Synchronous Interface Theories and Time Triggered Scheduling

Benoît Delahaye^{1*}, Uli Fahrenberg², Thomas A. Henzinger³, Axel Legay²¹, and Dejan Ničković^{4**}

Aalborg University, Denmark
 ² Irisa/INRIA Rennes, France
 ³ IST Austria, Klosterneuburg, Austria
 ⁴ Austrian Institute of Technology, Vienna, Austria

Abstract. We propose synchronous interfaces, a new interface theory for discrete-time systems. We use an application to time-triggered scheduling to drive the design choices for our formalism; in particular, additionally to deriving useful mathematical properties, we focus on providing a syntax which is adapted to natural high-level system modeling. As a result, we develop an interface model that relies on a guardedcommand based language and is equipped with shared variables and explicit discrete-time clocks. We define all standard interface operations: compatibility checking, composition, refinement, and shared refinement. Apart from the synchronous interface model, the contribution of this paper is the establishment of a formal relation between interface theories and real-time scheduling, where we demonstrate a fully automatic framework for the incremental computation of time-triggered schedules.

1 Introduction

Interface models and theories were developed with the aim to provide a theoretical foundation for compositional design. Interface models describe both input assumptions on a component and its output guarantees and they support *incremental design* and *independent implementability*, two central concepts in component-based design. Interface theories [1, 7, 13, 16, 18, 21, 23, 30] and related approaches [9, 29] have been subject of active research in the past years, and today provide a strong and stable foundation for component-based design. However, although the theoretical foundations of interface theories can be now considered to be quite solid, the practical applicability of the framework has remained rather limited. One of the reasons is the fact that little attention has

^{*} Research partially supported by the Danish-Chinese Center for Cyber Physical Systems (Grant No.61061130541) and VKR Center of Excellence MT-LAB.

^{**} The research leading to these results has received funding from ARTEMIS Joint Undertaking under grant agreement number 269335 and from national fundings (Federal Ministry for Transport, Innovation and Technology and Austrian Research Promotion Agency).

been given to adapt the modeling languages to the actual engineering needs in the target application domains.

In this paper, we propose synchronous interfaces (SI), a new interface theory motivated by an application to time-triggered scheduling and thus providing features that make our model closer to real-life needs of the engineers. In time-triggered communication scheduling, one allocates message transmissions to shared communication channels in a way that respects application-imposed and real-time constraints. The time-triggered scheduling problem can be naturally specified within an interface theory framework, by modeling scheduling constraints as interface guarantees, and considering the environment to be the scheduler. Even though our model has been developed with an eye to timetriggered scheduling, the application domain of the theory is much broader.

Incorrect scheduling of communication messages leads to violation of realtime and contention-freedom constraints, thus resulting in *timing incompatibilities*. This is in contrast to the standard interface theories that are untimed and are focused on reasoning about *value incompatibilities*. Continuous-time extensions of interface theories [15, 20] were developed to tackle this problem. While those are of clear interest and can solve interesting problems [14], they suffer from the complexity of handling continuous time in an explicit manner that is often unnecessary in practical application areas. We believe that discrete time provides the right level of abstraction for many application areas, and demonstrate it with the time-triggered scheduling application.

We base the syntax of SI on the model of *reactive modules*, a high-level and general-purpose modeling language that provides a syntax close to procedural guarded-command languages. In addition, we extend our model with *shared variables* that allow simple specification of contention freedom constraints, and explicit discrete-time *clocks* that facilitate modeling timing constraints.

Semantically, a SI is a set of concurrent processes whose behavior is evolving in discrete time. We equip our theory with operations that support incremental design and independent implementability: (1) *well-formedness check* that computes the set of environment choices for which the interface meets its guarantees; (2) *composition* that allows to combine two interfaces and compute the assumptions under which they interact in a compatible way and (3) *refinement* and *shared refinement* that are used to compare behaviors of different interfaces.

The second contribution of this paper is the *incremental* computation of timetriggered schedules, that is resolved as an incremental design problem with SI. We model scheduling constraints as SI guarantees and consider the environment to be the (unknown) scheduler. We apply well-formedness checking to restrict the environment to those schedules that satisfy the scheduling constraints. The composition operator allows to solve scheduling problems incrementally, by decomposing them into subproblems whose restricted environments are combined into a full schedule (using the well-formedness check again).

Related Work. Compositional scheduling for hierarchical real-time systems has been extensively studied in [22, 32] and other papers by the same authors, but in a setting which in a sense is complementary to ours. The focus of that work is on computing bounds on resource use under some (simple) schedulers, and on inferring resource bounds for complex systems in a compositional manner, whereas we focus on schedulability, i.e., computation of schedules under given task dependencies and resource bounds. Incremental time-triggered scheduling was also studied in [33], using an approach that computes schedules with an SMT solver, but may miss a feasible schedule.

Another area of related work, similar in spirit but different in methods, is the recent application of timed-automata based formalisms to schedulability problems. In [2], simple job-shop scheduling problems are solved using timed automata, and in [11, 31], priced timed automata and games are used for schedulability under resource constraints. Another work in this area is [24], which is using timed automata extended with tasks for solving scheduling problems under uncertainty. Other approaches to solve worst-case scheduling problems are reported in [12, 28, 34]. A synchronous relational interface theory was proposed in [35], but without the notion of shared variables. Interface theories with shared variables were also proposed in [13] and [16, 17]. However, unlike in SI, the information about ownership of the shared variables by individual components within a composed system is not preserved, thus not making them suitable to express time-triggered scheduling problems.

