Iterative decoding-based phase estimation for OFDM systems at low operating SNR

A. Sh. Fayziyev, Kwonhue Choi
Dept. of Information and Communication Engineering, Graduates school, Yeungnam University 214-1, Dae-Dong, Gyeongsan-Si, Gyeongbuk, 712-749, Korea
fayziyev@mail.ru, gonew@yu.ac.kr

Abstract. This paper proposes a new phase estimation algorithm for turbo-coded OFDM systems at low SNR. In the proposed algorithm, phase estimation process is implemented jointly with iterative decoding process. This algorithm uses extrinsic information at each decoding iteration to estimate OFDM carrier phase rotations instead of using pilots. The proposed algorithm iteratively estimates phase rotation at each OFDM carrier by averaging soft decision extrinsic information carrier by carrier. The algorithm significantly reduces pilot insertion and thus, it is suitable for low operating SNR systems. Simulation results show that the proposed algorithm achieves significantly improved phase estimation compared to the conventional pilot assisted phase estimation and it converges to perfect phase estimation case.

1 Introduction

OFDM signaling is an efficient way to overcome multipath fading channel effects. However, OFDM systems are very sensitive to carrier frequency and phase offset. Therefore, frequency and phase synchronization are most important problems in OFDM based systems.

In the presence of frequency selective channels, each carrier of OFDM signal will have different phase offset. The problem is how to estimate the carrier phase offset from the received signal, and how to implement it relatively in short time. So far, comparatively big attention is given to development of carrier phase recovery technique. Therefore, a number of methods and algorithms of carrier phase synchronization had been developed. Nowadays, pilot assisted carrier phase synchronization method [1] is widely being used. The basic idea is to use additional high power pilot signals with less sensitivity to noise, for carriers phase estimation. This method has simple realization, but due to addition of redundant signals, data transmission rate is considerably decreased. Moreover, the method is considered as not suitable for the low operating SNR systems, due to using high power pilot signals.

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The same is true for [2], where performance of the conventional pilot assisted estimation algorithm increased by using the iterative property of turbo decoders. The algorithm proposed in [2] uses soft bit estimates together with the already known pilot symbols. Such kind of operations can increase the performance of conventional pilot signal assisted phase estimation algorithm, but it still has redundancy in terms of time and power. To alleviate this problem, we should propose carrier phase estimation algorithm without pilot signals. In [3] and [4], [5] carrier phase compensation is implemented jointly with turbo decoding process. Algorithms proposed in those papers are free from the redundancy caused by additional pilot signals, but they are designed only for single carrier systems, not for multicarrier systems. Furthermore, these algorithms are designed not for channel estimation in a fading channel, but for recovering phase rotations caused by imperfect synchronization. Different method is implemented in [7], where phase estimation is achieved by applying modifications into forward and backward recursions of the SISO detection/decoding scheme. The algorithm proposed in [7] is not free from the redundancy, due to using additional pilot signals at each frame. As far as carrier phase estimation concerned, [8] gives an example of very low-complexity algorithm. However, there is no consideration about application to multicarrier systems like OFDM.

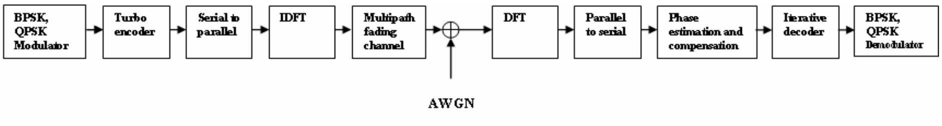


Fig. 1 Turbo coded OFDM transmission system

The main objective of this paper is to investigate new, relatively simple iterative carrier phase estimation algorithm for OFDM based systems. The proposed carrier phase estimation algorithm is based on Maximum-Likelihood approach, and compensates the carrier phase rotations iteratively, i.e., decoding process, phase estimation and correction processes are executed jointly. It makes use of the extrinsic information provided by the SISO decoder to compensate phase error. Iteratively implementing phase estimation at each iteration cycle increases the likelihood of the estimated phase, which gives the high reliable estimation of carrier phase in the end of the iterations.

In comparison with conventional pilot assisted phase estimation, the proposed algorithm does not demand redundant energy at each OFDM frame. In this paper, we investigated the BER performance of the conventional pilot assisted phase estimation algorithm in the presence of frequency selective fading channel. We considered that the frequency offset is already compensated and we showed the performance in comparison with pilot assisted phase estimation.

