Regulating Wireless Access Costs for Not Vertically Integrated Content Providers

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***Abstract*—We consider a single, unaffiliated streaming content provider (CP) and another that is vertically integrated (affiliated) with a cellular wireless ISP. We formulate a non-cooperative game between these two CPs involving, e.g., linear demand-response to price by the end-users with long-duration sessions (e.g., streaming video), and a model as amplified noise of additional network delay jitter and reduced responsiveness to changing channel conditions by the unaffiliated CP. The effect of effective additional side-payments from the unaffiliated CP to the ISP, as may be set by a government regulator, is studied at Stackelberg equilibrium both analytically and numerically.**

I. INTRODUCTION

Network neutrality continues to be debated as its core economic issues as described in, e.g., [3], [24], [6], and has not yet been resolved. The debate concerns all participants in the enormous and growing Internet economy: Internet service (access) providers (ISPs), content providers/streamers (CPs, more generally providers of application services), content creators, advertisers, end-users/consumers, and government regulators. Network neutrality regulations presently prevent ISPs from demanding *direct* side payments from content providers for access to the ISP's end-users/subscribers (origin neutrality). With regard to the latter, neutrality rules do not preclude asymmetric Service-Level Agreements (SLAs) at network-to-network interfaces (NNIs, or peering points) that are (neutrally) based on traffic *aggregates*. In this way, an "eyeball" ISP[1] can effectively demand additional payment from the transit ISP of a large (e.g., video) content provider. The transit ISP will naturally pass on these costs to the CP or be squeezed out of business forcing the CP to directly engage with the ISP - either way, a side-payment from the CP to the eyeball ISP effectively ensues, e.g., [13], [12]. There were certain exceptions to the neutrality rules for cellular wireless access in the USA [2], [10]. Note that end-users tolerate data-plan quotas [5] and overages (including overages priced based on usage) for wireless access, likely owing to the value they placed on convenience.

In January 2014, a US federal appeals court struck down parts of the FCC's net-neutrality rules [25]. Shortly thereafter, Comcast and Netflix announced an agreement in which Netflix would directly pay Comcast for a faster access to Comcast's subscribers, i.e., "fast lanes" [26][2]. Some cellular wireless ISPs in the USA are offering "sponsored data" agreements[3] wherein content providers pay so that their media is not applied to end-user data-plan quotas, or in exchange for an advertisement *from the cellular access provider* a certain amount of free content will not be applied to dataplan quota (select free content). More recently, the FCC reclassified the Internet as a utility [20] in order to shore-up neutrality regulations, while the coming Trump administration is apparently inclined to favor the interests of the ISPs.

A primary objective of network neutrality regulations is to address antitrust concerns, promote competition and reduce costs for consumers. Complicating the role of eyeball ISPs [16] with respect to the content it handles is the fact that many are themselves also content providers (over "managed services" they provide to their end-users) and creators, and thus in competition with some CPs that they enable over their commodity Internet service[4]. Though this is a growing trend, considering AT&T's current attempt to acquire Time-Warner and the growth of Google Fi in the US and Facebook's Express WiFi e.g., in India, some content providers/producers, e.g., Netflix, are not (yet) themselves network providers.

Since the onset of the neutrality debate, researchers have extensively studied parsimonious models of the Internet marketplace to gain insight into the economic forces in play, especially to study the effects on competition (including barriers to market entry) and costs-to-end-users of non-neutral actions by ISPs. Performance is often assessed based on the Bertrand-Nash equilibria of noncooperative, decentralized games, and in terms of dynamical convergence to these equilibria, often considering limited resources (particularly bandwidth) as in classical Cournot games. The role of the regulator can be considered using a Stackelberg (leader - follower) game framework. For example, games involving end-

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[1]Here, ISPs that service end-users.

[2]Apparently, Comcast shortly backed out of this agreement possibly because it was not neutral considering that the fastlanes were not set up upon end-user requests.

[3]*E.g.,* AT&T's Sponsored Data, Verizon Feebee, T-Mobile Binge On. Note that in traditional broadcast TV, commercial breaks are ads from the national broadcaster or local affiliate while the provider of video content often embeds ads, i.e., both parties engage in advertising not just the content provider.