Other examples of component-design based methodologies include the BIP toolset [5, 6] and its timed extension [3]. However, while BIP proposes features that are definitively beyond the scope of our work (generation of code, compilers, invariant-based verification), the approach does not permit to reason easily on shared variables, and does not provide (shared) refinement or pruning operators. Observe that several BIP-based approaches [8, 25] capable of restraining the behaviors of a distributed system by avoiding deadlocks have the potential to solve scheduling problems. However, a detailed study of (incremental) scheduling problems has not been considered in the mentioned papers, hence it is not clear whether TTEthernet scheduling would easily translate into the BIP framework.

2 Synchronous Interfaces

A synchronous interface comes equipped with a finite set X of typed variables which is partitioned into sets $X = extX \cup ctrX \cup sharedX$ of external, controlled, and shared variables. External variables, also called *input* variables in interface theories [19], are controlled by the environment. At each round, the environment sets the values of external variables; the interface can read, but not modify them. Controlled, or *output* variables, are controlled by the interface: in each round, the interface assigns new values to all controlled variables. We further partition $ctrX = intfX \cup privX$ into *interface* and *private* variables. Interface variables can be seen by other SI, while private variables are local; hence private variables do not influence the communication behavior of a SI, and we can safely ignore them. We let $obsX = X \setminus privX$ denote the set of observable variables. We use *unprimed* symbols, such as x, to denote a *latched* value, and *primed* symbols, such as x', to denote an *updated* value of the variable x. We naturally extend this notation to sets of variables. The function type(x) returns the type of variable x. In particular, clock variables have the type \mathbb{C} .

We follow the approach in [13] and introduce shared variables in the model, to facilitate communication using shared resources. We let the environment ensure the mutual exclusion property. Contrary to [13], we keep additional information on which individual component in the system owns the shared variable at each step of computation. In every computation step, the environment gives write access to a shared variable to at most one interface active in the system. We will define interface semantics following a game-oriented approach, hence this assumption is not a restriction.

Definition 1. A guarded command γ from variables X to Y consists of a guard p_{γ} and an action Act_{γ} . The guard p_{γ} is a predicate over X, and Act_{γ} is either a discrete action: an expression α_{γ} from X to Y, or a wait action, using the keyword wait.

We use $\gamma[p_{\gamma} \setminus p'_{\gamma}]$ for the operation that consists in replacing the predicate p_{γ} by the predicate p'_{γ} . Controlled and shared variables are collected into *atoms*, which additionally contain guarded commands which specify rules for initializing and updating variables.

In interface theories, non-determinism reflects the fact that, given all the available information at a given step of the execution of an interface, several behaviors are possible for its next step. We let the environment resolve non-deterministic choices; this is implemented by assuming that for each $x \in sharedX$, there exists a non-empty set $isCtr_x = \{isCtr_x^A\}$ of external variables, one for each atom A potentially controlling x. A variable $isCtr_x^A$ indicates whether atom A can safely write x at a given step of computation.

Definition 2. An X-atom A consists of a declaration and a body of guarded commands. We distinguish between atoms defined on controlled variables ctr(A) and those defined on shared variables shared(A).

- The atom declaration for ctr(A) consists of sets ctrX_A \subseteq ctrX, readX_A \subseteq X, and waitX_A \subseteq X\ctrX_A of controlled, read, and awaited variables. The atom body for ctr(A) consists of a set Init(A) of initial discrete guarded commands from waitX'_A to ctrX'_A and a set Update(A) of update guarded commands from readX_A \cup waitX'_A to ctrX'_A.
- The atom declaration for shared(A) consists of sets shared $X_A \subseteq$ sharedX, read $X_A \subseteq X$, and wait $X_A \subseteq X \setminus \text{shared} X_A$ of shared, read, and awaited variables, with $isCtr_x^A \in waitX_A$ for all $x \in \text{shared} X_A$. The atom body for shared(A) consists of a set Init(A) of initial discrete guarded commands from wait X'_A to shared X'_A and a set Update(A) of update guarded commands from read $X_A \cup waitX'_A$ to shared X'_A .

We denote by $P_{Init(A)} = \{p_{\gamma} \mid \gamma \in Init(A)\}$ and $P_{Update(A)} = \{p_{\gamma} \mid \gamma \in Update(A)\}$ the sets of predicates declared in initial and in update guarded commands of A. We say that a variable y awaits x, denoted $y \succ_A x$, if $y \in ctrX_A \cup sharedX_A$ and $x \in waitX_A$.

```
1 module M_{ex}
   2 external r : \mathbb{B}, b : \mathbb{N}, c : \mathbb{N}
                          isCtr_x^b : \mathbb{B}, isCtr_x^c : \mathbb{B};
   3
                                                                                                                        12 atom c reads c, x awaits r, isCtr_x^c
   4 shared x : \mathbb{N};
                                                                                                                                     init
                                                                                                                       13
                                                                                                                                           [] \neg isCtr_r^{c'} \rightarrow;
                                                                                                                       14
   5 atom b reads b, x awaits r, isCtr_x^b
                                                                                                                                       update
                                                                                                                        15
                                                                                                                                            \begin{array}{l} [] isCtr_x^{c'} \land \neg r' \to x' := x + c; \\ [] isCtr_x^{c'} \land r' \to x' := 0; \\ [] \neg isCtr_x^{c'} \to ; \end{array}
   6
              init
                                                                                                                        16
                    [] \neg isCtr_x^{b'} \rightarrow;
   7
                                                                                                                       17
               update
   8
                    \begin{array}{l} \begin{array}{l} \begin{array}{l} \begin{array}{l} \operatorname{isCtr}_{x}^{b'} \wedge \neg r' \rightarrow x' := x + b; \\ \end{array} \\ \begin{array}{l} \operatorname{isCtr}_{x}^{b'} \wedge r' \rightarrow x' := 0; \\ \end{array} \\ \begin{array}{l} \begin{array}{l} \neg \operatorname{isCtr}_{x}^{b'} \rightarrow ; \end{array} \end{array}
                                                                                                                        18
   9
 10
11
```

Fig. 1: An example of a SI

Definition 3. A synchronous interface (SI) M consists of a declaration X_M and a body \mathcal{A}_M , where X_M is a finite set of variables, and $\mathcal{A}_M = ctr(\mathcal{A}_M) \cup$ shared (\mathcal{A}_M) is a finite set of X_M -atoms for which $\bigcup_{A \in ctr(\mathcal{A}_M)} ctrX_A = ctrX_M$ and $\bigcup_{A \in shared(\mathcal{A}_M)} sharedX_A = sharedX_M$, $ctrX_{A_1} \cap ctrX_{A_2} = \emptyset$ for all atoms $A_1, A_2 \in ctr(\mathcal{A}_M)$ with $A_1 \neq A_2$, and such that the transitive closure $\succ_M = (\bigcup_{A \in \mathcal{A}_M} \succ_A)^+$ is asymmetric.

