Reproducing Kernel Hilbert Space Methods to Reduce Pulse Compression Sidelobes

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Abstract. Since the development of pulse compression in the mid-1950's the concept has become an indispensable feature of modern radar systems. A matched filter is used on reception to maximize the signal to noise ratio of the received signal. The actual waveforms that are transmitted are chosen to have an autocorrelation function with a narrow peak at zero time shift and the other values, referred to as sidelobes, as low as possible at all other times. A new approach to radar pulse compression is introduced, namely the Reproducing Kernel Hilbert Space (RKHS) method. This method reduces sidelobe levels significantly. The paper compares a second degree polynomial kernel RKHS method to a least squares and L_{2P}-norm mismatched filter, and concludes with a presentation of the representative testing results.

1 Introduction

Since the development of pulse compression in the mid-1950's [1,2] the concept has become an indispensable feature of modern radar systems. Pulse compression gives radar designers the ability to obtain sufficient energy from a target for detection without degrading the range resolution of the system or resorting to the use of very high transmitter power levels. Pulse compression therefore allows for the use of lower power transmitters but with longer pulse lengths to increase the energy content of a pulse. A matched filter is used on reception to maximize the signal to noise ratio (SNR) of the received signal. The actual waveforms that are transmitted are chosen to have an autocorrelation function (ACF) with a narrow peak at zero time shift and sidelobe levels as low as possible at all other times. The sidelobes have the undesirable effect of masking smaller targets which are in close proximity to large targets, such as clutter returns. It is therefore desirable to have a main pulse lobe as narrow as possible.

This paper introduces the Reproducing Kernel Hilbert Space (RKHS) methods in radar pulse compression. It is shown that the RKHS methods reduce the sidelobe levels significantly compared to the results in [3].

In the section 2 we discuss the matched filter used in radar pulse compression and the least squares technique to solve the mismatched filter coefficients. Section 3 discusses the RKHS methods to be used in pulse compression. Results obtained in testing the algorithms are presented in section 4. The paper ends with a conclusion.

2 Problem Formulation

2.1 Matched Filter

In the discrete time domain the transmit pulse of a radar could be represented by a sequence of complex transmit coefficients $\{a_n\}$. Digital pulse compression is performed by the convolution of the received signal, which is assumed to be a time delayed and scaled version of the transmitted pulse, with the complex receive filter coefficients $\{z_n\}$. For the purpose of analyzing the sidelobe response of the pulse compressor, a zero time-delay and unity scaling factor can be assumed without loss of generality. In this paper a P4 code [4] (i.e. sampled linear frequency chirp) is used for the transmit coefficients.

For the transmit pulse $\{a_n\}$ the matched filter is given by $h_n = \overset{*}{a}_{N-n}$ where * denotes the complex conjugate and N is the number of transmit pulse samples. If the matched filter is used, the output of the pulse compressor will be the ACF of $\{a_n\}$ which is equivalent to the discrete convolution

$$b_i = \sum_i a_{i+1-k} h_k. \tag{1}$$

The convolution sequence $\{b_i\}$ for the matched filter has the maximum attainable SNR at zero time shift.

The sidelobe values of the convolution result can be minimized by introducing cost functions which map the set of sidelobes to a single real value. By minimizing the cost functions, mismatched receive filters with reduced sidelobe responses can be found. In the next section the mismatched filter solutions for the least-squares sidelobe measure, which is equivalent to the L_2-norm solution, and the generalized L_{2P}-norm are briefly discussed. In [3] the L_2- and L_{2P}-norms are presented as methods to reduce the sidelobes of the pulse compressor output. These methods will be compared to the RKHS pulse compressor.

