hold strategy, that is, the strategy the investor will never make trades on the market. The performance of the investors are evaluated by the excess returns.

[2] For more detail, refer to Black/Litterman[6].

2.3 Equity Price Forecasting Method

In this research, we analyze several kinds of forecasting methods such as (1) forecasting based on fundamental values, (2) forecasting based on trends (4 types), and (3) forecasting based on past averages (4 types). The details of each estimation are explained below.

Fundamentalist In this paper, we refer to the investors who make investment decisions based on fundamental values as "fundamentalists". We adopt the dividend discount model(DDM), which is the most basic derivation model for the fundamental value of stocks. The fundamentalists are supposed to know that the corporate profit accrues according to Brownian motion. Fundamentalists estimate the forecast stock price ($P^{f,i}_{t+1}$) and forecast profit ($y^{f,i}_{t+1}$) from profit for the term (y_t) and discount rate of stock (δ) respectively as $P^{f,i}_{t+1} = y_t/\delta$, $y^{f,i}_{t+1} = y_t$.

Forecasting based on trends The conventional asset pricing theories insist that the fundamentals are reflected in the prices so that the prices in the past do not affect the current price. However, the real markets and societies are flooded with information about the prices, and the price itself may have the meaning in real markets as Shiller pointed [20]. Furthermore, the analyses of the experiments on human being indicate that the people tend to find out the trends from a random sequence [5][14] which means there are good chances that investors find out some trends from the random fluctuation of stock prices.

With such underlying arguments, we formulate a model of the investor who finds out the trends from randomly fluctuate stock prices. Forecasting based on trends involves forecasting next term equity prices and profit through extrapolation of the most recent stock value fluctuation trends. In this research we deal with 4 types of trend measurement period: 1 day, 5 days, 10 days, and 20 days for trend measurements. The trend predictors estimate the next step's stock price and profit from the trend at t-1 (a^i_{t-1}) as $P^{f,i}_{t+1} = P_{t-1} \cdot (1 + a^i_{t-1})^2$ and $y^{f,i}_{t+1} = y_t \cdot (1 + a^i_{t-1})$, where $a^i_{t-1} = (P_{t-1}/P_{t-2} - 1)$ [1day], $a^i_{t-1} = (1/5) \sum_{i=1}^{5} (P_{t-1}/P_{t-i-1} - 1)$ [5 days], $a^i_{t-1} = (1/10) \sum_{i=1}^{10} (P_{t-1}/P_{t-i-1} - 1)$ [10 days] and $a^i_{t-1} = (1/20) \sum_{i=1}^{20} (P_{t-1}/P_{t-i-1} - 1)$ [20 days], respectively. Predicted price ($P^{f,i}_{t+1}$) and profit ($y^{f,i}_{t+1}$) are different when the trend measurement period is different.

Forecasting based on past averages Forecasting based on past averages involves estimating next term equity prices and profit based on the most recent average stock value.In this research we deal with 4 types of average measurement period: 1 day, 5 days, 10 days, and 20 days. The investors based on past averages estimate the next step's stock price and profit from the historical average of stock prices at t-1 as $P^{f,i}_{t+1} = P_{t-1}, y^{f,i}_{t+1} = y_t$ [1day], $P^{f,i}_{t+1} = (1/5) \sum_{i=1}^{5} (P_{t-i}), y^{f,i}_{t+1} = (1/5) \sum_{i=1}^{5} (y_{t-i+1})$ [5 days], $P^{f,i}_{t+1} = (1/10) \sum_{i=1}^{10} (P_{t-i}), y^{f,i}_{t+1} = (1/10) \sum_{i=1}^{10} (y_{t-i+1}$ [10 days] and $P^{f,i}_{t+1} = (1/20) \sum_{i=1}^{20} (P_{t-i}), y^{f,i}_{t+1} = (1/20) \sum_{i=1}^{20} (y_{t-i+1}$ [20 days], respectively.

2.4 Risk Estimation Method

In the area of decision making theory, it is reported that human being tends to be overconfident in his/her own ability[5]. Also in real markets, we often find that each investor talks about different future prospects with confidence. It seems like all investors tend to have overconfidence in varying degrees. With such background, we formulate the model of investors who are overconfident in their own predictions by assuming that they underestimate the risk of the stock.

