Performance Analysis for Distributed Classification Fusion Using Soft-Decision Decoding in Wireless Sensor Networks

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Abstract. Distributed Classification Fusion using Error-Correcting Codes (DCFECC) has recently been proposed for wireless sensor networks. It adopts the Minimum Hamming Distance (MHD) fusion rule and performs much better than traditional classification approaches when the network has faulty sensors. Different fusion rules were proposed later. One of them is Distributed Classification fusion using Soft-decision Decoding (DCSD). The DCSD fusion rule has a considerably lower misclassification probability than the MHD fusion rule. This work analyzes the performance of the DCSD fusion rule. Asymptotic performance approximation of the DCSD fusion rule is derived based on the Central Limit Theorem. Furthermore, an asymptotic upper bound on the misclassification probability is obtained. Finally, numerical simulations are conducted to verify our analysis results.

Keywords: Wireless sensor networks, distributed detection, soft-decision decoding, Central Limit Theorem.

1 Introduction

Wireless sensor networks (WSNs) comprise many tiny, low-cost, battery-powered sensors in a small area. The sensors detect environmental variations and then transmit the detection results to other sensors or a base station. The base station or a sensor, serving as a fusion center, collects all detection results, and determines what phenomenon has occurred [1, 2]. The WSN sometimes must be able to function under severe conditions, such as in a battlefield, fireplace or polluted area. The transmission channel, as well as the environmental phenomenon observed by the sensor, is noisy. Furthermore, the observation signal to noise ratio (OSNR) and the channel signal to noise ratio (CSNR) may change quickly and be difficult to estimate accurately. Some sensors may even have unrecognized faults in the harsh environment. Therefore, a fault-tolerant system must be developed to make the received local decisions error-resistant [3, 4].

Wang *et al.* [5] proposed Distributed Classification Fusion using Error-Correcting Codes (DCFECC) to solve this problem by combining the distributed detection theory [6] with the concept of error-correcting codes in communication systems [7]. DCFECC with the Minimum Hamming Distance (MHD) fusion rule has a much lower probability of misclassification when some sensors are faulty than the traditional distributed classification method. DCFECC outperforms the method even when CSNR is not correctly estimated. Its performance analysis is given in [8].

Three fusion rules were proposed and compared [9, 10] later. One is the maximum *a posteriori* probability (MAP) fusion rule, one is the Minimum Euclean Distance (MED) fusion rule, and the other is Distributed Classification fusion using Soft-decision Decoding (DCSD) fusion rule. The MAP and DCSD fusion rules have a considerably misclassification probability than the MED one. Moreover, the DCSD has a lower computational complexity than the MAP with little performance loss when no faulty sensor appears. If some sensors are defective, the DCSD outperforms the MAP when the misclassification probability is lower than 0.2. Therefore, the DCSD fusion rule is a more practical choice than the other ones. However, its performance analysis have not been provided.

In this work, we analyze the performance of the DCSD fusion rule without assuming no errors in local decisions and wireless channels. Asymptotic performance approximations are obtained by the Central Limit Theorem. Asymptotic upper bounds on the misclassification probability are derived. These results can be utilized for the optimal code matrix design in the future. Computer simulations show the performance approximation is accurate and the upper bound is tight when the misclassification probability is lower than 0.2.

The remainder of this work is organized as follows. Section 2 briefly addresses the distributed detection problem in WSNs and the DCSD fusion rule. The performance analysis of the DCSD fusion rule is derived in Section 3. Section 4 shows simulation results. Concluding remarks and suggestions for future works are given in Section 5.