2.2 Least Squares and L_{2P} Sidelobe Minimisation

In matrix form the output of the pulse compressor could be written as [3]

$$\mathbf{b} = \mathbf{A}_F \mathbf{z}, \tag{2}$$

where

$$\mathbf{b} = [b_1, b_2, \cdots, b_{2N-1}]^T, \tag{3}$$

$$\mathbf{z} = [z_1, z_2, \cdots, z_N]^T \tag{4}$$

and

$$\mathbf{A}_F = \begin{bmatrix} a_1 & a_2 & \cdots & a_N & 0 & \cdots & 0 \\ 0 & a_1 & a_2 & \cdots & a_N & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & a_1 & a_2 & \cdots & a_N \end{bmatrix}^T, \tag{5}$$

where ^{T} denotes the transpose of a vector or matrix and that $\mathbf{A}_F$ is the full convolution matrix.

The sidelobe measure function for a compressed pulse can now be formulated by defining a new matrix, $\mathbf{A}$, which is similar to $\mathbf{A}_F$, except that the row in $\mathbf{A}_F$ which produces the compression peak is removed. The sidelobe measure cost function to be minimised can therefore be written as

$$\begin{aligned} f(\mathbf{z}) &= \mathbf{b}^H \mathbf{b} \\ &= \mathbf{z}^H \mathbf{A}^H \mathbf{A} \mathbf{z} \\ &= \mathbf{z}^H \mathbf{C} \mathbf{z}, \end{aligned} \tag{6}$$

with

$$\mathbf{C} = \mathbf{A}^H \mathbf{A}, \tag{7}$$

and ^{H} denotes the complex conjugate transpose. The row in $\mathbf{A}_F$ that is removed could now be written as a constraint

$$\mathbf{a}\mathbf{z} = b_{peak}, \tag{8}$$

where $\mathbf{a} = [a_1, a_2, \cdots, a_N]$. This optimisation problem could now be solved using Lagrange multipliers.

The generalised L_{2P}-norm sidelobe cost function could now in a similar way be derived as [3]

$$\begin{aligned} f(\mathbf{z}) &= \left(\sum_{i=1}^{2N-1} \left[\|b_i\|^2 \right]^P \right)^{\frac{1}{2P}} \\ &= \left(\sum_{i=1}^{2N-1} \left[\mathbf{z}^H C_i \mathbf{z} \right]^P \right)^{\frac{1}{2P}}, \end{aligned} \tag{9}$$

where

$$C_i = \mathbf{a}_i^H \mathbf{a}_i, \tag{10}$$

and $\mathbf{a}_i$ is the ith row of $\mathbf{A}$. Using the same constraint as in (8), Lagrange multipliers could be used to solve the minimisation problem.

3 RKHS-based Filter

The idea of a function space reproduced by a single kernel function as well as the question of whether or not there exists a kernel which will reproduce a specific function space has received attention since the beginning of the 20th century, and even before. Aronszajn [5], however, was the first to formally define the notion of a Reproducing Kernel Hilbert space during the decade 1940 to 1950.

Today the applications of the theory of reproducing kernels are widely spread in mathematical statistics and engineering applications. In the 1960's (refer to [6],[7] and [8]) Parzen applied the theory of Reproducing Kernel Hilbert spaces to time series analysis. In the early 1970's Kailath ([9],[10],[11]) and his coworkers applied this theory to problems encountered in detection and estimation. More recently, the theory of reproducing kernel Hilbert spaces has found applications in generalised sampling theory, in wavelets and in graph matching (see [12],[13] and [14] as well as references therein).

In its simplest form a RKHS is a Hilbert space H equipped with an inner product $(\cdot,\cdot)$ and a kernel $K(\cdot,\cdot) : \mathrm{R}\times\mathrm{R} \to \mathrm{R}$ such that $K(t,\cdot) \in H$ for all $t \in \mathrm{R}$ and which has the reproducing property, i.e.

$$(F(\cdot), K(t,\cdot)) = F(t)$$

for all $t \in \mathrm{R}$. A consequence of the reproducing property is that $(K(s,\cdot), K(t,\cdot)) = K(s,t)$.

