# Duopoly Price Competition in Secondary Spectrum Markets

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***Abstract*—In this paper, we consider the problem of spectrum sharing in a Cognitive Radio Network (CRN) with spectrum holder, two secondary operators and secondary users (SUs). In the system model under consideration, the spectrum allocated to the two secondary operators can be shared by SUs, which means that secondary operators buy spectrum from spectrum holder and then sell spectrum access service to SUs. We model the relationship between secondary operators and SUs as a two-stage stackelberg game, where secondary operators make spectrum channel quality and price decisions in the first stage, and then the SUs make their spectrum demands decisions. The backward induction method is employed to solve the stackelberg game. Numerical results are performed to evaluate our analysis.**

***Index Terms*—Pricing, CRN, secondary operators, SUs**

## I. Introduction

Wireless spectrum is considered as one of the scare and precious radio resources in communication networks, and it is conventionally controlled by government via static license-based allocations. Some recent works have reported that many spectrum bands are largely under-utilized even in densely populated urban areas [1]–[3]. Besides, the demand for wireless data service is growing exponentially in recent years. According to a recent report released by Cisco, the monthly global mobile data traffic will be 49 exabytes by 2021 [4]. The paradox between rapidly growing demand for wireless services and under-utilized spectrum allocation indicates that current static spectrum allocation policy has some shortcomings.

Cognitive Radio (CR), also known as dynamic spectrum access (DSA), has been proposed as a novel approach to improve the efficiency utilization of spectrum. In Cognitive Radio Networks (CRNs), unlicensed secondary users (SUs) can dynamically access unused part of legacy spectrum bands that used by primary (licensed) users (PUs).

Today, mobile virtual network operators (MVNOs), also called secondary operators, have received successful operations in many countries, which is one of the main motivations of our study. MVNOs do not own the physical infrastructure and lease spectrum from spectrum holders to provides services to SUs. For example, IIJmio and LINE MOBILE are MVNOs in Japan, both of whom provide services to users by paying to lease spectrum from DOCOMO.

Unlike most of the existing works focus on the technical aspects of spectrum sharing (e.g., designing power control method), in this paper we study from the economic aspect. Moreover, different from previous works that simply analyze homogeneous SUs, that is all SUs have homogeneous valuation for the spectrum service, we divide SUs into different types based on their different preferences for the spectrum quality. For example, some SUs who are watching videos may have higher valuations of the spectrum while some other SUs who are just phoning have lower valuations on the spectrum [5].

In this paper, we investigate the competition between two

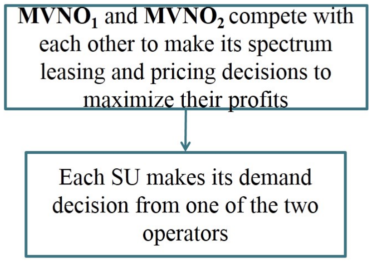


Fig. 1. Two-level structure between operators and secondary users.

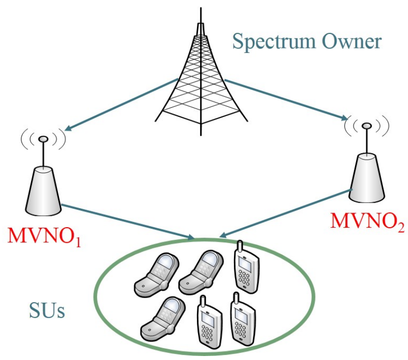


Fig. 2. System Model.

secondary operators in spectrum leasing and pricing to provision service access to a common pool of SUs. The two operators lease spectrum from spectrum owners, and then compete with each other to sell the resource to SUs with the goal of maximizing their own profits. We model the interaction between two operators and SUs as a Stakelberg (leader-follower) game, where the two operators first set service access prices to maximize their profits and then each SU will decide which operator to select based on prices and spectrum quality, as illustrated in Fig.1. In particular, we model competition between the two operators as Stakelberg Game (SG) where one operator sets price firstly, and the other operator sets price later.