2 Fault-Tolerant Distributed Detection and DCSD Fusion Rule

Figure 1 depicts a wireless sensor network for distributed detection with N sensors deployed for collecting environment variation data and a fusion center for making a final decision of detections. At the jth sensor, one observation y_j is undertaken for one of phenomena H_i, where $i = 1, 2, \ldots, M$. The observation is normally a real number represented by many bits. Transmitting the real number to the fusion center would consume too much power, so a local decision, u_j, is

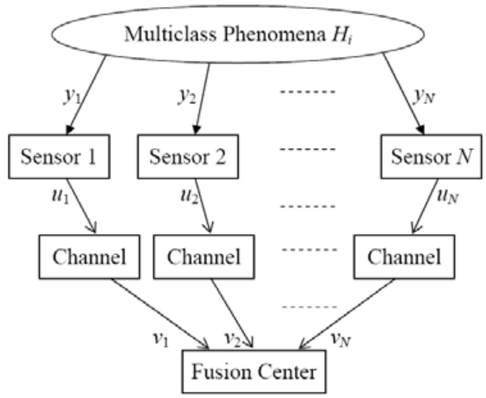

Fig. 1. Structure of a wireless sensor network for distributed detection using N sensors

made instead. For a phenomenon, if only L bits are allowed to send the local decision from the sensor to the fusion center, then the L bits are used to represent the decision.

The DCFECC approach [5] sets $L = 1$, and designs an $M \times N$ code matrix $\mathbf{T}$ not only to correct transmission errors, but also to resist faulty sensors. The application of the code matrix is derived from error-correcting codes. Table 1 lists an example of $\mathbf{T}$, which is the optimal code matrix found through the criterion in [11]. Row i of the matrix represents a codeword $\mathbf{c}_i = (c_{i,1}, c_{i,2}, \ldots, c_{i,N})$ corresponding to hypothesis H_i, and $c_{i,j}$ denotes a 1-bit symbol corresponding to the decision of sensor j. Notably, sensors 1 to 10 have the same decision pattern and sensors 11 to 20 have the same decision pattern. As a result, there are two decision patterns for the code matrix in Table 1.

Table 1. The 4×20 optimal code matrix

H_1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
H_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
H_3	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
H_4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Let v_j be the received local decision at the fusion center, where $v_j \in \{0, 1\}$. A cost function is then defined as

$$C_{\mathbf{v},\mathbf{c}_i} = \begin{cases} 1 - \frac{1}{q}, & \mathbf{c}_i \text{ is one of } q \text{ solutions of} \\ & \arg\min_{\mathbf{c}_k} d_H(\mathbf{v}, \mathbf{c}_k); \\ 1, & \text{else.} \end{cases}$$

Notably, $d_H(\mathbf{v}, \mathbf{c}_k)$ denotes the Hamming distance between a received vector, $\mathbf{v} = (v_1, v_2, \ldots, v_N)$, and a codeword, $\mathbf{c}_k$. Hence, the Bayes risk function [6] represents the probability of misclassification,

$$P_e = \sum_{i,\mathbf{v}} \int_{\mathbf{y}} p(\mathbf{v}, \mathbf{y}, H_i) C_{\mathbf{v},\mathbf{c}_i} d\mathbf{y}, \tag{1}$$

where $\mathbf{y} = (y_1, y_2, ..., y_N)$. Set $\mathbf{u} = (u_1, u_2, \ldots, u_N)$, and make the following assumptions:
Assumption 1: Observations at all sensors are conditionally independent, i.e.,

$$p(\mathbf{y}|H_i) = p(y_1, y_2, \ldots, y_N|H_i) = \prod_{j=1}^{N} p(y_j|H_i).$$

Assumption 2: The jth local decision, u_j, only depends on the jth observation, y_j.
Assumption 3: The jth received local decision, v_j, only depends on the jth local decision, u_j.
Equation (1) can then be recast as

$$\begin{aligned} P_e = \sum_{i,\mathbf{u},\mathbf{v}-v_j} \int_{\mathbf{y}} & P(H_i)[P(\mathbf{v}_{j=1}|\mathbf{u})p(\mathbf{u}|\mathbf{y})p(\mathbf{y}|H_i)C_{\mathbf{v}_{j=1},\mathbf{c}_i} \\ & + P(\mathbf{v}_{j=0}|\mathbf{u})p(\mathbf{u}|\mathbf{y})p(\mathbf{y}|H_i)C_{\mathbf{v}_{j=0},\mathbf{c}_i}]d\mathbf{y}, \end{aligned}$$

where $\mathbf{v}_{j=b_v} = (v_1, \ldots, v_{j-1}, b_v, v_{j+1}, \ldots, v_N)$, $b_v \in \{0, 1\}$, and $\mathbf{v} - v_j$ represents the elements of $\mathbf{v}$ except v_j.