Simulation results show that the performance improvement of the proposed iterative phase estimation algorithm over conventional pilot assisted phase estimation is significant.

More specifically, in section 2, we described conventional pilot assisted phase estimation algorithm. In section 3, the general architecture of OFDM turbo-coded transmission system is shown. In section 4, we described the channel model used for simulations. Section 5 devoted to describing the technique of proposed iterative phase estimation algorithm. Section 6 contains simulation results, and last section has conclusions of this paper.

2 Conventional pilot assisted phase estimation

As opposed to single carrier systems, most of the OFDM systems use pilot assisted phase estimation algorithm with uniformly position of their pilot symbols in the frequency domain and equalize the channel by using simple multiplication after the FFT.

In OFDM modulation, pilots have to be inserted both in time and frequency domains. The spacing between pilot symbols in the time and frequency domains is chosen according to the sampling theorem. For our simulations, we assumed that the channel parameters are constant during the one OFDM frame, therefore spacing between pilot symbols in a time domain equal to OFDM frame duration

As we consider sufficiently short OFDM frame, we can safely assume that the channel parameters are constant within the one frame. Therefore, it does not matter how we distribute pilots in a frame. We consider the simple realization and inserted pilot signals only to first symbol of the OFDM frame.

The analytical selection of optimum pilot pattern and OFDM frame structure under the constant Eb/N0 is constraint is the rather difficult. This is caused by the fact that any increase of the number of pilot improves the channel estimation but, at the same time reduces the available energy for code transmission. In Fig. 2 has depicted the BER performance dependence of the conventional pilot assisted phase estimation from the number of pilot signal bits. Simulation results have gotten for 16 carriers, information bits in OFDM frame 1024, code rate ½.

In order to derive a channel estimate let us denote the received faded signal as *r(n)*, and pilot symbols in the received *r(n)* signal as *p(m).*

$$r_k(n) = A_k s_k(n) e^{j\phi_k} + w(n) \tag{1}$$

where A_k and ϕ_k , amplitude fluctuation and phase rotations at k-th carrier.

If the total number of subcarriers is *K* and their durations in time domain is *M*, the estimation of the phase offset can be implemented as follows:

$$S_k = \sum_{m=1}^{M} p_k(m) \tag{2}$$

where $p_k(m)$ - is the already known *m-th* pilot symbol corresponding to the *k-th* OFDM carrier.

In this case the phase offset ϕ_k at k-th OFDM carrier is:

$$\overline{\phi}_k = \arg(S_k) \tag{3}$$

The channel estimation implemented in conventional pilot assisted phase estimation algorithm inserts pilot symbols to each OFDM frame and then applies a linear algorithm to interpolate the resultant channel estimates (see Fig. 5). The performance of the pilot assisted phase estimation algorithm depends on the number of pilot signals used in an OFDM frame.

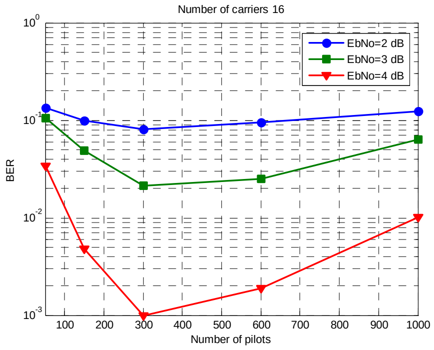


Fig. 2 BER performances vs. Number of Pilot signal bits

3 Turbo-coded OFDM system model

The Baseband architecture of the Turbo coded OFDM system is shown in Fig. 1. The binary information data bits are modulated by baseband modulator, which are then fed into *r*-rate turbo encoder. Turbo encoder consists of two RSC encoders [6]. The parity bits, outgoing from two encoders are punctured to reduce the overhead.