These conditions ensure that the atoms in M control exactly the variables in $ctrX_M \cup sharedX_M$, that each variable in $ctrX_M$ is controlled by exactly one atom in \mathcal{A}_M , and that the await dependencies between variables in \mathcal{A}_M are acyclic. A linear order A_1, \ldots, A_n of the atoms in \mathcal{A}_M is consistent if for all $1 \leq i < j \leq n$, the awaited variables in A_i are disjoint from the control variables in A_j . The asymmetry of \succ_M guarantees the existence of a consistent order of atoms in \mathcal{A}_M . We denote by $P_M = \bigcup_{A \in \mathcal{A}_M} P_{Init(A)} \cup \bigcup_{A \in \mathcal{A}_M} P_{Update(A)}$ the set of all predicates declared in the guarded commands of M. Remark that in our examples, we name atoms by the set of variables they control. This is only possible when all atoms have disjoint sets of controlled variables.

Example 1. Consider the SI M_{ex} given in Figure 1. M_{ex} consists of two external integer variables b, c, three external Boolean variables $r, isCtr_x^b, isCtr_x^c$, and a shared integer variable x. Intuitively, M_{ex} models a simple additive controller that works as follows. The shared variable x is either incremented or reset at each time step in which the module controls x. M_{ex} controls x whenever $(isCtr_x^b \lor isCtr_x^c = t)$. In this case, if r = t, then x is reset. Else, if atom b (resp. c) controls x ($isCtr_x^b = t$), then x is incremented by the value of b (resp. c). If none of these atoms assign a value to the variable, then the environment will do.

3 Semantics

The intuition about the semantics of a SI is as follows: in each round, the environment assigns arbitrary values of correct type to external variables. Then, the atoms are executed in a (abitrary) static consistent order. As we do not assume that modules are input-enabled, there may be valuations of the external variables for which one or several atoms cannot be executed. Such configurations result in *deadlock states*. A given valuation of the variables is *reachable* if there exists a succession of rounds of the atom ending in this valuation.

Formally, the semantics of a SI is given by a labeled transition system (LTS). Given a SI M with set of variables X_M , we denote by $V[X_M]$ the set of valuations on variables in X_M . A state s of an interface M is a valuation in $V[X_M]$. We write $\Sigma_M = \Sigma_{X_M}$ for the set of states of M. Given a state $s \in \Sigma_M$ and $Y \subseteq X_M$, we denote by s[Y] the projection of the state s to the valuations of variables in Y. Note that we will define the semantics in a way which keeps enough information about the syntax to be able to go back from semantics to syntax; this is important for several of the operations which we define in the next section, as these are defined only at the semantics level.

Given a state s of an interface M, we denote by $\operatorname{safe}_{M}^{sv}(s)$ the predicate that indicates whether the state is safe with respect to shared variables in M; formally, $\operatorname{safe}_{M}^{sv}(s) = \operatorname{t} \operatorname{iff} \forall x \in \operatorname{shared} X_{M}, \bigwedge_{A,A' \in M, A \neq A'} s[\operatorname{isCtr}_{x}^{A}] \wedge s[\operatorname{isCtr}_{x}^{A'}] = \operatorname{f}$. Intuitively, a state s of M is safe if and only if, for all shared variables x, there is at most one atom that controls x.

Definition 4. Let X, Y, and $Z \subseteq Y$ be sets of variables and γ a guarded command from X to Y. We define the semantics $[\![\gamma]\!] \subseteq \Sigma_X \times \Sigma_Y$ of γ as follows:

- If γ is of the form $p_{\gamma} \to \alpha_{\gamma}$, where $\alpha_{\gamma} : V[X] \to V[Z]$, then $(s,t) \in [\gamma]$ iff (1) $s \models p_{\gamma}; (2) \ \forall z \in Z, t[z] = \alpha_{\gamma}(s)[z]; (3) \ \forall y \in Y \setminus Z \text{ such that } type(y) \neq \mathbb{C},$ $t[y] = s[y] \text{ and } (4) \ \forall y \in Y \setminus Z \text{ such that } type(y) = \mathbb{C}, t[y]. = s[y] + 1$
- If γ is of the form $p_{\gamma} \to \text{wait}$, then $(s,t) \in [\![\gamma]\!]$ iff (1) $s \models p_{\gamma}$, (2) $\forall y \in Y$ such that $type(y) = \mathbb{C}$, t[y] = s[y] + 1, (3) $\forall y \in Y$ such that $type(y) \neq \mathbb{C}$, t[y] = s[y], and (4) $t \models p_{\gamma}$.

Let A be an atom from X to Y and let Γ_A be either Init(A) or Update(A), i.e., a finite set of guarded commands. Then, Γ_A defines a relation $\llbracket \Gamma_A \rrbracket \subseteq \Sigma_X \times \Sigma_Y$ such that $(s,t) \in \llbracket \Gamma_A \rrbracket$ iff $(s,t) \in \llbracket \gamma \rrbracket$ for some $\gamma \in \Gamma_A$.