The DCSD is applied as follows. Set $\mathbf{u} = (u_1, u_2, \ldots, u_N)$. The local decision $\mathbf{u}$ is transmitted for the final decision to the fusion center. The received data at the fusion center are $\tilde{\mathbf{v}} = (\tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_N)$, where

$$\tilde{v}_j = (-1)^{u_j} \sqrt{\frac{E_s}{L}} + n_j. \tag{2}$$

Notice that E_s is the total transmission energy per sensor, and n_j is the additive white Gaussian noise (AWGN) with the two-sided power spectral density $N_0/2$. The received data are decoded as hypothesis i if

$$p(\tilde{\mathbf{v}}|\mathbf{c}_i) \geq p(\tilde{\mathbf{v}}|\mathbf{c}_k) \text{ for all } \mathbf{c}_k, \text{ where } k = 1, \ldots, M. \tag{3}$$

For simplicity, let $L = 1$. Since $\tilde{v}_j$ does not depend on $c_{i,j}$ given u_j, and according to Assumptions 2 and 3, (3) can be rewritten as

$$\prod_{j=1}^{N} \sum_{b_u=0}^{1} p(\tilde{v}_j|u_j = b_u) p(u_j = b_u|c_{i,j}) \geq \prod_{j=1}^{N} \sum_{b_u=0}^{1} p(\tilde{v}_j|u_j = b_u) p(u_j = b_u|c_{k,j}),$$

$$\Rightarrow \sum_{j=1}^{N} \ln \frac{\sum_{b_u=0}^{1} p(\tilde{v}_j|u_j = b_u) p(u_j = b_u|c_{i,j})}{\sum_{b_u=0}^{1} p(\tilde{v}_j|u_j = b_u) p(u_j = b_u|c_{k,j})} \geq 0. \tag{4}$$

Because $c_{i,j}$ and $c_{k,j}$ are binary, the bit logarithm-likelihood ratio of the received data at the fusion center can be defined as

$$\lambda_j = \ln \frac{\sum_{b_u=0}^{1} p\left(\tilde{v}_j | u_j = b_u\right) p\left(u_j = b_u | c_{i,j} = 0\right)}{\sum_{b_u=0}^{1} p\left(\tilde{v}_j | u_j = b_u\right) p\left(u_j = b_u | c_{k,j} = 1\right)}. \tag{5}$$

(4) is then equivalent to

$$\sum_{j=1}^{N} \left[\lambda_j - (-1)^{c_{i,j}}\right]^2 \leq \sum_{j=1}^{N} \left[\lambda_j - (-1)^{c_{k,j}}\right]^2. \tag{6}$$

3 Performance Analysis

Assume that the wireless channel between the fusion center and the sensor is influenced by AWGN with zero mean and variance σ_c^2. Namely,

$$p\left(\tilde{v}_j | u_j = b_u\right) = \frac{1}{\sqrt{2\pi\sigma_c^2}} \exp\left\{-\frac{\left(\tilde{v}_j - (-1)^{b_u}\right)^2}{2\sigma_c^2}\right\}. \tag{7}$$

For simplicity, let

$$\begin{aligned} P_{j,0|0} &= p\left(u_j = 0 | c_{k,j} = 0\right) \\ P_{j,1|1} &= p\left(u_j = 1 | c_{k,j} = 1\right). \end{aligned} \tag{8}$$