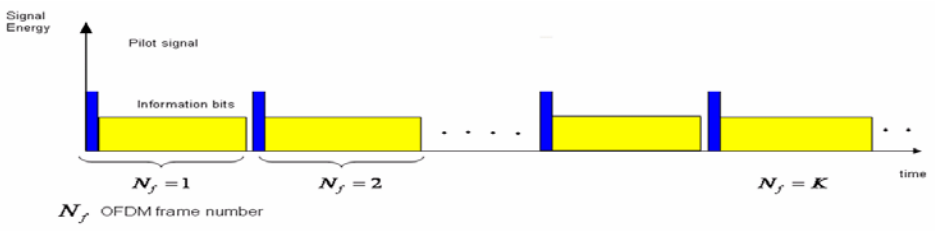


Fig. 3 Structure of the OFDM frame for pilot assisted phase estimation

The received turbo coded bits then converted from serial to parallel and modulated by OFDM modulation technique. The OFDM modulated signal then transferred through Multipath fading and AWGN channels.

Assuming that the symbol timing and frequency offset has already known and compensated the complex envelope of the *t*-th OFDM frame can be expressed as:

$$r^k(t) = A_k s^k(t) e^{j\phi_k} + n^k(t) \tag{4}$$

where k- carrier number, A_k -amplitude degradation at *k*-th carrier, ϕ_k - phase offset at *k*-th carrier, and finally $n^k(t)$ - is complex valued zero mean white Gaussian noise.

For decoding these received symbols MAP-algorithm which minimizes the symbol error probability is used. For each transmitted symbol, algorithm generates its hard estimate and soft output in the form of the a posteriori probability on the basis of the received sequence ***r*****.** It computes the log likelihood ratio by:

$$\Lambda(s) = \log \frac{P_r\{s = 1 \mid r\}}{P_r\{s = 0 \mid r\}} \tag{5}$$

The decoder determines the hard estimate of *s* as:

$$s = 1 \text{ if } \Lambda(s) \geq 0$$

$$\text{and } s = 0 \text{ otherwise}$$

Value $\Lambda(s)$ represents the soft information associated with the hard estimate of s. The key role in this iterative decoding process plays extrinsic information received from SISO decoders. Each decoder updates extrinsic information and transfers it to another one as *a priory* information. Such kind of operations improves the bit reliability iteration by iteration. The process is stopped after some iteration, and the hard decisions of the last received $\Lambda(s)$ are output.

4 Channel Model

The effect of the fading channel can be different. For simulations, frequency selective Rayleigh fading channel is used. In general, the faded signal in a multipath fading channel can be expressed as a summation of the reflected paths:

$$r(t) = \sum_{n=1}^{N_m} \alpha_n(t) s(t - \tau_n(t)) + n(t) \tag{6}$$

where *Nm*- number of paths, *s(t)*- is the transmitted signal, $\tau_n(t)$ - is the time variant delay for *n-th* path, $\alpha_n(t)$ -is the time variant multiplicative factor which represents signal fluctuations for *n-th* path, and finally *n(t)* is the time variant additive white Gaussian noise.

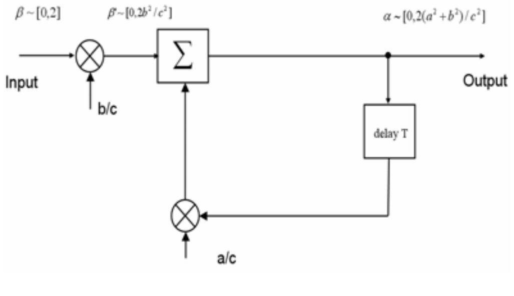


Fig. 4 Time variant multiplicative factor generator at each path

The effect of the fading channel can be represented by the $\alpha_n(t)$ parameter. In our simulation channel model this parameter considered as a constant for one OFDM frame, and generated by scheme depicted in Fig. 4. Where β -is the randomly generated complex variable with mean equal to 0 and variance equal to 2. The output is the multiplicative factor α . α is the complex valued variable and can be expressed as:

$$\alpha = Ae^{j\phi} \tag{7}$$

where A , ϕ - amplitude fluctuation and phase rotations of the path.

In this proposed channel model each next generated multiplicative factor depends on previous one. The multiplicative factor is generated for each transmitted OFDM frame. In practical case, the effect of the fading channel changes with some correlation. Parameters *a*, *b* and *c* used to make correlation between previous and current generated values of the multiplicative factor.

Each next value of the α parameter is summation of the previous one and randomly generated variable. How high the value of the *a*, so slow the fading rate. The values of *a* and *b* should be chosen by taking into account conditions, which are needed to correlative change of the generated multiplicative factor α :

$$\begin{cases} a + b = c \\ a > b \end{cases} \tag{8}$$

The output of the proposed channel is the multiplicative factor with mean equal to 0, and variance $\frac{2(a^2 + b^2)}{c^2}$.