Suppose now we are given a data set of input-output training patterns $\mathcal{T} = \{t_i, f_i\}_{i=1}^N$ where $f_i = F(t_i) + \epsilon_i$ are noisy measurements of some unknown function $F(\cdot) : \mathrm{R} \to \mathrm{R}$. The following approximation problem is of interest: given $\mathcal{T}$ find the minimum norm approximation $\tilde{F}(\cdot)$ of $F(\cdot)$ in the RKHS H subject to the constraints $(\tilde{F}(\cdot), K(t_i,\cdot)) = f_i$. It can be shown that $\tilde{F}(\cdot)$ is of the form [15]

$$\tilde{F}(\cdot) = \sum_{i=1}^{N_c} c_i \, K(\tilde{t}_i, \cdot), \tag{11}$$

where usually $N_c \leq N$ due to the presence of noise and the kernel centres $\tilde{t}_i$ are inferred from $\mathcal{T}$ by means of some data reduction scheme [14]. The solution of this approximation problem is then obtained as

$$\mathbf{c} = \mathbf{G}^{\dagger}\mathbf{f} \tag{12}$$

where $\mathbf{c} = (a_i)$, $\mathbf{f} = (f_i)$ and $\mathbf{G} = (K(\tilde{t}_i, t_j))$. Here $\mathbf{G}^{\dagger}$ denotes the pseudo inverse of the matrix $\mathbf{G}$.

For the application discussed here we have chosen the polynomial kernel [16] namely

$$K(\mathbf{s},\mathbf{t}) = \left(1 + \mathbf{s}^T\mathbf{t}\right)^d, \tag{13}$$

where d is the degree of the polynomial. One could also use other RKHS kernels, for example the Dirichlet kernel (which is periodic) [14]

$$K(\mathbf{s},\mathbf{t}) = \frac{\sin\left(\left(n+\frac{1}{2}\right) 2\pi\,(\mathbf{s}-\mathbf{t})\,u\right)}{\sin\left(\frac{2\pi(\mathbf{s}-\mathbf{t})u}{2}\right)}, \tag{14}$$

or the *sinc* kernel

$$K(\mathbf{s},\mathbf{t}) = \frac{\sin\left(2\pi\,(\mathbf{s}-\mathbf{t})\,u\right)}{2\pi\,(\mathbf{s}-\mathbf{t})\,u}, \tag{15}$$

where n is the harmonic number and u is the width (or dilation) parameter of the kernel. However, for this application the polynomial kernel gave much better results, and therefore only the polynomial kernel will be considered when presenting the results.

Once the interpolator coefficients $\mathbf{c}$ are solved, we can define a template [16]

$$\tilde{\mathbf{t}} = \sum_{i=1}^{N_c} c_i \, \bar{\mathbf{t}_i}, \tag{16}$$

where

$$\bar{\mathbf{t}_i} = \left[\begin{bmatrix} 1 & \mathbf{t}_i^T \end{bmatrix} \otimes \begin{bmatrix} 1 & \mathbf{t}_i^T \end{bmatrix} \otimes \cdots \otimes \begin{bmatrix} 1 & \mathbf{t}_i^T \end{bmatrix} \right]^T, \tag{17}$$

$\otimes$ denotes the Kronecker Tensor Product, $\mathbf{t}_i$ is an input vector, and the term $\begin{bmatrix} 1 & \mathbf{t}_i^T \end{bmatrix}$ in (17) is repeated d times. For example if a second order polynomial kernel is used, then

$$\bar{\mathbf{t}_i} = \left[\begin{bmatrix} 1 & \mathbf{t}_i^T \end{bmatrix} \otimes \begin{bmatrix} 1 & \mathbf{t}_i^T \end{bmatrix} \right]^T. \tag{18}$$

Once the RKHS pulse compressor is trained, the template $\tilde{\mathbf{t}}$ is used to calculate the ith output of the RKHS pulse compressor as $\tilde{\mathbf{t}}^T \bar{\mathbf{t}_i}$.