Substituting (7) and (8) into (5), we can rewritte logarithm-likelihood ratio as

$$\begin{aligned} \lambda_j &= \ln \frac{\exp\left\{\frac{\tilde{v}_j}{\sigma_c^2}\right\} P_{j,0|0} + \exp\left\{-\frac{\tilde{v}_j}{\sigma_c^2}\right\} \left(1 - P_{j,0|0}\right)}{\exp\left\{\frac{\tilde{v}_j}{\sigma_c^2}\right\} \left(1 - P_{j,1|1}\right) + \exp\left\{-\frac{\tilde{v}_j}{\sigma_c^2}\right\} P_{j,1|1}} \\ &= \ln \frac{\exp\left\{\frac{2\tilde{v}_j}{\sigma_c^2}\right\} P_{j,0|0} + \left(1 - P_{j,0|0}\right)}{\exp\left\{\frac{2\tilde{v}_j}{\sigma_c^2}\right\} \left(1 - P_{j,1|1}\right) + P_{j,1|1}}. \end{aligned} \tag{9}$$

Thus, the Cumulative Density Function (CDF) of λ_j can be expressed as

$$\begin{aligned} \Pr(\lambda_j < x | c_{i,j}) &= \Pr\left\{ \ln \frac{\exp\left\{\frac{2\tilde{v}_j}{\sigma_c^2}\right\} P_{j,0|0} + \left(1 - P_{j,0|0}\right)}{\exp\left\{\frac{2\tilde{v}_j}{\sigma_c^2}\right\} \left(1 - P_{j,1|1}\right) + P_{j,1|1}} < x \middle| c_{i,j} \right\} \\ &= \Pr\left\{ \tilde{v}_j < \frac{\sigma_c^2}{2} \ln \frac{e^x P_{j,1|1} + P_{j,0|0} - 1}{P_{j,0|0} + e^x \left(P_{j,1|1} - 1\right)} \middle| c_{i,j} \right\}. \end{aligned} \tag{10}$$

We denote

$$\zeta_j(x) = \frac{\sigma_c^2}{2} \ln\left(\frac{e^x P_{j,1|1} + P_{j,0|0} - 1}{P_{j,0|0} + e^x (P_{j,1|1} - 1)} \right). \tag{11}$$

Because the Probability Density Function (PDF) of $\tilde{v}_j$ can be represented by

$$\begin{aligned} f_{\tilde{v}_j}(x|c_{i,j}) = & \frac{P_{j,c_{i,j}|c_{i,j}}}{\sqrt{2\pi\sigma_c^2}} \exp\left\{ -\frac{x-(-1)^{c_{i,j}}}{2\sigma_c^2} \right\} \\ & + \frac{(1-P_{j,c_{i,j}|c_{i,j}})}{\sqrt{2\pi\sigma_c^2}} \exp\left\{ -\frac{x-(-1)^{(1-c_{i,j})}}{2\sigma_c^2} \right\}, \end{aligned} \tag{12}$$

where $P_{j,c_{i,j}|c_{i,j}}$ represents the probability of correct local decision for the sensor j, (10) can be rewritten as

$$\begin{aligned} & \Pr(\tilde{v}_j < \zeta_j(x)|c_{i,j}) \\ & = \int_{-\infty}^{\zeta_j(x)} f_{\tilde{v}_j}(x|c_{i,j}) dx \\ & = P_{j,c_{i,j}|c_{i,j}} \times \Phi\left(\frac{\zeta_j(x) - (-1)^{c_{i,j}}}{\sigma_c} \right) \\ & \quad + (1-P_{j,c_{i,j}|c_{i,j}}) \times \Phi\left(\frac{\zeta_j(x) - (-1)^{c_{i,j}}}{\sigma_c} \right), \end{aligned} \tag{13}$$

where $\Phi(\cdot)$ is the CDF of a random variable with normal distribution, i.e.,

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left\{ -\frac{x^2}{2} \right\} dx.$$

Therefore, the PDF of λ_j can be given by

$$\begin{aligned} f_{\lambda_j}(x|c_{i,j}) = & \frac{d}{dx} \int_{-\infty}^{\zeta_j(x)} f_{\tilde{v}_j}(t|c_{i,j}) dt \\ = & \frac{1}{2} \left[\frac{P_{j,c_{i,j}|c_{i,j}}}{\sqrt{2\pi\sigma_c^2}} \exp\left\{ -\frac{\zeta(x)-(-1)^{c_{i,j}}}{2\sigma_c^2} \right\} \right. \\ & \left. + \frac{(1-P_{j,c_{i,j}|c_{i,j}})}{\sqrt{2\pi\sigma_c^2}} \exp\left\{ -\frac{\zeta(x)-(-1)^{(1-c_{i,j})}}{2\sigma_c^2} \right\} \right] \\ & \times \frac{\sigma_c^2 e^x (P_{j,1|1} + P_{j,0|0})}{(e^x P_{j,1|1} + P_{j,0|0} - 1)(e^x P_{j,1|1} + P_{j0|0} - e^x)}. \end{aligned} \tag{14}$$