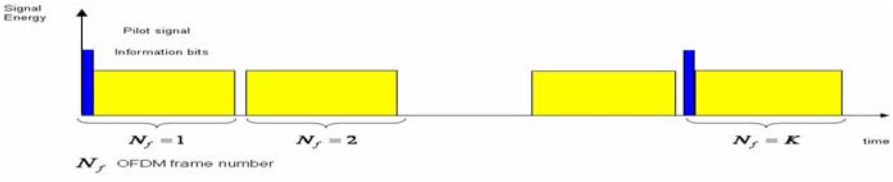


Fig. 5 Structure of the OFDM frame for proposed algorithm

In simulations, the multiplicative factor α is generated for each path by this channel model. The resulting received signal is the summation of the delayed and multiplied by generated complex valued α paths.

5 Proposed iterative carrier phase estimation algorithm

The main idea of this paper is to provide an efficient phase estimation algorithm for Turbo-coded OFDM systems. In this section we described the iterative phase estimation algorithm which gives such kind of efficiency.

In proposed algorithm phase estimation is implemented iteration by iteration and jointly with decoding process. We assumed that the phase offset during the frame is constant and absolute value of phase offset difference between two frames is smaller than $\pi/2$. In first iteration of the decoding process, the received code samples are delivered into turbo decoder as conventional decoding process. From the second iteration, we recursively update the received samples and compensate the phase error based on the *a priory* information. Which means, we replace the received codeword sample, which has been used in previous iteration, with more reliable updated codeword symbol prior to current iteration. Therefore, as iteration process goes on, the phase estimation becomes more reliable and in the end gives more reliable estimation.

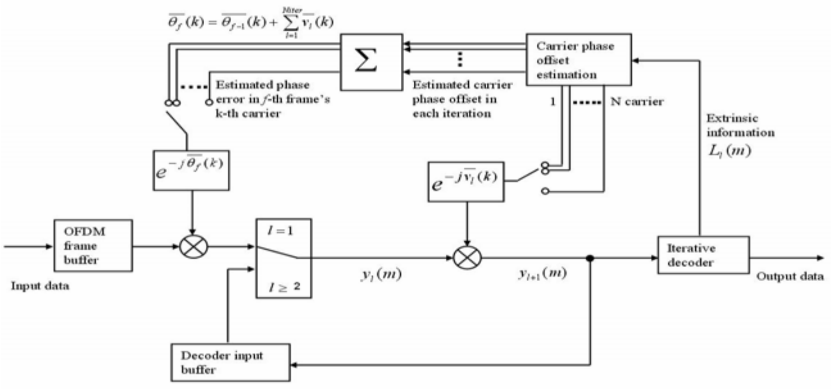


Fig. 6 Block diagram of proposed iterative phase recovery algorithm.

As the frame has a *K* carriers, and each of them has a different phase offset, we use parameter *k* to define which carrier the given *m*-th symbol is concerns. So, we can express that $y_l^k(m)$ denotes the sample for *m*-th codeword symbol used in the *l*-th iteration. In this case, the phase error in $y_l^k(m)$ is iteratively estimated and compensated as follows:

$$y_1^k(m) = r^k(m)$$

$$.$$

$$y_{l+1}^k(m) = y_l^k(m) e^{-j\bar{v}_l(k)} \tag{9}$$

where $\bar{v}_l(k)$ - is phase error in *l*-th iteration for *k*-th carrier.

If we assume that *h*-th sample corresponds to *k*-th carrier it is calculated as:

$$\bar{v}_l(k) = \angle\left[\sum_h y_l^k(h) \cdot \alpha_l^k(h)\right] \tag{10}$$

Where we average compensated soft decision received samples carrier by carrier to get more reliable phase estimation.

For BPSK modulation α_l^k is given as follows [3]:

$$\alpha_l^k(m) = \tanh \frac{L_{l,i}^k(m)}{2} \tag{11}$$

where $L_l^k(m)$ is an extrinsic information of the *m*-th codeword bit at the *l*-th iteration.