The semantics of SI is a LTS whose states represent valuations of variables, and whose transitions correspond to complete rounds of updates for all atoms A_i in a static consistent order A_1, \ldots, A_n .

Definition 5. The semantics of a SI M is the LTS $\llbracket M \rrbracket = (S_M, S_M^0, \rightarrow_M, L_M)$ with $S_M = V[X_M] \cup \{s_{init}\}, S_M^0 = \{s_{init}\}, L_M \subseteq P_M^{\mathcal{A}_M}$ the set of all functions $l: \mathcal{A}_M \rightarrow P_M$ for which $l(A) \in P_A$ for all $A \in \mathcal{A}_M$, and \rightarrow_M defined as follows: $-(s_{init}, l, t) \in \rightarrow_M$ iff $\mathtt{safe}_M^{sv}(t)$ and there exist $\gamma_1, \ldots, \gamma_n$ such that for all $1 \leq i \leq n, \gamma_i \in Init(\mathcal{A}_i), l(\mathcal{A}_i) = p_{\gamma_i}$ and there exists $s^0 \in V[X_M]$ such that $t = \llbracket Init(\mathcal{A}_n) \rrbracket \circ \cdots \circ \llbracket Init(\mathcal{A}_1) \rrbracket (s^0).$

 $- (s, l, t) \in \to_M \text{ iff } \mathtt{safe}_M^{sv}(s), \, \mathtt{safe}_M^{sv}(t), \text{ and there exist } \gamma_1, \dots, \gamma_n \text{ such that} \\ \text{for all } 1 \leq i \leq n, \, \gamma_i \in Update(A_i), l(A_i) = p_{\gamma_i} \text{ and } t = \llbracket Update(A_n) \rrbracket \circ \cdots \circ \llbracket Update(A_1) \rrbracket(s).$

Note that we label each transition by the predicates of the guarded commands that are effectively executed during the round, hence we preserve full syntactic

$$\begin{array}{c|c} \mbox{module } M \\ \mbox{external } a: \mathbb{N}; \\ \mbox{interface } b, c: \mathbb{N}; \\ \mbox{atom } b \mbox{ awaits } a \\ \mbox{initupdate} \\ \m$$

Fig. 2: An example of guard strengthening. Left: M, right: GS(M)

information about the interface in its semantics. In the following, we may omit this labelling in our notations when we do not need the information.

A trajectory of a SI M is a finite sequence of states s_0, s_1, \ldots, s_n in $\llbracket M \rrbracket$ such that: (1) $s_0 = s_{init}$; (2) $(s_i, s_{i+1}) \in \to_M$ for all $0 \le i < n$; and (3) no deadlock states are reachable from s_n . The sequence $s_1[obsX_M], \ldots, s_n[obsX_M]$ of observable valuations is called a *trace* of M; the *trace language* L(M) of M is the set of traces of M. In our optimistic approach, computing environments that cannot result in deadlock states amounts to projecting L(M) onto the external variables. Note that we can compute these environments if the interface has a finite representation of its trace language, i.e. if $\llbracket M \rrbracket$ has a finite state space. We say that M is *well-formed* if $L(M) \neq \emptyset$.

4 Operations

We now describe some operations on SI which will allow us to use SI as an interface theory. Note that we will also use some of these operations for incremental scheduling in Section 5; but notably shared refinement is not used in incremental scheduling, yet a necessary ingredient in any interface theory.

Guard strengthening. Given a well-formed interface M, we are interested in computing an equivalent module (in terms of infinite executions) in which no deadlock states are reachable. This *guard strengthening* GS(M) is computed by strengthening the guards of M in such a way that deadlocks are forbidden. An illustration of guard strengthening is given in Figure 2.

Semantically, the construction relies on a notion of recursively *pruning* deadlock states together with states which inevitably lead to them: Let M be a SI and let $\llbracket M \rrbracket = (S_M, S_M^0, \to_M, L_M)$ be its associated LTS. Define the function $\operatorname{Succ}_X : S_M \times V[X] \to 2^{S_M}$ by

$$\operatorname{Succ}_X(q,v) = \{q' \mid (q,q') \in \to_M \text{ and } q'[X] = v\}$$

This function gives all successors of q in $\llbracket M \rrbracket$ which for variables in X match the valuation v. Next we define a mapping Pred which outputs the controllable predecessors of a subset $B \subseteq S_M$:

$$\operatorname{Pred}(B) = \{s \mid \forall v \in V[extX_M] : \\ \operatorname{Succ}_{extX_M}(s, v) \neq \emptyset \Rightarrow \operatorname{Succ}_{extX_M}(s, v) \cap B \neq \emptyset \}.$$

Denote by Pred^{*} the closure of Pred, let $B = \{s \in S_M \mid \forall t \in S_M : (s,t) \notin \to_M\}$ be the deadlock states and $B^* = \text{Pred}^*(B)$. Intuitively, these are states from which the environment cannot prevent M from reaching a deadlock.

The pruning of $\llbracket M \rrbracket$ is then given by the LTS $\rho(\llbracket M \rrbracket) = (S_M \setminus B^*, S_M^0 \setminus B^*, \rightarrow'_M, L_M)$, where $\rightarrow'_M = \{(s, l, s') \in \rightarrow_M \mid s' \notin B^*\}$. Intuitively, pruning $\llbracket M \rrbracket$ removes all bad states and transitions leading to them, which reduces the state-space of M without affecting its language. Note that if M is not well-formed, then $\rho(\llbracket M \rrbracket)$ has no initial states; we then say that the pruning of M is empty.