The mean and the variance of λ_j can be found as

$$\mu_{\lambda_j} = \int_{-\infty}^{\infty} x f_{\lambda_j}(x|c_{i,j}) dx, \tag{15}$$

$$\sigma^2_{\lambda_j} = \int_{-\infty}^{\infty} (x - \mu_{\lambda_j})^2 f_{\lambda_j}(x|c_{i,j}) dx, \tag{16}$$

respectively.

The misclassification probability at the fusion center can be expressed by

$$P_e = \sum_{i=1}^{M} \Pr(H_i) \Pr\left(\bigcup_{k=1, k\neq i}^{M} \tilde{H}_k \middle| H_i \right). \tag{17}$$

When the DCSD fusion rule is employed, the misclassification probability given that H_i occurs can be derived as follows.

$$\begin{aligned} \Pr(\tilde{H}_k|H_i) &= \Pr\left(\sum_{j=1}^{N} [\lambda_j - (-1)^{c_{i,j}}]^2 \geq \sum_{j=1}^{N} [\lambda_j - (-1)^{c_{k,j}}]^2 \middle| H_i \right) \\ &= \Pr\left(\sum_{j=1}^{N} \lambda_j (c_{k,j} - c_{i,j}) \leq 0 \middle| H_i \right), \end{aligned} \tag{18}$$

where $k \neq i$. Let S be the number of the decision patterns for a code matrix and $\Omega(\ell), \ell = 1, 2, ..., S$, be the set of sensors with the same decision pattern ℓ. For example, $\Omega(1) = \{1, 2, ..10\}$ and $\Omega(2) = \{11, 12, ..20\}$ for the code matrix in Table 1. Since the sensors with the same decision pattern operate identically, the means and the variances of λ_j, where $j \in \Omega(\ell)$, have no difference and can be denoted as μ_ℓ and σ^2_ℓ, respectively. Moreover, define $d_H^{(\ell)}(\mathbf{c}_k, \mathbf{c}_i)$ as the partial Hamming distance between $\mathbf{c}_k$ and $\mathbf{c}_i$ at the set ℓ. For example, $d_H^{(1)}(\mathbf{c}_1, \mathbf{c}_2) = 0$ and $d_H^{(2)}(\mathbf{c}_1, \mathbf{c}_3) = 10$ for the code matrix in Table 1. The sensor sets $\Omega(\ell)$ satisfying $d_H^{(\ell)}(\mathbf{c}_i, \mathbf{c}_k) \neq 0$ are employed to differentiate H_k from H_i at the fusion center. When the information from two or more sensor sets are utilized for the final decision, the Hamming distance between $\mathbf{c}_i$ and $\mathbf{c}_k$ is large. Because of the large Hamming distance, the probability of misclassification is small. Therefore, when $\|\{\ell : d_H^{(\ell)}(\mathbf{c}_i, \mathbf{c}_k) \neq 0\}\| \geq 2$, (18) can be rewritten and approximated as

$$\begin{aligned} \Pr(\tilde{H}_k|H_i) &= \Pr\left(\sum_{\{\ell : d_H^{(\ell)}(\mathbf{c}_i, \mathbf{c}_k) \neq 0\}} \sum_{j \in \Omega(\ell)} \lambda_j (c_{k,j} - c_{i,j}) \leq 0 \middle| H_i \right) \\ &\approx \prod_{\{\ell : d_H^{(\ell)}(\mathbf{c}_i, \mathbf{c}_k) \neq 0\}} \Pr\left(\sum_{j \in \Omega(\ell)} \lambda_j (c_{k,j} - c_{i,j}) \leq 0 \middle| H_i \right). \end{aligned} \tag{19}$$

The following corollary can be obtained based on the Central Limit Theorem.