In this research, stock risk is measured as $\sigma^{s,i}_{t-1} = s_i \cdot \sigma^h_{t-1}$. In this case, σ^h_{t-1} is an index that represents stock volatility calculated from price fluctuation of the most recent 100 steps, and s_i is the degree of overconfidence. The presence of a strong degree of overconfidence can be concluded when the value of s_i is less than 1, as estimated forecast error is shown as lower than its actual value. The investors whoes value of s_i is less than 1 tend to invest more actively[3].

2.5 Deciding Transaction Prices

Transaction prices are set as the price where stock supply and demand converge[1].The investment ratio (w^i_t) is the decreasing function of the stock price, and the total number of the stock issued in the market (N) is constant. We derive the traded price where the demand meets the supply($\sum_{i=1}^{M}(F^i_t w^i_t)/P_t = N$).

2.6 Rules of Natural Selection

After 25 terms pass since the market has started, the rules of natural selection come into play in this market on the basis of cumulative excess return for the most recent 5 terms [11][23]. The rules of natural selection are composed of the 2 steps of (1) appointment of investors who alter their investment strategy(forecast type and degree of overconfident(s_i)), and (2) alteration of investment strategy.

At first step, the investor who obtain negative cumulative excess return changes the strategy at the following probability:$p_i = min(1, max(0.5 \cdot e^{-r^{cum}_i} - 0.5, 0))$, where p_i is probability at which investor i changes own strategy and r^{cum}_i is cumulative return of investor i during recent 5 terms.

At second step, the investors who change the strategy tend to select the strategy that has brought positive cumulative excess return[4]. The probability to select $strategy_i$ as new strategy is given as: $p_i = \frac{e^{(r^{cum}_i)}}{\sum_{j=1}^{M} e^{(r^{cum}_j)}}$, where r^{cum}_i is the cumulative excess return of each investor.

3 Experimantal Results

The traditional financial theories analyze the asset prices by considering the behavior of representative investors only. We analyze the influences of investors' behavior on the asset prices through the experiments in a virtual market that contains various types of investors. As explained in the previous section, we explicitly describe only the movement of fundamentals (the corporate profit) and the rules of investors' behavior in this virtual financial market, and the asset prices are determined bottom-up as a result of trading[5]. This analysis sets out to search for conditions by which the market value would reflect the fundamental value, after firstly undertaking a conditional search for investment strategy capable of acquiring excess return.

[3] e.g. When such investors predict that stock price will increase, they invest more in stock than ones whoes value of s_i is 1.

[4] We apply the method of genetic algorithm to the selection rule of new strategy[11]. In this analysis, r^{cum}_i corresponds to the fitness of genetic algorithm.

[5] The prices in real markets are also determined as a result of the autonomous behavior of each investor. In this context, our virtual market has the price determination mechanism that is closer to the real one.

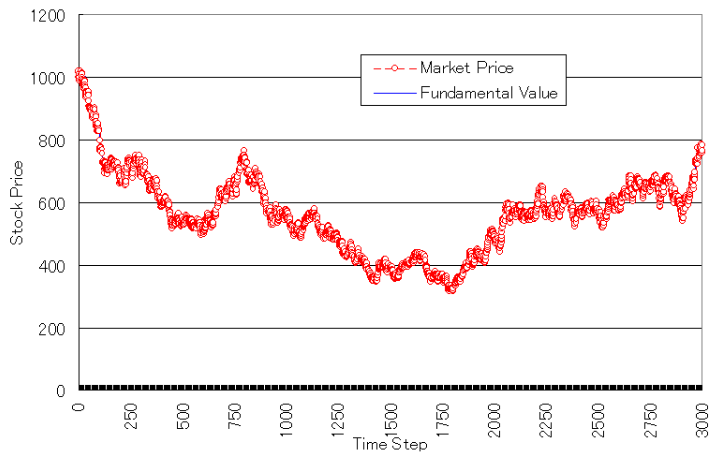

Fig. 1. Price history(Fundamentalist:Trend=500:500)

3.1 Searching for Investment Strategy

Firstly, we analyzed the market where there was a (1) high ratio of fundamental forecasting, and (2) a high ratio of trend forecasting. As the results of this analysis confirmed a strengthening degree of overconfidence in both cases, an analysis of (3) the random distribution of the initial ratio of each forecasting model was also undertaken to determine whether the same result could be obtained under different conditions. The results of this analysis are explained in detail below.