## II. Related Works

In this section, we review and discuss some notable related works that centered around price-based spectrum service access control in CRNs.

Traditionally, game theory based techniques have been widely used for resource management in wireless networks. Focusing on a duopoly femtocell communications market, Ren et al. studied the problem of long-term entry and spectrum sharing scheme decision from the perspective of an entrant network service provider [9]. Due to users have different preferences for different time slots, Zhang et al. studied time-dependent price competition in a duopoly wireless networks market [10].

In recent years, spectrum trading/sharing and resource management in CRNs have been extensively studied by using game theory. Some works related works analyze the interaction between the primary and secondary operators. The authors in [7] jointly address the problem of pricing and network selection in CRNs, where the primary operator who can provide higher guaranteed service and the secondary operator who provides cheaper best-effort secondary network service compete to serve a common pool of users. The problem under consideration is formulated as a a Stackelberg game, where the two operators first set the prices of network services to maximize their revenues. Then, users decide which operator to select. Kinoshita et al. proposed a spectrum sharing method aiming to achieve both users' higher throughput and operators' profit by setting appropriate pricing strategy [11].

However, the previous works ignore users' heterogeneous types or do not consider the channel information.

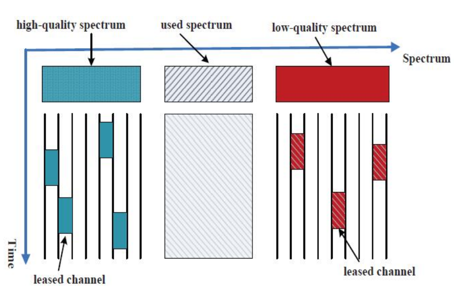


Fig. 3. A spectrum pool with two kinds of spectrum qualities [8].

## III. System Model

In this section, we introduce the system model where two secondary operators, denoted by $\text{MVNO}_1$ and $\text{MVNO}_2$ respectively, lease spectrum from spectrum owner and provide service to a number of SUs, as illustrated in Fig.2. The system model that we use is mainly inspired by [8] but with different objectives. As the radio spectrum allocated to spectrum holder remains largely unused even in densely populated urban areas [1], the unused spectrum can form a spectrum pool where the total available bands are divided into a lot of unit channels. These channels have different qualities due to interference levels, as shown in Fig.3. We assume that each SU purchases one channel and has its own preference for channel quality.

We assume that channel with high spectrum quality denoted as $C_1$ is leased to $\text{MVNO}_1$, and the one with low spectrum quality as $C_2$ is leased to $\text{MVNO}_2$. The channel quality $C_i$ is expressed as

$$C_i = Blog_2(1 + \frac{\rho}{I_i}), i = 1, 2 \tag{1}$$

where $B$ is bandwidth, $\rho$ is the power received by the SU, and $I_i$ is the interference of the channel.

### A. SUs' Model

In order to capture SUs' heterogeneous valuations of the spectrum service, we divide SUs into different types based on their different preferences for the spectrum quality. We assume that SU type is assumed to be uniformly distributed in [0, 1] with probability distribution function (PDF) $f(\cdot)$ and cumulative distribution function (CDF) $F(\cdot)$. One of the main reasons for uniform distribution is for convenience of analysis. For an SU type $\theta_k$, higher value of $\theta_k$ means this SU has a

TABLE I
NOTATIONS SUMMARY.

| Notation | Description |
| --- | --- |
| $i$ | $i \in \{1, 2\}$, which is MVNO set |
| $k$ | subscript of a SU |
| $p_i$ | the price of $\text{MVNO}_i$, for $i = 1, 2$ |
| $D_i$ | the demand of services from $\text{MVNO}_i$ |
| $C_i$ | spectrum capacity of $\text{MVNO}_i$, for $i = 1, 2$ |
| $\pi_i$ | the profit of $\text{MVNO}_i$ in NSG scenario, for $i = 1, 2$ |
| $\theta_k$ | SU $k$'s sensitivity to delay |
| $f(\cdot)$ | probability density function (PDF) of SUs' preferences parameter |
| $F(\cdot)$ | cumulative density function (CDF) of SUs' preferences parameter |
| $\mu$ | unit cost coefficient |
| $\theta_i$ | the marginal point where SUs switch from negative utility to positive for choosing $\text{MVNO}_i$, for $i = 1, 2$ |
| $\tilde{\theta}$ | the marginal point where SUs switch from one MVNO to the other |
| $U_{i,k}$ | the utility that type $\theta_k$ SU gets from $\text{MVNO}_i$, for $i = 1, 2$ |