For guard strengthening at the syntactic level, matching the pruning of the semantics, we proceed as follows: for each initial set of guarded commands Init(A) of an atom A in \mathcal{A}_M and $\gamma \in Init(A)$, the predicate $p_{\gamma} \in \gamma$ is replaced by

$$\tilde{p}_{\gamma} = \bigvee_{(s_{init}, t \in S_M \setminus B^*, (s, l, t) \in \to_M, l(A) = p_{\gamma})} \bigwedge_{(x \in X_M[waitX_A])} x' = t[x].$$

Similarly, for each update set of guarded commands Update(A) and $\gamma \in Update(A)$, the predicate $p_{\gamma} \in \gamma$ is replaced by

$$\tilde{p}_{\gamma} = \bigvee_{(s,t \in S_M \setminus B^*, (s,l,t) \in \to_M, l(A) = p_{\gamma})} \bigwedge_{(x \in X_M[waitX_A])} x' = t[x] \land \bigwedge_{(x \in X_M[readX_A])} x = s[x].$$

Intuitively, this method amounts to an enumeration, for every guarded command, of possible valuations of read and awaited variables that cannot reach a bad state. Replacing original predicates with associated enumerations prevents exactly bad behaviors. It follows, by construction, that $[[\mathsf{GS}(M)]] \equiv \rho([[M]])$, hence also that M is well-formed if and only if $[[\mathsf{GS}(M)]]$ is not empty. As the trace language L(M) by definition only includes traces which cannot be extended to a deadlock state, we also have $L(M) = L(\mathsf{GS}(M))$.

Parallel Composition. We introduce a synchronous parallel composition operation which combines two compatible SI. We say that SI M and N are *composable* if (1) the interface variables of M and N are disjoint; and (2) the await dependencies between the observable variables of M and N are acyclic, that is the transitive closure $(\succ_M \cup \succ_N)^+$ is asymmetric. Observe that the sets of shared variables may overlap, and that private variables are not taken into consideration here: these are not visible from the outside, hence in case of private variables with the same name, we consider that they are different and belong to different name spaces.

We say that two composable SI are *compatible* if there exists an environment in which they can be composed without reaching deadlock states. Informally, the synchronous composition P of M and N consists in the union of their atoms, where some controlled variables in M can constrain external variables in N, and vice-versa. An execution of P thus consists in an update of all the remaining external variables, followed by an update of the controlled and shared variables of M and N. **Definition 6.** Let M and N be two composable SI. Define an intermediate module P by $privX_P = privX_M \cup privX_N$, $intfX_P = intfX_M \cup intfX_N$, $extX_P =$ $extX_M \cup extX_N \setminus intfX_P$, $sharedX_P = sharedX_M \cup sharedX_N$, and finally, $\mathcal{A}_P =$ $\mathcal{A}_M \cup \mathcal{A}_N$. M and N are compatible if P is well-formed, in which case we define $M \parallel N = \mathsf{GS}(P)$.

The below theorem shows that parallel composition is *associative*, hence allowing incremental composition. The theorem follows directly from the fact that pruning does not affect the trace language.

Theorem 1. For composable SI M_1 , M_2 , and M_3 , $L(M_1 \parallel (M_2 \parallel M_3)) = L((M_1 \parallel M_2) \parallel M_3)$.

Refinement. Refinement of SI allows comparing interfaces. Informally, if N refines M, then N works in at least all the environments where M works, and all the behaviors of N defined in these environments are also behaviors of M. Hence refinement for SI is similar to alternating simulation for I/O automata [4]. For valuations $v \in V[extX_M]$, we define the set $ctr_M(v)$ of shared variables that are controlled by M according to v by $ctr_M(v) = \{x \in sharedX_M \mid (\bigvee_{A \in M} v[isCtr_x^A]) = t\}$, and we let $noctr_M(v) = extX_M \cup (sharedX_M \setminus ctr_M(v))$.

Definition 7. Let M and N be SI and $\llbracket M \rrbracket = (S_M, S_M^0, \rightarrow_M, L_M), \llbracket N \rrbracket = (S_N, S_N^0, \rightarrow_N, L_N)$. We say that N refines M, written as $N \leq M$, if $extX_M \subseteq extX_N$, $intfX_N \subseteq obsX_M$, $sharedX_N = sharedX_M$, and there exists a relation $R \subseteq S_N \times S_M$ such that $(s_{init}, t_{init}) \in R$ and for all $(s, t) \in R$, we have

- $-(s,t) \neq (s_{init}, t_{init}) \text{ implies that } s[extX_M \cup intfX_N \cup sharedX_M] = t[extX_M \cup intfX_N \cup sharedX_M]$
- for all $v \in V[extX_M]$ and $v' \in V[sharedX_M \setminus ctr_M(v)]$ it holds that if $\operatorname{Succ}_{noctr(v)}(t, v \cup v') \neq \emptyset$, then also $\operatorname{Succ}_{noctr(v)}(s, v \cup v') \neq \emptyset$, and then for all $s' \in \operatorname{Succ}_{noctr(v)}(s, v \cup v')$, there exists $t' \in \operatorname{Succ}_{noctr(v)}(t, v \cup v')$ such that s'Rt'.

The relation between refinement and trace languages is as follows: for a SI M, let $\operatorname{adm}(M) = \{w \in extX_M^* \mid \exists w' \in L(M).w' \downarrow_{extX_M} = w\}$ be the set of all *admissible external valuations*; here $w' \downarrow_{extX_M}$ denotes the projection of w' to external variables, hence we are collecting all traces of valuations of external variables which *do not block* execution of M. Then:

Theorem 2. For SI N, M with $N \leq M$ we have $\{w \in L(N) \mid w \downarrow_{extX_M} \in adm(M)\} \subseteq L(M)$.

The next theorem shows that SI theory supports *independent implementability*: Refinement is compatible with parallel composition in the sense that components may be refined individually.