For QPSK modulation case α_l^k is given as follows [3]:

$$\alpha_l^k(m) = \tanh\frac{L_{l,i}^k(m)}{2} + j\tanh\frac{L_{l,q}^k(m)}{2} \tag{12}$$

The estimated phase error for current frame can be expressed as the sum of the phase errors at each iteration:

$$\bar{\theta}_f(k) = \bar{\theta}_{f-1}(k) + \sum_{l=1}^{L}\bar{v}_l(k) \tag{13}$$

where *L*-denotes number of iterations, *f*-frame number

For each next frame we use the estimated phase at previous frame. In this case for each next frame $r^k(m)$ is compensated with estimated phase error $\bar{\theta}(k)$ (see Fig. 6) of previous frame before delivering turbo-decoder. If consider that the phase offset for first frame estimated accurately, and the absolute value of phase offset difference between neighbor frames is smaller than $\pi/2$, the process gives high-accurate phase error estimation, which was proved by simulation results.

The main weakness of using currently estimated phase error for the next frame is that, if the estimation for some frame is incorrect all the next frames can be erroneous. To avoid such kind of propagation errors, we should periodically estimate the phase by pilots (Fig. 5). Period of pilot insertion depends on SNR value. Increasing the pilot insertion period decreases the redundant power used for pilot signals, at the same time increasing the probability of propagation errors.

6 Simulation Results

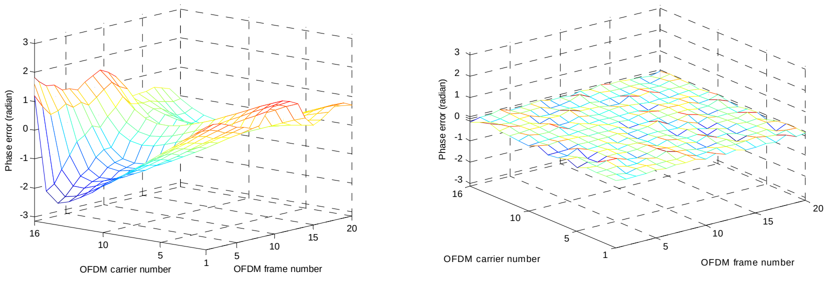
Following table shows system parameters used for simulations:

Table 1. System parameters used for simulation

Parameters	Value
Information bits in OFDM frame	1024
RSC generator	$[1\ \frac{D^2+D+1}{D^2+1}]$
Number of bits in OFDM frame	2048
Number of carriers	16,32,64
Code rate	½
Number of iterations	5

Modulation type	BPSK
Bit rate, (Mbps)	0.32,0.64, 1.28
Number of paths	4,6
Max. Doppler frequency	70,120 Hz
Maximum time delay	0.1 μsec
Number of information bits per carrier	64, 32, 16
Number of Pilots (optimized)	300
Number of Pilots (non optimized)	200

In figures Fig. 7 (a) and (b) the phase errors in OFDM carriers before and after compensation are depicted. In a fading channel, the number of reflected paths can be changed. In figures below (see Fig 8 (a) and (b)) there are shown the simulation results, for fading channels with number of paths equal to 4 and 6 respectively. In order to get these results we used channel model described in section 4. We assumed that there is a line-of-sight signal component in the channel. The results show that any increase of number of paths causes BER performance degradations in low-SNR.

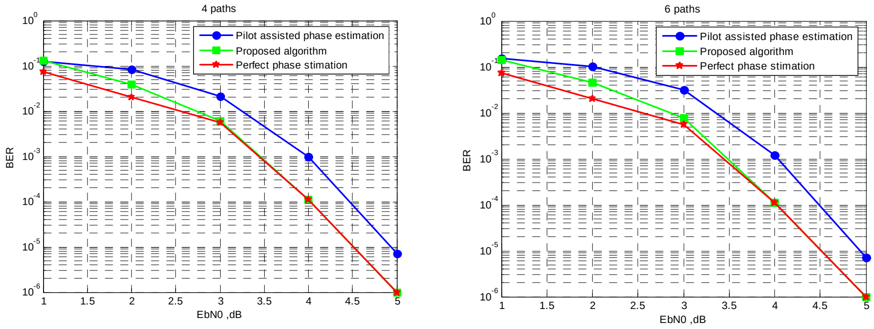


(a) (b)

Fig. 7 Phase error in one OFDM carrier (in radian) for EbN0=3dB, Maximum Doppler frequency $f_d = 70Hz$ before (a) and after (b) compensation

Also we should take into account that the performance of the pilot assisted and proposed iterative phase estimation algorithms depends on number of carriers. Due to increasing number of carriers, information bits at each carrier decrease, which means algorithm uses fewer bits to estimate the carrier phase offset (see Fig 10, Fig 8 (a)). Increasing the number of bits in OFDM frame can avoid this degradation. In figure (Fig. 10) we also depicted BER performance of the non optimized pilot assisted case.