higher preference for the quality of channel. For the $\theta_k$ SU that selects access service from $\text{MVNO}_i$, its utility function is given as

$$U_{i,k} = \theta_k C_i - p_i, i = 1, 2 \tag{2}$$

where $C_i$ denotes the spectrum quality of $\text{MVNO}_i$ and $p_i$ is channel price.

### B. Secondary Operators' Model

We assume that the two secondary operators, denoted as $\text{MVNO}_1$ and $\text{MVNO}_2$, set prices of their network services as $p_1$ and $p_2$ for channel quality $C_1$ and $C_2$ respectively to compete for a number of SUs, with the objective of maximizing their profits.

The notations used throughout this paper are summarized in Table 1.

## IV. DUOPOLY PRICE COMPETITION

In this section, we analyze a competitive market where two secondary operators compete with each other by setting optimal prices of their services to maximize their profits. The relationship between secondary operators and SUs can be characterized as the two-stage Stakelberg game, which can be solved by employing the backward induction method [14], [15]. We first analyze the demand decisions of SUs in Stage II. Then, we investigate how the two operators set their prices in Stage I.

Besides, we model the competition between the two operators as Stakelberg Game where one operator sets price firstly, and the other operator sets price later.

### A. SUs' Demand Decision

Based on the prices of the two MVNOs $(p_1, p_2)$, each of the SU will make a demand decision to choose service from one of them, or neither. We denote the demands of SUs for services from $\text{MVNO}_1$ and $\text{MVNO}_2$ as $D_1(p_1, p_2)$ and $D_2(p_1, p_2)$, respectively.

We consider two critical types of SUs $\theta_1$ and $\theta_2$, such that

$$U_{1,k} = \theta_1 C_1 - p_1 = 0 \tag{3}$$

$$U_{2,k} = \theta_2 C_2 - p_2 = 0 \tag{4}$$

From which we get

$$\theta_1 = \frac{p_1}{\alpha C_1} \tag{5}$$

$$\theta_2 = \frac{p_2}{\alpha C_2} \tag{6}$$

We also denote an indifferent user by $\tilde{\theta}$ such that $U_{1,k} = U_{2,k}$, that is

$$\tilde{\theta} C_1 - p_1 = \tilde{\theta} C_2 - p_2 \tag{7}$$

Then we have

$$\tilde{\theta} = \frac{p_1 - p_2}{C_1 - C_2} \tag{8}$$

SUs are assumed to be self-interested, which means that they choose service access of $\text{MVNO}_i$ $(i = 1, 2)$ if their utilities are not only positive but also higher than the other one. Therefore, we have the following result.

**Proposition 1.** A type $\theta_k$ SU will make the following decision such that

- It will chooose Operator 1 if $U_{1,k}(\theta_k, p_1) > U_{2,k}(\theta_k, p_2)$, and $U_{1,k}(\theta_k, p_1) > 0$, which requires $\theta_k < \tilde{\theta}$ and $\theta_k < \theta_1$;
- It will choose Operator 2 if $U_{2,k}(\theta, p_2) > U_{1,k}(\theta_k, p_1)$, and $U_{2,k}(\theta_k, p_2) > 0$, which requires $\tilde{\theta} < \theta_k < \theta_2$;
- It will choose neither if $U_{1,k}(\theta_k, p_1) < 0$, and $U_2(\theta_k, p_2) < 0$, which requires $\theta_k > \theta_1$ and $\theta >_k \theta_2$.