Theorem 3. Given SI M_1, M'_1, M_2, M'_2 with M_1 and M_2 compatible, if $M'_1 \leq M_1$ and $M'_2 \leq M_2$, then M'_1 is compatible with M'_2 and $M'_1 \parallel M'_2 \leq M_1 \parallel M_2$.

Shared Refinement. We finish this section by mentioning that there is also a notion of *shared refinement* for SI which supports component reuse in different parts of a design. The shared refinement of two SI M_1 and M_2 is the SI $M = M_1 \wedge M_2$ which is the product of the state spaces in LTSs of M_1 and M_2 , with appropriate transitions ensuring that M_1 and M_2 evolve synchronously along the same transitions. Hence M accepts inputs that satisfy any of the assumptions from M_1 and M_2 , and it provides outputs that satisfy both guarantees of M_1 and M_2 . In particular, M can be used to implement two different aspects of a single component. Moreover, M is the *smallest* such SI in the sense of the theorem below.

Theorem 4. Given two SI M_1 and M_2 , we have that: (1) $M_1 \wedge M_2 \leq M_1$; (2) $M_1 \wedge M_2 \leq M_2$; and (3) for all SI M' such that $M' \leq M_1$ and $M' \leq M_2$, also $M' \leq M_1 \wedge M_2$.

5 Incremental TTEthernet Scheduling with SI

In this final section we present a methodology for solving scheduling problems using the synchronous interface theory developed in this paper. We concentrate on the particular application of *TTEthernet* scheduling [26], but our framework is sufficiently general that it also allows application to other scheduling and job-shop problems.

A specification of a TTEthernet network consists of a *physical topology*, a set of *frames* and a set of time-triggered scheduling *constraints*. The physical topology is an undirected graph consisting of a set of vertices, corresponding to communicating *devices* (end-systems or switches), and edges, representing bidirectional communication links, called *data-flow links*, between devices. A frame specifies a message that is sent over the network, and is represented by a tree that defines the route for the message delivery from a sender device to a set of receiver devices. Every edge in the tree represents the frame on a particular data-flow link, and is characterized by its *period* (relative deadline for the frame arrival from the sender to its receiver), *length* (value denoting frame delivery duration on the data-flow link) and offset (actual time slot at which the frame is sent from a sender to a receiver device). Like in [33], we assume, without loss of generality, that the frame period *Period* is the same for all frames on all data-flow links in the specification. Finally, time-triggered scheduling constraints are defined over the offset values of frames on data-flow links. To simplify presentation, we consider only two most common types of TT scheduling constraints: (1) contention free (CF) constraints that impose to any reasonable schedule to forbid simultaneous presence of two frames on the same data-flow link; and (2) pathdependent (PD) constraints that impose correct flow of a frame through dataflow links, ensuring that a device cannot send a frame before receiving it.

In a TTE there network specification, the only non-fixed values are the offset parameters of frames in data-flow links. A schedule that satisfies the specification corresponds to an assignment of concrete values to offset parameters

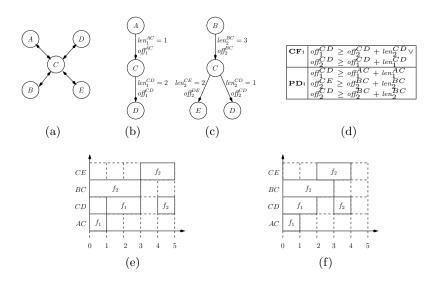


Fig. 3: Specification of a TTEthernet network: (a) network topology N; (b), (c) specification of frame routes f_1 and f_2 ; (d) constraints on offset values; (e) feasible schedule; (f) infeasible schedule

which satisfies all the constraints. The TT scheduling problem consists in computing such a schedule from a specification.

We introduce our methodology by way of an example below, but in essence, it proceeds as follows:

- 1. Introduce a SI *Clock* which keeps track of time within a period.
- 2. Model each frame as a SI, including transmission length, path dependency, and shared resources.
- 3. Use parallel composition and well-formedness check to incrementally reject all non-feasible offset values.
- 4. If any feasible offset values remain after the preceding step, then any of these constitutes a feasible schedule. Otherwise the problem is unschedulable.

Figure 3 depicts an example of a time-triggered scheduling problem for a particular TTEthernet network specification. The input to the scheduling problem consists of the network topology N (Figure 3(a)) and two frames f_1 , f_2 (Figures 3(b) and (c)). The contention-freedom and path-dependency constraints induced by the frames are depicted in Figure 3(d). Solving the scheduling problem specified in Figure 3 consists in computing the feasible schedules that satisfy all the requirements of the specification. Figures 3(e) and (f) depict two schedules, one that satisfies and another that violates the specification.

module $Clock$ interface $clk_P : \mathbb{C}$; atom clk_P reads clk_P
init
[] $t \rightarrow clk_P' := 0;$
update
$[] (clk_P \ge P - 1) \to clk_P' := 0; [] clk_P < P - 1 \to wait;$
Fig. 4: SI Clock

To solve the example scheduling problem, we first introduce a SI *Clock* as depicted in Figure 4 which measures the relative time within every period using an explicit clock variable clk_P . This clock is visible to all other interfaces in the system. Then we model the two frames f_1 and f_2 as two independent interfaces M_1 and M_2 ; this will allow to solve the problem incrementally. The application of the well-formedness check operator on the composition $Clock||M_i|$ computes the set

of all feasible partial schedules that are consistent with the scheduling (pathdependency) constraints of the frame f_i . We then use the parallel composition of *Clock* with M_1 and M_2 to combine compatible partial schedules for f_1 and f_2 , effectively removing all schedules that violate the contention-freedom constraint.