Corollary 1. *If $d_H^{(\ell)}(\mathbf{c}_k, \mathbf{c}_i)$ is sufficiently large, the misclassification probability can be approximated as*

$$\Pr(\tilde{H}_k|H_i) \approx \prod_{\{\ell : d_H^{(\ell)}(\mathbf{c}_i, \mathbf{c}_k) \neq 0\}} \Phi\left(-\frac{\sqrt{d_H^{(\ell)}(\mathbf{c}_k, \mathbf{c}_i)} \times (c_{k,j} - c_{i,j}) \, \mu_\ell}{\sigma_\ell} \right). \tag{20}$$

If the size of the code matrix is large, it is difficult to calculate the approximation according to (20). Since the probability of the union in (17) can be approximated as

$$\sum_{k=1,k\neq i}^{M} \Pr\left(\tilde{H}_k|H_i\right), \tag{21}$$

we can obtain the following approximation.

Corollary 2. *If $d_H^{(\ell)}(\mathbf{c}_k, \mathbf{c}_i)$ is sufficiently large, the misclassification probability can be approximated as*

$$P_e \approx \sum_{i=1}^{M} \sum_{k=1,k\neq i}^{M} \Pr(H_j) \times \prod_{\{\ell: d_H^{(\ell)}(\mathbf{c}_i,\mathbf{c}_k)\neq 0\}} \Phi\left(-\frac{\sqrt{d_H^{(\ell)}(\mathbf{c}_k, \mathbf{c}_i)} \times (c_{k,j} - c_{i,j})\,\mu_\ell}{\sigma_\ell}\right).$$

Define

$$P_e^* = \sum_{i=1}^{M} \sum_{k=1,k\neq i}^{M} \Pr(H_j) \times \prod_{\{\ell: d_H^{(\ell)}(\mathbf{c}_i,\mathbf{c}_k)\neq 0\}} \Phi\left(-\frac{\sqrt{d_H^{(\ell)}(\mathbf{c}_k, \mathbf{c}_i)} \times (c_{k,j} - c_{i,j})\,\mu_\ell}{\sigma_\ell}\right).$$

Next, we propose a corollary to derive the upper bound of the approximation.

Corollary 3. *For all ℓ and all pair $\{i, k\}$, if*

$$\Phi\left(-\frac{\sqrt{d_H^{(\ell)}(\mathbf{c}_k, \mathbf{c}_i)} \times (c_{k,j} - c_{i,j})\,\mu_\ell}{\sigma_\ell}\right) \leq \Pr\left(\sum_{j\in\Omega(\ell)} (c_{k,j} - c_{i,j})\,\tilde{v}_j \leq 0\right), \tag{22}$$

then

$$P_e^* \leq \sum_{i=1}^{M} \sum_{k=1,k\neq i}^{M} \Pr(H_i) \times \prod_{\{\ell: d_H^{(\ell)}(\mathbf{c}_i,\mathbf{c}_k)\neq 0\}} \Pr\left(\sum_{j\in\Omega(\ell)} (c_{k,j} - c_{i,j})\,\tilde{v}_j \leq 0\right). \tag{23}$$

From (12), the characteristic function of $\tilde{v}_j$ is

$$\begin{aligned}\varphi(z) = {} & P_{j,c_{i,j}|c_{i,j}} \exp\left\{\mathbf{j}\,(c_{k,j} - c_{i,j})\,z - \frac{\sigma_c^2 z^2}{2}\right\} \\ & + (1 - P_{j,c_{i,j}|c_{i,j}}) \exp\left\{-\mathbf{j}\,(c_{k,j} - c_{i,j})\,z - \frac{\sigma_c^2 z^2}{2}\right\}\end{aligned} \tag{24}$$

and the characteristic function of a random variable which is the summation of $\tilde{v}_j, j = 1, 2, ..., n$, is