When there is a High Ratio of Fundamental Forecasting Fig. 1 shows the history of the stock price obtained as a result of experiments in the financial market. In this case, the number of Fundamentalist is 500 and the number of trend chaseer is 500. As fundamentalists enforce a strong influence on the market value under these conditions, the market value is almost in concord with the fundamental value (Fig. 1). It can be confirmed that the number of fundamentalists is on the increase due to the rules of natural selection in regard to the transition of investor numbers (Fig. 2). Looking at transition in the degree of overconfidence, a strengthening degree of overconfidence can be confirmed in the remaining investors as market transactions go forward(Fig. 3).

When there is a High Ratio of Trend Forecasting Fig. 4 shows the history of the stock price obtained as a result of experiments in the financial market. In this case, the number of Fundamentalist is 100, and the number of trend chaser is 900. When there is a high ratio of investors using trend forecasting, the market value deviated greatly from the fundamental value. It was confirmed that the number of investors using trend forecasting also increases as such investors enforce a strong influence on the market value(Fig. 5). This is thought to be the result of an investment environment in which different forecasting methods were applied to obtain excess return. On the other hand, it was confirmed that investors with a strong degree of overconfidence survive in the market even under these conditions(Fig. 6).

When the Initial Ratio is Applied Randomly A case in which the initial ratio of investors is applied randomly was analyzed next. Although the case example (Fig. 7) shown here

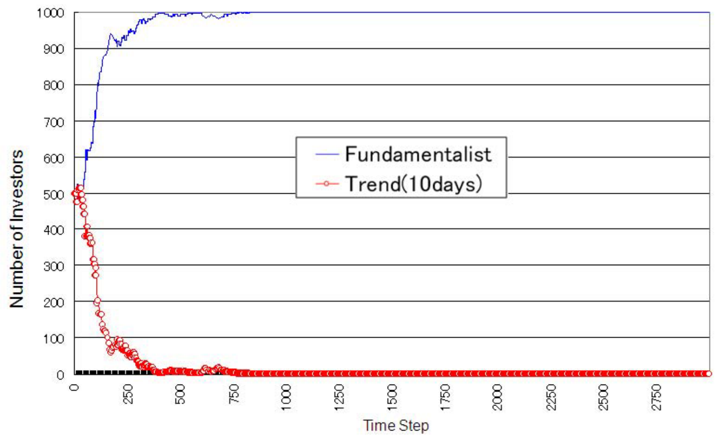

Fig. 2. History of the number of Investors(Fundamentalist:Trend=500:500)

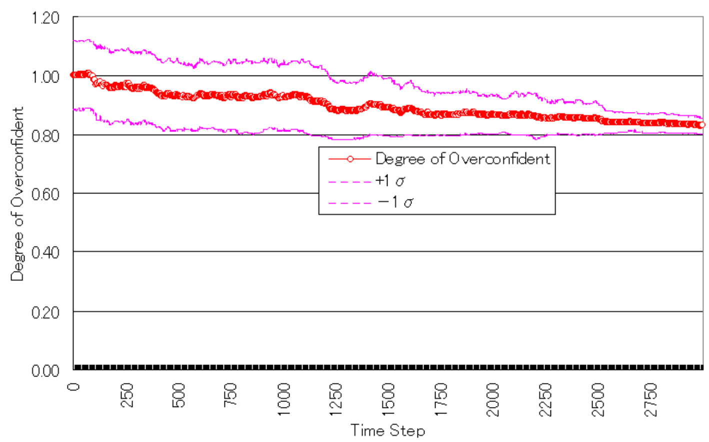

Fig. 3. History of the average degree of overconfidence (Fundamentalist:Trend=500:500)