We now encode the frame f_1 as a SI M_1 , shown in Figure 5. The environment (scheduler) owns the variables $soff_1^{AC}$ and $soff_1^{CD}$ (line 2), that are used to propose in the initial state the offset values for the message of frame f_1 on the data flow AC and CD, respectively. The interface M_1 checks in line 8 whether the proposed values satisfy the path-dependency constraint, and accordingly rejects the offsets, or accepts them and copies them into the controlled variables off_1^{AC} and off_1^{CD} . The atom depicted in lines 6-10 controls a local clock clk_1^{AC} , that measures the time length of the message transmitted on the data flow link ACby the frame f_1 . The clock clk_1^{AC} is reset when the corresponding offset value is reached, and the atom ensures that the transmission of the message is finished before the end of the period P. The atom that controls the local clock clk_1^{CD} , depicted in lines 18-23, does the same monitoring of the message transmitted by f_1 on the data flow link CD. Finally, the last atom controls the shared variable x_{CD} , that models the shared resource (data flow link) CD. It ensures that when the frame f_1 is given access to the data flow link CD (via the external variable $isCtr_{x_{CD}}^1$), it is not preempted before the message transmission is done.

In order to compute the partial feasible schedules for f_1 , one needs to apply the well-formedness check on $Clock || M_1$, which amounts to generating the pruned semantics graph of this composition (also shown (in parts) in Figure 5). The wellformedness checking results in pruning all states that lead to a deadlock, i.e., it removes all states where the offsets that are proposed by the environment result in a violation of a scheduling constraint. The partial feasible schedules are encoded as the valuations of off_1^{AC} and off_1^{CD} in the remaining initial states.

The encoding of the scheduling problem for f_2 into a synchronous interface M_2 , and the corresponding computation of the partial feasible schedules for f_2 is done in a similar way. Given the pruned transition systems $\rho([[Clock]|M_1]])$ and $\rho([[Clock]|M_2]])$, the parallel composition combines the two systems and removes the joint behaviors that are not compatible. In our example, it amounts to remove all the behaviors in which the mutual exclusion property on the access to the shared variable x_{CD} is violated, thus falsifying the contention-freedom scheduling constraint. The pruned transition system of the composition encodes exactly all

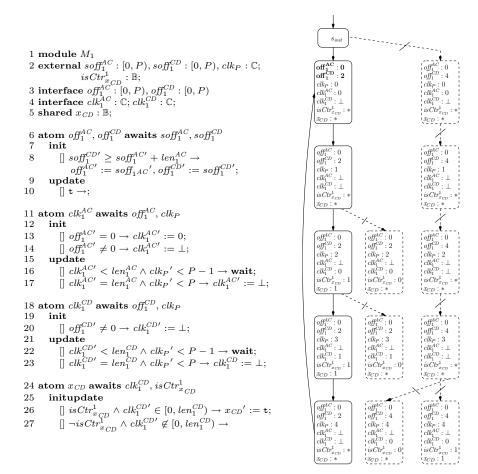


Fig. 5: Synchronous interface M_1 : (a) syntax; (b) part of its pruned semantics

feasible schedules of the original problem. Figure 6 depicts two fragments of the transition systems for $Clock || M_1$ and $Clock || M_2$ and of the pruned semantics of their composition.

6 Conclusion and Further Work

We present in this paper a simple yet powerful model for *synchronous interfaces*. Contrary to most other interface models one finds in the literature, the modeling language we use is inspired by a specific application domain, resulting in a model that resembles a high-level programming language. At the same time, we allow explicit use of time and of shared variables that are treated in a flexible way, resulting in a rich model which satisfies most common engineering needs. We develop our model into an *interface theory*, allowing for high-level reasoning and

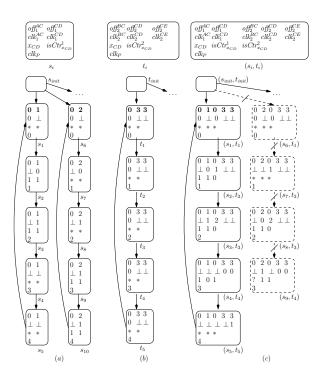


Fig. 6: Parallel composition of M_1 and M_2 : fragments of (a) $\rho([Clock||M_1])$; (b) $\rho([Clock||M_2])$ and (c) $\rho(\rho([Clock||M_1])||\rho([Clock||M_2]))$

component-based design using (shared) refinement, composition and pruning. We propose to use our interface theory as an elegant solution for incremental computation of time-triggered schedules.

In the future, we plan to implement the SI framework and apply it to different scheduling problems. We also believe that the state-based type of analysis on SI makes our approach a good candidate for development of efficient and flexible heuristics, by assigning value functions to states and restricting the search space to the assigned values. Finally, we plan to extend our approach in order to incorporate deeper information about the platform on which the system is running, like in the spirit of recent works [10, 27] done in the context of the untimed BIP and UPPAAL frameworks.

References

- F. Aarts and F. W. Vaandrager. Learning I/O automata. In CONCUR, vol. 6269 of LNCS, pp. 71–85. Springer, 2010.
- Y. Abdeddaïm, E. Asarin, and O. Maler. Scheduling with timed automata. TCS, 354(2):272–300, 2006.