Based on the above joining decision policy, the demands of SUs for services from $\text{MVNO}_1$ and $\text{MVNO}_2$ are respectively given as

$$D_1(p_1, p_2) = F_1(\theta) = \int_{max\{\theta_1, \tilde{\theta}\}}^{1} f(\theta) d\theta \tag{9}$$

$$D_2(p_1, p_2) = F_2(\theta) = \int_{\theta_2}^{\tilde{\theta}} f(\theta) d\theta \tag{10}$$

Based on Eqs.(9) and (10), we get the following results.

**Proposition 2.** For a given pair of prices ($p_1$, $p_2$), there exits a unique pair of equilibrium demands $D_1^e$ and $D_2^e$ at $\text{MVNO}_1$ and $\text{MVNO}_2$ respectively, such that

1) If $\theta_2 > \theta_1$, then $\theta_1 > \tilde{\theta}$ and $\theta_2 > \tilde{\theta}_1$. We have $F_1(\theta) = 0$ and $F_2(\theta_2) = F(\theta_2)$;

2) If $\theta_1 > \theta_2$, then $\tilde{\theta} > \theta_1$ and $\tilde{\theta}_1 > \theta_2$. We have $F_1(\theta) = 1 - F(\tilde{\theta})$ and $F_2(\theta_2) = F(\theta_2) - F(\tilde{\theta})$;

2) corresponds to the duopoly secondary market where $\text{MVNO}_1$ and $\text{MVNO}_2$ coexist. Therefore, the equilibrium demands for services from Operator 1 and Operator 2 are given as

$$D_1(p_1,p_2) = 1 - F_1(\tilde{\theta}) = 1 - \frac{p_1 - p_2}{C_1 - C_2} \tag{11}$$

$$D_2(p_1,p_2) = F_2(\tilde{\theta}) - F_2(\theta_2) = \frac{p_1 - p_2}{C_1 - C_2} - \frac{p_2}{C_2} \tag{12}$$

### *B. Competition Between Two MVNOs*

Based on the demands of SUs, the two MVNOs will compete to set optimal prices to maximize their profits, which is denoted as

$$\pi_1 = (p_1 - \mu C_1) D_1(p_1,p_2) = (p_1 - \mu C_1)(1 - \frac{p_1 - p_2}{C_1 - C_2}) \tag{13}$$

$$\pi_2 = (p_2 - \mu C_2) D_2(p_1,p_2) = (p_2 - \mu C_2)(\frac{p_1 - p_2}{C_1 - C_2} - \frac{p_2}{C_2}) \tag{14}$$

where $\mu$ is cost coefficient.

The competition between two MVNOs can be modelled as the following one shot game.

**Players**: $\text{MVNO}_1$ and $\text{MVNO}_2$,

**Strategies**: Prices $p_i > 0, i = 1, 2,$

**Payoff**: Profits $\pi_i, i = 1, 2.$

We next investigate the competition between two MVNOs, which is modelled as an Stackelberg game, where $\text{MVNO}_1$ is the leader, whereas $\text{MVNO}_2$ is the follower. $\text{MVNO}_1$ has the first-move advantage, which means that it sets optimal $p_1$ to maximize its profit by anticipating the choice on $p_2$ of $\text{MVNO}_2$.

The profits maximization problem of and $\text{MVNO}_1$ is expressed as **Problem1:**

$$\max_{p_1} \pi_1 = (p_1 - \mu C_1) D_1(p_1,p_2) \quad \text{s.t.} \quad p_1 \geq 0 \tag{15}$$

where $D_1(p_1,p_2)$ is given in Eq.(9).

After knowing and $\text{MVNO}_1$'s best response price $p_1$, $\text{MVNO}_2$ determines its optimal price $p_2$ by solving the following profit optimization problem,

**Problem2:**

$$\max_{p_2} \pi_2 = (p_2 - \mu C_2) D_2(p_1,p_2) \quad \text{s.t.} \quad p_2 \geq 0 \tag{16}$$

where $D_2(p_1,p_2)$ is given in Eq.(10).

By solving **Problem1** and **Problem2**, we have the following results.