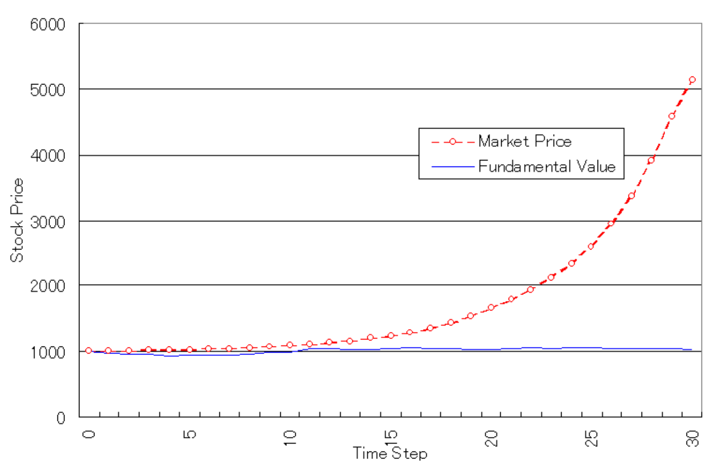

Fig. 4. Price history(Fundamentalist:Trend=100:900)

indicates that numerous investors employ investment strategy based on the fundamental value, the forecasting model employed by market investors is dependant on the ratio of each type of investor etc, changing along with circumstances such as trend forecasting and the average value of past equity prices. In contrast, it has been confirmed that overconfident investors survive in the market even when a random initial value is applied for the degree of overconfidence (Fig. 8).

In this analysis, we are also able to confirm that overconfident investors emerges from the bottom up. This interesting analysis result suggests the possibility of universality when survival trends of overconfident investors are compared with the forecasting model[6].

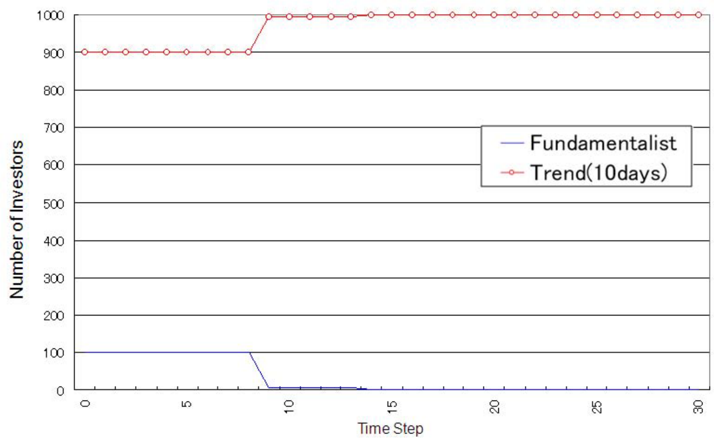

Fig. 5. History of the number of Investors(Fundamentalist:Trend=100:900)

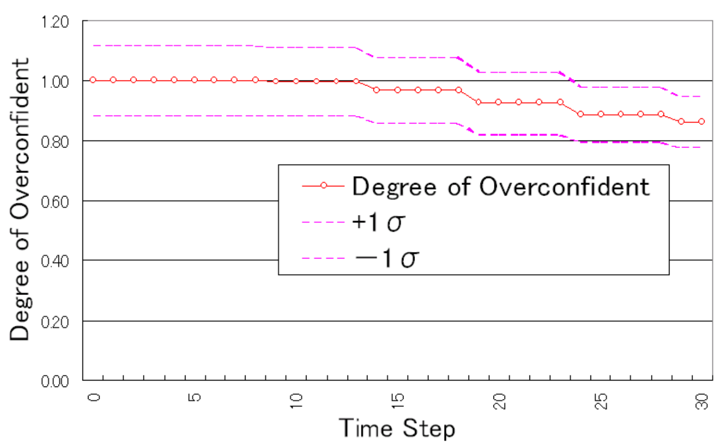

Fig. 6. History of the average degree of overconfidence (Fundamentalist:Trend=100:900)

[6] In this mean, overconfidence in this paper is modeled neither ad-hoc nor conveniently.

3.2 Exploring Market Conditions

This analysis endeavors to determine the conditions necessary for transaction prices to reach the fundamental value. In order to discuss the problem, we employed Inverse simulation analysis method.