4 Numerical Results

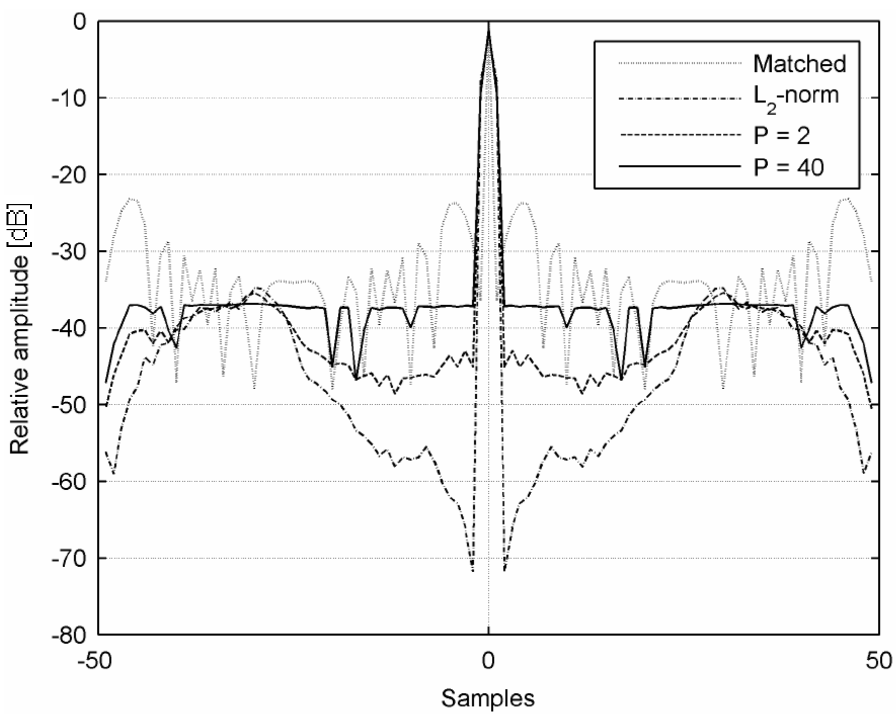

Fig. 1. Matched filter, L_2- and L_{2P}- norm pulse compression response for a linear chirp transmit pulse with a TBWP of 50 (Borrowed from [3])

For this experiment we used a second order RKHS polynomial kernel. The MATLAB programming environment [17] was used for implementation. A linear chirp pulse

$$y(t) = Ae^{j2\pi ft}, \tag{19}$$

where A is the amplitude, f is frequency and t is time, is used to simulate the signal that should be transmitted by the radar. The generated signal had a time-bandwidth product (TBWP) of 50.