**Proposition 3.** There exists a unique Nash Equilibrium price pair ($p_1$, $p_2$) Stackelberg game scenario.

**Proof:**

By taking the derivative of $\pi_1$ with respective to $p_1$, and setting the equality to zero,

$$\frac{\partial \pi_1}{\partial p_1} = 0 \tag{17}$$

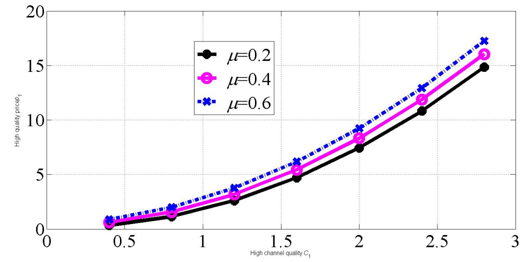


Fig. 4. The optimal price of $\text{MVNO}_1$ versus its channel quality.

From which, we get

$$p_1 = \frac{C_1 - C_2 + p_2 + \mu C_1}{2} \tag{18}$$

By substituting Eq.(18) into Eq.(14), we have

$$\pi_2 = (p_2 - \mu C_2)[\frac{C_1 - C_2 - p_2 + \mu C_1}{2(C_1 - C_2)} - \frac{p_2}{C_2}] \tag{19}$$

By taking the derivative of $\pi_2$ with respective to $p_2$, and setting the equality to zereo, we get

$$p_2 = \frac{C_2(C_1 - C_2) + \mu C_2(3C_1 - C_2)}{2(2C_1 - C_2)} \tag{20}$$

By substituting Eq.(20) into Eq.(18), we get

$$p_1 = \frac{(C_1 - C_2)(4C_1 - C_2) + \mu(4C_1^2 + C_1C_2 - C_2^2)}{4(2C_1 - C_2)} \tag{21}$$

Accordingly, by substituting Eqs.(20) and (21) into Eqs.(11) and (12) respectively, we can get users' demands $D_1(p_1,p_2)$ and $D_2(p_1,p_2)$.

Therefore, we have the following corollary.

**Corollary 1.** The profits of MVNOs in Stackelberg game scenario are denoted as:

$$\pi_1 = p_1 D_1(p_1,p_2) \tag{22}$$

$$\pi_2 = p_2 D_2(p_1,p_2) \tag{23}$$

## V. Numerical Results

In this section, we present numerical results to validate the performance of our analysis. Based on [8], the parameters are set as follows: $\mu = 0.2$, $0.1 \leq C_2 \leq C_1 \leq 3(bps)$.

Fig.4 shows how the price of $\text{MVNO}_1$ varies as its channel quality. Fig.5 shows how the price of $\text{MVNO}_2$ varies with its channel quality. From the two figures we can observe that the prices of the two MVNOs increase with their channel quality increasing. The two figures reveal that MVNOs have to set higher prices if the marginal costs increase.

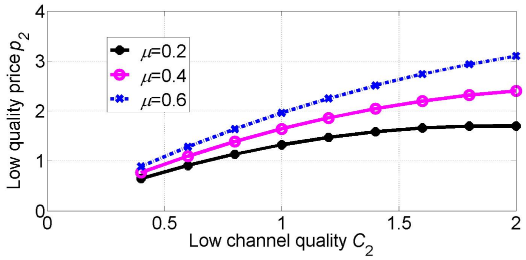


Fig. 5. The optimal price of $MVNO_2$ versus its channel quality.

## VI. Conclusions and Future Works

In this paper, we investigate the price competition in a duopoly secondary spectrum market, where the idle spectrums with different qualities are leased to secondary operators who provide service access to SUs with the objective of maximizing secondary operators' profits. The numerical results show that MVNOs can set optimal prices if they have high channel qualities, and they have to set higher prices for the channel quality if the marginal costs increase. Our study can be further studied in several directions. In the first place, we will study another competition scenario where two secondary operators set prices simultaneously to maximize their profits. Another research direction is the investigation of the oligopoly case where there are multiple secondary operators compete to provide spectrum service to SUs.

## VII. Acknowledgement

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