One of other most important parameters of the fading channel is the Doppler frequency. Doppler frequency shows how fast the channel is changes. The higher the Maximum Doppler frequency, the worse the performance of the algorithms, which proved by simulations (Fig. 9).



(a) (b)

Fig. 8 BER performances for 4 and 6 paths respectively (16 carriers, Max. Doppler frequency $f_d = 70Hz$)

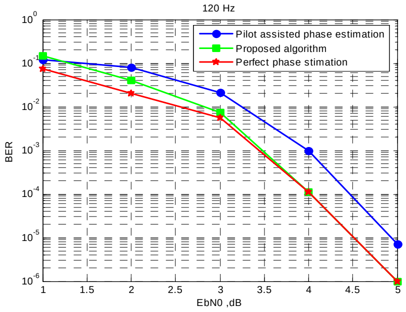


Fig. 9 BER performance for channel with Maximum Doppler frequency 120 Hz and 16 carriers (number of paths 4)

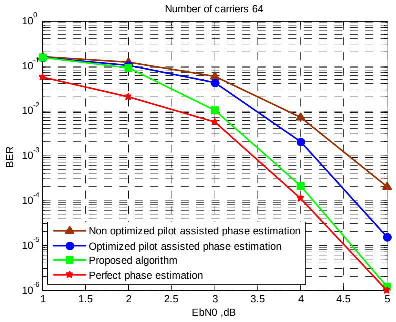


Fig. 10 BER performances for number of carriers 64 (Max. Doppler frequency $f_d = 70Hz$, number of paths 4)

Conclusions:

In this paper we proposed new carrier phase estimation algorithm for OFDM systems. By simulations it is shown that the proposed algorithm achieves BER performance very close to perfect phase estimation case and shows much more better performance in comparison with pilot assisted phase compensation algorithm. The proposed algorithm can be very useful for low operating SNR systems, because it significantly decreases the power redundancy caused by inserted pilot signals. Proposed algorithm shows significantly better performance even for low SNR in comparison with pilot assisted phase compensation. Results showed that algorithm can perform well in severe frequency selective channels.

References:

[1] F. Tufvesson, Tmaseng, “Pilot assisted channel estimation for OFDM in mobile cellular systems” *VTC 97*, pp. 1639-1643, Phoenix, Arizona, 1997

[2] M. Flament, B. Mielczarek and A. Svenson “Joint Channel Estimation and Turbo decoding for OFDM-based Systems” . *Chalmers University of Technology, SE-412 96 Gotheburg, Sweden, available at: http://db.s2.chalmers.se/download/publications/flament_1109.pdf*

[3] V. Lottici and M. Luise “Embedding Carrier Phase Recovery Into Iterative Decoding of Turbo-Coded Linear Modulation” *IEEE Transactions on Communications*, vol.52, pp. 661-669, April 2004.

[4] Li Zhang and Alister G. Burr “Iterative Carrier Phase Recovery Suited to Turbo-Coded Systems” *IEEE Transactions on Communications, vol.3, N0. 6, November 2004.*

[5] Kwonhue Choi, Sun-Heui Ryoo and Do-Seob Ahn, “Joint Iterative Frequency and Phase Recovery for Turbo Code”, *IEICE Technical Report SAT2005-41(2005-10).*

[6] A Turbo Code Tutorial, W.E.Ryan. (1998).
http://www.eecis.udel.edu/~fu/images/tc_tutorial.pdf

[7] A. Anastasopoulos, K.M. Chugg, “Adaptive iterative detection for phase tracking in turbo-coded systems”, *IEEE Transactions on Communications*, vol.49,pp. 2135-2144, Dec. 2001.

[8] C. Morlet, M.L.Boucheret, I.Buret, “Low-complexity carrier –phase estimator suited on-board implementation”, *IEEE Transactions on Communications*, vol. 48, pp. 1451-1454, Sept. 2000