$$\varphi^n(z) = \sum_{t=0}^{n} \binom{n}{t} (P_{j,c_{i,j}|c_{i,j}})^t (1 - P_{j,c_{i,j}|c_{i,j}})^{(n-t)} \times \exp\left\{ \mathbf{j}(2t-n)\left(c_{k,j} - c_{i,j}\right) - \frac{z^2}{2}(n\sigma_c^2) \right\}, \tag{25}$$

where $\mathbf{j} = \sqrt{-1}$. Then, the PDF of a random variable which is the summation of $\tilde{v}_j, j = 1, 2, ..., n$, is

$$f_{\sum \tilde{v}_j}(x) = \frac{1}{\sqrt{2\pi n\sigma_c^2}} \sum_{t=0}^{n} \binom{n}{t} (P_{j,c_{i,j}|c_{i,j}})^t (1 - P_{j,c_{i,j}|c_{i,j}})^{(n-t)} \times \exp\left\{ -\frac{\left(x - (2t-n)\left(c_{k,j} - c_{i,j}\right)\right)^2}{2n\sigma_c^2} \right\}. \tag{26}$$

Let $w_\ell = d_H^{(\ell)}(\mathbf{c}_k, \mathbf{c}_i)$. According to Corollary 3, when (22) holds, the upper bound can be expressed as

$$P_e^* \leq \sum_{i=1}^{M} \sum_{k=1, k\neq i}^{M} \Pr(H_i) \prod_{\{\ell : w_\ell \neq 0\}} \sum_{t=0}^{w_\ell} \binom{w_\ell}{t} (P_{j,c_{i,j}|c_{i,j}})^t \times (1 - P_{j,c_{i,j}|c_{i,j}})^{w_\ell - t} \varPhi\left(-\frac{(2t - w_\ell)\left(c_{k,j} - c_{i,j}\right)}{\sqrt{w_\ell}\sigma_c} \right). \tag{27}$$

4 Numerical and Simulation Results

The proposed approximations and the upper bound are verified by simulations with 10^6 Monte Carlo tests. A fusion center and $N = 20$ sensors are deployed to detect and classify four hypotheses H_1, H_2, H_3 and H_4. We also assume that the local observations are interfered by the Gaussian noise with the same standard deviation σ_o and mean $0, 1, 2,$ and 3, respectively. In addition, wireless channels are interfered by AWGN and CSNR is $10 \times \log_{10}(E_s/N_0)$. The code matrix in Table 1 was utilized.

The first and second approximations are stated in Corollary 1 and Corollary 2, respectively. The first set of figures shows the approximations and the simulation result when CSNR is set to be 0 and 10, respectively. In this case, M is small and the probability of the union in (17) is obtainable. As shown in Fig. 2 and 3, both approximations are accurate when the misclassification probability is lower than 0.2. The first approximation is better than the second one. However, the computational complexity of the first approximation is higher than the second one, as we pointed in the previous section. When OSNR is low, the probability of the union in (17) cannot be approximated by (21). Thus, the difference between the approximation and the simulation result is large at -3 dB. Figures 4 and 5 show that the upper bound in (27) is very close to the simulation result when the misclassification probability is lower than 0.2.

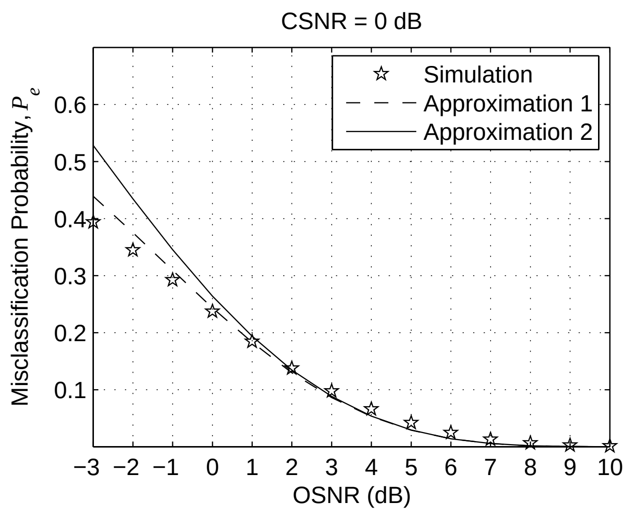

Fig. 2. Proposed approximations and simulation results when CSNR = 0 dB

5 Conclusions and Future Works

This work analyzes the performance of the distributed detection using the DCSD fusion rule. Two approximations and an upper bound of the misclassification probability are presented. The analysis is based on the Central Limit Theorem. The simulation results showed that the approximation and the upper bound are accurate for the network with only twenty sensors. In the future, we will employ the analysis result to design the optimal code matrix for the DCSD fusion rule.