Inverse Simulation Analysis Method Inverse Simulation Analysis consists of the following 3 steps. (1) Carry out 100 times a simulation with an investment period of 100 terms. (2) Calculate the index of deviation between transaction prices and the fundamental value for each simulation. (3) Set the calculated index as the adaptive value and select 100 simulation conditions (investors' forecasts, confidence). This analysis is undertaken through repetition of these 3 steps. The index (q) of deviation between transaction prices and the fundamental value expresses the deviation ratio with the fundamental value and is specifically calculated as $q = E[x]^2 + Var[x]$, where P_t^0 is the fundamental value for term t and $x_t = (P_t - P_t^0)/P_t^0$.

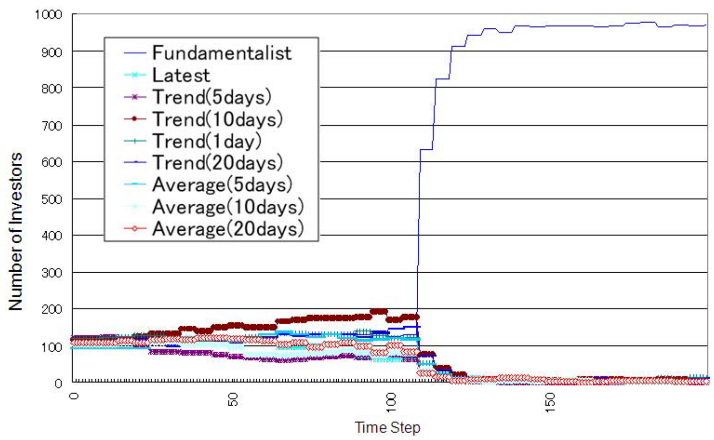

Fig. 7. History of the number of Investors(Random)

3.3 Conditional Search Results

Fig.9-11 show experimental results. It can be seen from analysis results that transaction prices tend to approach the fundamental value (Fig. 9) when there is a high percentage of fundamentalist investors (Fig. 10) coupled with a strong degree of investor confidence (Fig. 11). In addition, transaction prices almost match the fundamental value in this case.

Traditional finance argues that investors who are able to swiftly and accurately estimate both the risk and rate of return on stock survive in the market, and such investment behaviors contribute to market efficiency[7]. However, analysis results obtained here regarding the influence irrational investors have on prices suggests a different situation, pointing to the difficulty of market modeling which takes real conditions into account. These results indicate that overconfident investors have pricing power and they can contribute to efficient market when assumptions of traditional financial theory are extended to the ones closer to the reality.

[7] Efficiency of the market is one of the most important hypothesis in Financial Economics[8]. In efficienct markets, asset prices swiftly and accurately reflect fundamental values.

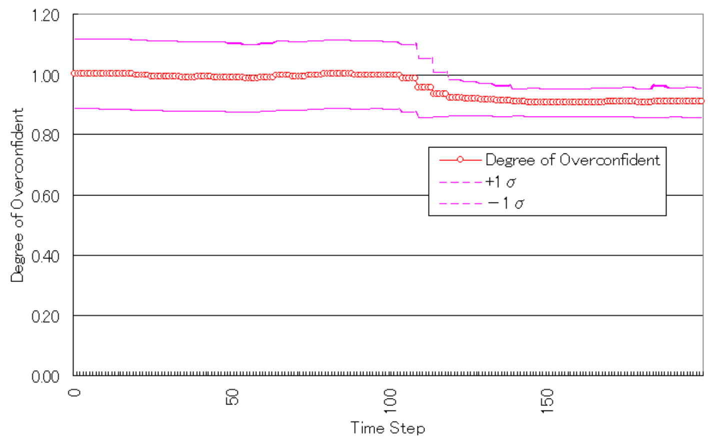

Fig. 8. History of the degree of overconfidence(Random)

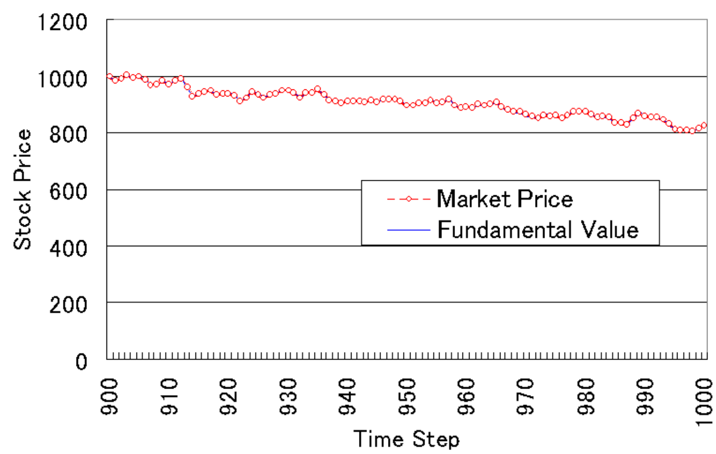

Fig. 9. Price History(Inverse Simulation)