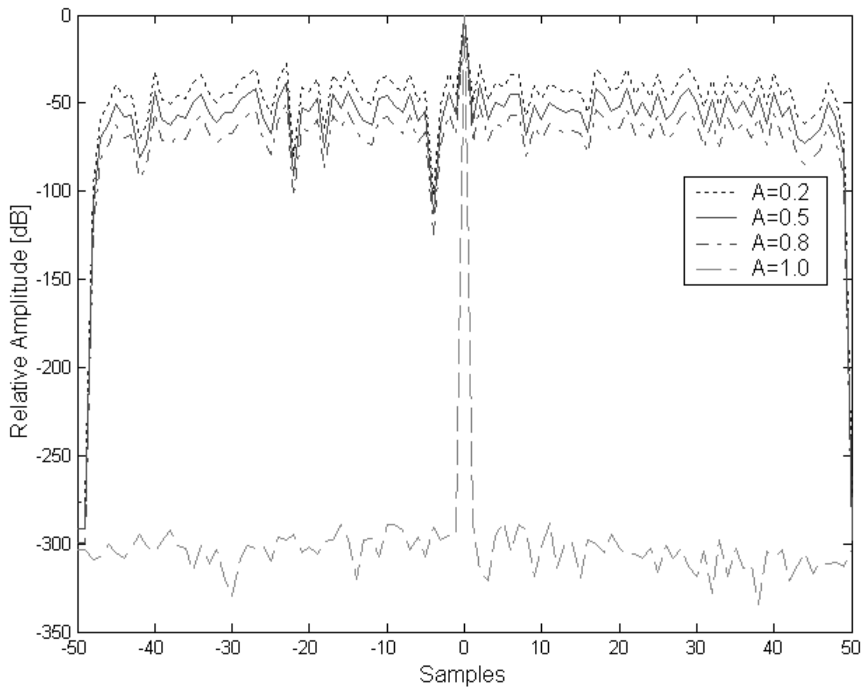

Fig. 2. RKHS pulse compressor output as the input chirp signal amplitude varies

The results from [3] are shown in Fig. 1. The matched filter response, mismatched least squares and two L_{2P}-norm filter responses ($P = 2$, $P = 40$) are shown. This figure shows only the output of the compressor for the input signal that was used determine ("train") the filter coefficients. The best sidelobe levels achieved were around -35dB. The value of the output in dB is calculated as

$$\text{Output}_{dB} = 20 \log_{10} (\|\text{Output}\|) . \tag{20}$$

Fig. 2 to Fig. 4 show the results for the proposed RKHS pulse compressor. A second degree polynomial kernel was used for this RKHS method, and a chirp pulse with amplitude $A = 1$ was used to train the system. Fig. 2 shows the output of the RKHS pulse compressor for four different values of the input chirp pulse, namely $A = 0.2$, $A = 0.5$, $A = 0.8$ and $A = 1$. Each output signal was scaled by its maximum value and then converted to the dB scale. This forces the maximum value (in dB) for each output equal to 0dB, which enables us to see the different outputs of the different input amplitudes in perspective of the output amplitude of the training chirp pulse. For the training signal with amplitude $A = 1$ it is clear that the RKHS pulse compressor performs very well compared to the least squares method, and as the amplitude deviates from 1, the sidelobe levels start to increase.