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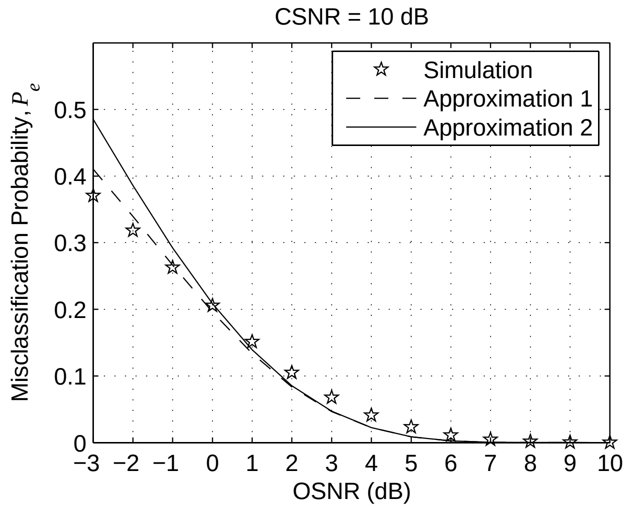

Fig. 3. Proposed approximations and simulation results when CSNR = 10 dB

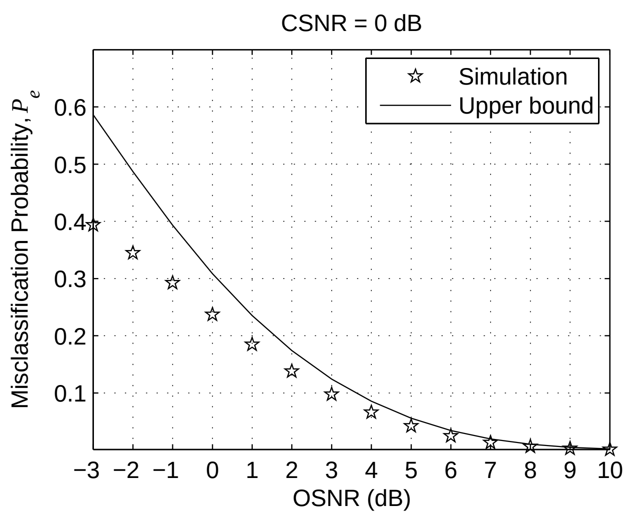

Fig. 4. Proposed upper bounds and simulation results when CSNR = 0 dB

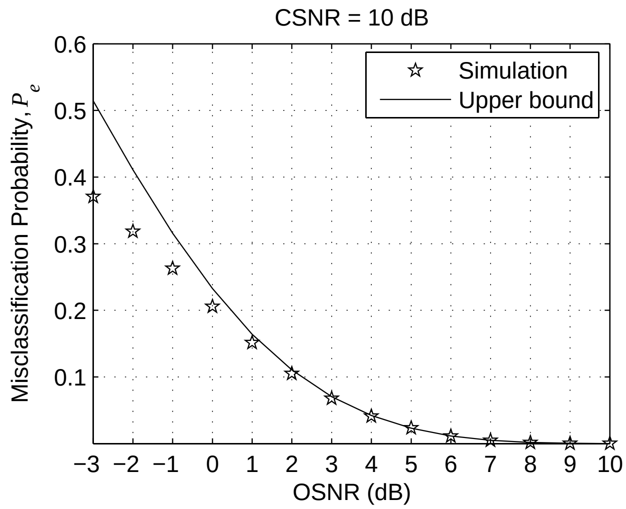

Fig. 5. Proposed upper bounds and simulation results when CSNR = 10 dB

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