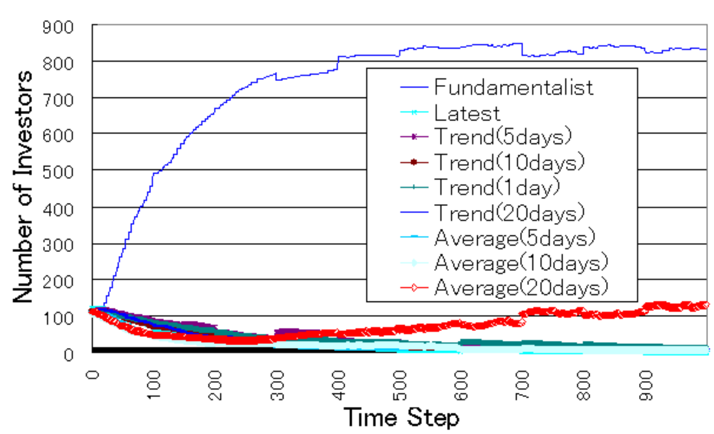

Fig. 10. History of the average number of Investors(Inverse Simulation)

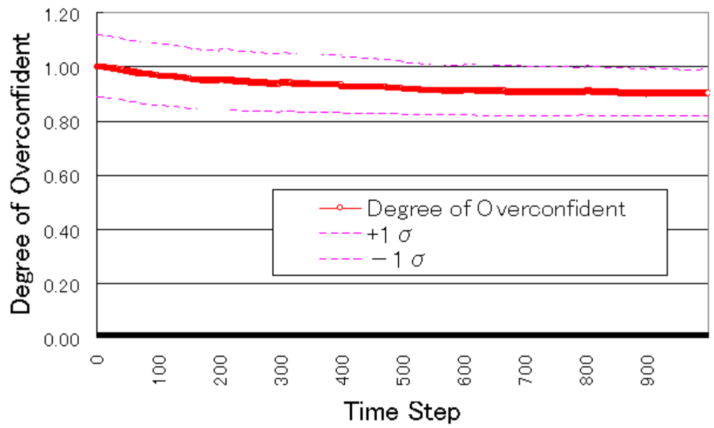

Fig. 11. History of the average degree of overconfidence(Inverse Simulation)

4 Summary

This paper utilizes the Agent-Based Modeling to analyze both microscopic and macroscopic associations in the financial market. In the process, it has been found that overconfident investors are generated in a bottom-up fashion in the market. Showing the existence of a survival mechanism as a characteristic feature of overconfidence in decision making is one of the significant achievements of this research. Furthermore, this research has also succeeded in showing that such characteristic features have the ability to contribute to a market which reflects fundamentals. Future issues include market modeling which takes more realistic conditions into account.

A Prameter List

List of the principle parameters used in this analysis.

M: Number of Investors (1000)
N: Number of shares (1000)
F_t^i: Total Assets value of investor i for Term t ($F_0^i = 2000$: common)
W_t: Benchmark equity stake for term t ($W_0 = 0.5$)
w_t^i: Equity stake of investor i for term t ($w_0^i = 0.5$: common)
y_t: Profits generated during term t ($y_0 = 0.5$)
σ_y: Standard deviation of profit fluctuation ($0.2/\sqrt{200}$)
δ: Discount rate of shares (0.1/200)
λ: Invstors' degree of risk avoidance (1.25)
r_t^{im}: Expected rate of share return as estimated from risk etc
c: adjustment coefficient (0.01)
σ_t^s: Assessed value of standard deviation of share fluctuation
σ_t^h: Historical stock volatility
P_t: Transaction prices for term t
$P_t^{f(,i)}$: Forecast value of transaction prices (of investor i) for term t
$y_t^{f(,i)}$: Forecast value of profits (of investor i) for term t