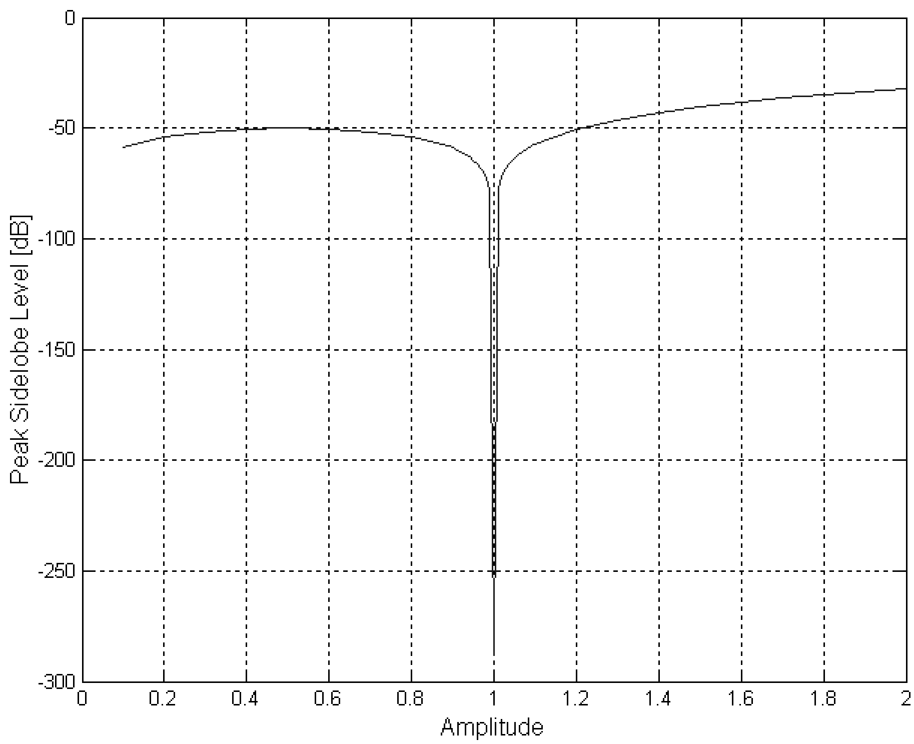

Fig. 3. RKHS maximum sidelobe levels as the input chirp signal amplitude varies

Fig. 3 shows a graph of the unscaled maximum sidelobe levels as the chirp pulse amplitude varies from $A = 0.1$ to $A = 2$. For a chirp signal with the training amplitude $A = 1$, the RKHS pulse compressor achieved sidelobe levels of maximum -287dB, which is far better than the -35dB of the pulse compressors shown in Fig. 1. Then as the input amplitude decreases or increases from that of the training pulse, the sidelobe levels varied from -60dB to -30dB. This is still better (or comparable) to the results from the L_{2P}-norm and least squares methods in Fig. 1, which only shows results for the training signal and not for

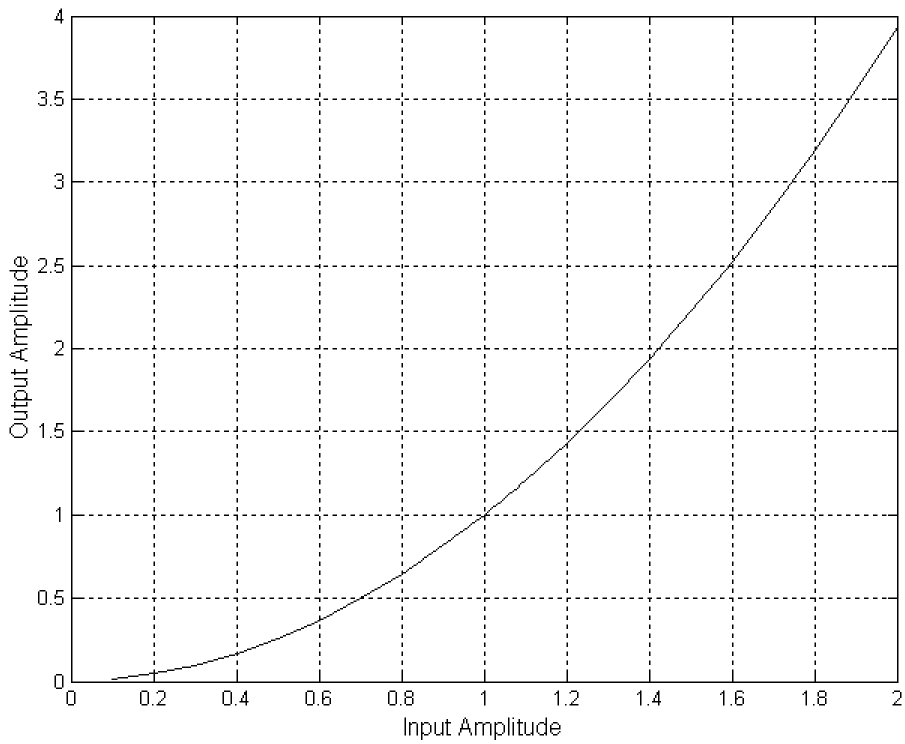

Fig. 4. RKHS pulse compressor output when a pulse is detected as the input chirp signal amplitude varies

input signals different than that of the training set. Fig. 4 shows the output peak amplitude (when a pulse was actually detected) in terms of the input amplitude. Since a second degree polynomial kernel was used, the output has a quadratic relation to the input amplitude.

5 Conclusion

In this paper we have presented a RKHS method to be applied in radar pulse compression. We compared our proposed method to the least squares and L_{2P}-norms for minimising pulse compression sidelobes. The RKHS method has superior performance over the other methods and showed significant sidelobe reduction of between −30dB and −287dB.

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