- T. Abdellatif, J. Combaz, and J. Sifakis. Model-based implementation of real-time applications. In *EMSOFT*, pp. 229–238. ACM, 2010.
- 4. R. Alur, T. A. Henzinger, O. Kupferman, and M. Y. Vardi. Alternating refinement relations. In *CONCUR '98*, vol. 1466 of *LNCS*, pp. 163–178. Springer, 1998.
- A. Basu, S. Bensalem, M. Bozga, J. Combaz, M. Jaber, T.-H. Nguyen, and J. Sifakis. Rigorous component-based system design using the BIP framework. *IEEE Software*, 28(3):41–48, 2011.
- A. Basu, L. Mounier, M. Poulhiès, J. Pulou, and J. Sifakis. Using BIP for modeling and verification of networked systems – a case study on TinyOS-based networks. In NCA, pp. 257–260. IEEE Computer Society, 2007.
- S. S. Bauer, P. Mayer, A. Schroeder, and R. Hennicker. On weak modal compatibility, refinement, and the MIO workbench. In *TACAS*, pp. 175–189, 2010.
- S. Bensalem, M. Bozga, S. Graf, D. Peled, and S. Quinton. Methods for knowledge based controlling of distributed systems. In *ATVA*, vol. 6252 of *LNCS*, pp. 52–66. Springer, 2010.
- A. Benveniste, B. Caillaud, A. Ferrari, L. Mangeruca, R. Passerone, and C. Sofronis. Multiple viewpoint contract-based specification and design. In *FMCO*, vol. 5382 of *LNCS*, pp. 200–225. Springer, 2008.
- P. Bourgos, A. Basu, M. Bozga, S. Bensalem, J. Sifakis, and K. Huang. Rigorous system level modeling and analysis of mixed HW/SW systems. In *MEMOCODE*, pp. 11–20. IEEE, 2011.
- P. Bouyer, U. Fahrenberg, K. G. Larsen, N. Markey, and J. Srba. Infinite runs in weighted timed automata with energy constraints. In *FORMATS*, pp. 33–47, 2008.
- A. Burns. Preemptive priority based scheduling: An appropriate engineering approach. *PRTS*, pp. 225–248, 1994.
- A. Chakrabarti, L. de Alfaro, T. A. Henzinger, and F. Y. C. Mang. Synchronous and bidirectional component interfaces. In *CAV*, vol. 2404 of *LNCS*, pp. 414–427, 2002.
- A. David, K. G. Larsen, A. Legay, U. Nyman, and A. Wasowski. ECDAR: An environment for compositional design and analysis of real time systems. In *ATVA*, vol. 6252 of *LNCS*, pp. 365–370. Springer, 2010.
- A. David, K. G. Larsen, A. Legay, U. Nyman, and A. Wasowski. Timed I/O automata: a complete specification theory for real-time systems. In *HSCC*, pp. 91–100. ACM ACM, 2010.
- L. de Alfaro, L. D. da Silva, M. Faella, A. Legay, P. Roy, and M. Sorea. Sociable interfaces. In *FroCos*, vol. 3717 of *LNCS*, pp. 81–105. Springer, 2005.
- 17. L. de Alfaro and M. Faella. An accelerated algorithm for 3-color parity games with an application to timed games. In *CAV*, vol. 4590 of *LNCS*, pp. 108–120. Springer, 2007.
- L. de Alfaro and T. A. Henzinger. Interface automata. In ESEC / SIGSOFT FSE, pp. 109–120, 2001.
- L. de Alfaro and T. A. Henzinger. Interface theories for component-based design. In *EMSOFT*, vol. 2211 of *LNCS*. Springer, 2001.
- L. de Alfaro, T. A. Henzinger, and M. Stoelinga. Timed interfaces. In *EMSOFT*, vol. 2491 of *LNCS*, pp. 108–122. Springer, 2002.
- L. Doyen, T. A. Henzinger, B. Jobstmann, and T. Petrov. Interface theories with component reuse. In *EMSOFT*, pp. 79–88. ACM, 2008.
- A. Easwaran, I. Shin, O. Sokolsky, and I. Lee. Incremental schedulability analysis of hierarchical real-time components. In *EMSOFT*, pp. 272–281. ACM, 2006.

- M. Emmi, D. Giannakopoulou, and C. S. Pasareanu. Assume-guarantee verification for interface automata. In *FM*, vol. 5014 of *LNCS*, pp. 116–131. Springer, 2008.
- E. Fersman, P. Krčál, P. Pettersson, and W. Yi. Task automata: Schedulability, decidability and undecidability. *I&C*, 205(8):1149–1172, 2007.
- S. Graf, D. Peled, and S. Quinton. Monitoring distributed systems using knowledge. In *FMOODS/FORTE*, vol. 6722 of *LNCS*, pp. 183–197. Springer, 2011.
- H. Kopetz, A. Ademaj, P. Grillinger, and K. Steinhammer. The time-triggered ethernet (TTE) design. In *ISORC*, pp. 22–33. IEEE Computer Society, 2005.
- M. Mikučionis, K. G. Larsen, J. I. Rasmussen, B. Nielsen, A. Skou, S. U. Palm, J. S. Pedersen, and P. Hougaard. Schedulability analysis using Uppaal: Herschel-Planck case study. In *ISoLA* (2), vol. 6416 of *LNCS*, pp. 175–190. Springer, 2010.
- S. Palm. Herschel-Planck ACC ASW: sizing, timing and schedulability analysis. Tech. rep., Terma A/S, 2006.
- S. Quinton and S. Graf. Contract-based verification of hierarchical systems of components. In SEFM, pp. 377–381. IEEE Computer Society, 2008.
- J.-B. Raclet, E. Badouel, A. Benveniste, B. Caillaud, A. Legay, and R. Passerone. Modal interfaces: unifying interface automata and modal specifications. In *EM-SOFT*, pp. 87–96. ACM, 2009.
- J. I. Rasmussen, K. G. Larsen, and K. Subramani. On using priced timed automata to achieve optimal scheduling. *FMSD*, 29(1):97–114, 2006.
- I. Shin and I. Lee. Compositional real-time scheduling framework. In *RTSS*, pp. 57–67. IEEE Computer Society, 2004.
- W. Steiner. An evaluation of SMT-based schedule synthesis for time-triggered multi-hop networks. In *RTSS*, pp. 375–384, 2010.
- 34. Terma A/S. Software timing and sizing budgets. Tech. rep., Terma A/S, Issue 9.
- S. Tripakis, B. Lickly, T. A. Henzinger, and E. A. Lee. A theory of synchronous relational interfaces. ACM Trans. Program. Lang. Syst., 33(4):14, 2011.