$r^{f(,i)}$: Short term expected rate of return on shares (of investor i)
σ_n: Standard deviation of data dispersion for short term expected rate of return on shares (0.01)
a_t: Price trend on stock until term t
r_i^{cum}: Cumulative excess return of investor i for most recent 5 term
p_i: Probability that investors' who alter their strategy will adopt investor i's strategy
s_i: Coefficient to express degree of confidence (uniform random number of 0.8-1.2)
a: Coefficient that expresses the degree of investment strategy selectivity (20)

References

1. Arthur, W. B., Holland, J.H., Lebaron, B., .Palmer, R.G. and Taylor, P.: Asset Pricing under Endogenous Expectations in an Artificial Stock Market, *The Economy as an Evolving Complex System II*, Addison-Wesley, pp.15-44 (1997).
2. Axelrod, R.: *The Complexity of Cooperation -Agent-Based Model of Competition and Collaboration*,Princeton Uniersity Press (1997)
3. Axtell, R.: Why Agents? On the Varied Motivation For Agent Computing In the Social Sciences, *the Brookings Institution Center on Social and Economic Dynamics Working Paper*, November, No.17 (2000).
4. Barber, B. and Odean, T.: Trading is Hazardous to Your Wealth: The Common Stock Investment Performance of Individual Investors, *Journal of Finance*, 55, pp.773-806 (2000).
5. Bazerman, M.: *Judgment in Managerial Decision Making*,John Wiley & Sons (1998).
6. Black, F. and Litterman, R.: Global Portfolio Optimization, *Financial Analysts Journal*, September-October, pp.28-43 (1992).
7. Epstein, J. M. and Axtell, R.(1996),*Growing Artificial Societies Social Science From The Bottom Up*,MIT Press.
8. Fama, E.: Efficient Capital Markets: A Review of Theory and Empirical Work, *Journal of Finance*, 25, pp.383-417 (1970).
9. Fuller, R. and J. Farrel, Jr.: *Modern Investment and Security Analysis*, McGraw-Hill (1987).
10. Gervais, S. and Odean, T.: Learning to be overconfident, *Review of Financial Studies*, vol.14, 1, pp.1-27 (2001).
11. Goldberg, D.: *Genetic Algorithms in Search, Optimization,and Machine Learning*, Addison-Wesley (1989).
12. Hirshleifer, D.:Investor Psychology and Asset Pricing, *Journal of Finance* 56,pp.1533-1597 (2001).
13. Kahneman, D. and Tversky, A.: Prospect Theory of Decisions under Risk, *Econometrica*, 47, pp.263-291 (1979).
14. Kahneman, D., Slovic, P. and Tversky, A., Judgment under uncertainty: Heuristics and biases, *Cambridge University Press*,(1982).
15. Kahneman, D. and Tversky, A.: Advances in prospect Theory : Cumulative representation of Uncertainty, *Journal of Risk and Uncertainty*, 5 (1992).
16. Kyle, A.S. and Wang, A.: Speculation Duopoly with Agreement to Disagree: Can Overconfidence Survive the Market Test?, *Journal of Finance*, 52, pp.2073-2090 (1997).
17. Levy, M., Levy, H. and Salomon, S.: *Microscopic Simulation of Financial Markets*, Academic Press (2000).
18. O'Brien, P.: Analysts' Forecasts as Earnings Expectations, *Journal of Accounting and Economics*, January, pp.53-83 (1988).
19. Sharpe, W.F.: Integrated Asset Allocation, *Financial Analysts Journal*,September-October (1987).
20. Shiller, R.J.: *Irrational Exuberance*, Princeton University Press [2000]
21. Shleifer, A.: *Inefficient Markets*,Oxford University Press[2000].
22. Stein, J.C.: Agency, Information and Corporate Investment, *NBER Working Paper*, No.8342 [2001]
23. Takahashi, H. and Terano, T.: Agent-Based Approach to Investors' Behavior and Asset Price Fluctuation in Financial Markets, *Journal of Artificial Societies and Social Simulation*, 6, 3 (2003).
24. Tesfatsion, L.: Agent-Based Computational Economics, *Economics Working Paper*, No.1, Iowa Sate University (2002).