[4]Somewhat conversely, in the context of the Digital Millennium Copyright Act (DMCA) and associated debate over illegal file-sharing (again, the traffic volume of which was a trigger for the network neutrality debate), some have argued to hold ISPs accountable for associated loss of revenue to holders of copyrighted content.

users and content providers on an ISP platform were studied in [7], [18]. Shapley values, indicating fair division of revenue with a coalition (or cooperative game), are used to argue for side-payments between ISPs and CPs in [16], [17]. Pricing congestible commodities has been extensively studied [23], [5], e.g., in [9] a demand model is based on a "cost" that is the sum of a price and latency term. Extensive prior work has also used simple models to study the impact of: direct CP-to-ISP side payments for access or differential service to end-users, advertising revenue and/or direct subscription payments from end-users to CPs[5], network caching [13], [14], and competition among ISPs and/or among CPs including CPs affiliated/integrated with ISPs, see e.g., [15] and the references therein.

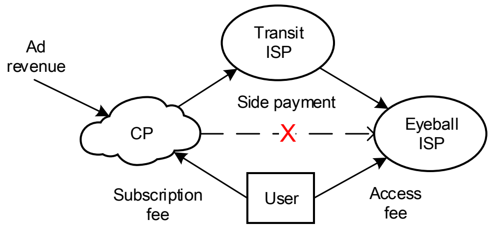

Fig. 1. Side-payment from remote/unaffiliated CP

In this paper, we consider a noncooperative game between

- a single, unaffiliated CP (or cooperating collective) and
- a single cellular-wireless Internet-access provider (ISP, or ISP oligopoly) with an affiliated CP in competition with the unaffiliated CP (i.e., CP-ISP coalition).

That is, a two-sided content-streaming market where one party is "vertically integrated" [4] with the cellular-wireless ISP. We assume the content-delivery latency associated with the affiliated CP is smaller, e.g., owing to content caching at edge cloud (fog) facilities operated by the ISP.

We formulate the noncooperative game in one cell, and don't consider cross-cell interference. We use a model of medium access control by CDMA (3G) because of it has an interesting power-limited channel capacity resulting from intra-cell interference, giving a concave Pareto frontier. The methodology presented in the following can be readily adapted to simplified models of cellular wireless access technologies of 4G, which we also describe. For improved tractability and to reduce the potentially vast number of model parameters in play (as is commonly required when contemplating regulatory policy) we simplify the model in several ways, including assuming a single aggregative, long-duration (e.g., streaming video) end-user session per content provider with an associated linear demand-response to subscription price[6]. We also model as amplified noise the additional network delay jitter of content from the unaffiliated CP (and less responsiveness to changing channel conditions). As seen in the following, the resulting game retains interesting constraints that complicate analysis and, we believe, will yield useful "high level" results to network operators and regulators. In particular, a Stackelberg game is used to demonstrate how a regulator may set an effective side-payment from the unaffiliated CP to the ISP (a side-payment which is larger than that from the vertically integrated CP to the ISP).

[5]ISPs have pruned ads from delivered content arguing that they were not requested by the end-users [11], while CPs argued that (otherwise free) content or services are monetized through embedded advertisements the receipt of which end-users implicitly request.

[6]See [22] for a survey on pricing for Internet access.

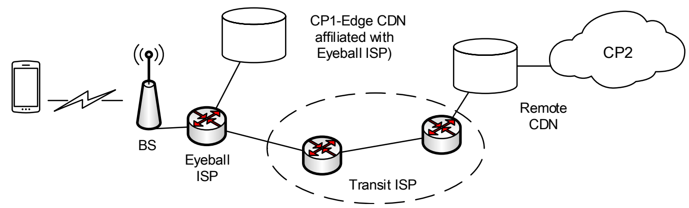

Fig. 2. local server and remote server

This paper is organized as follows. Section II, we detail our model and simplifications. In Section III, we derive Nash and Stackelberg equilibria. Numerical results are given in Section IV. In Section V, we discuss queueing based models that are relevant to 4G and future work. Finally, we conclude with a short summary in Section VI.

II. Modeling a non-cooperative game between two CPs

Consider the two CPs shown in Fig. 2: CP1 is affiliated with the cellular ISP and uses an edge CDN, whereas CP2 is not affiliated with ISP and uses a remote CDN. Let π_i be the user-subscription price per unit throughput and S_i be the subscriber pool for CPi, where $S = S_1 \cup S_2$ is the set of all subscribers. Other possible CP pricing schemes involve a fixed price per video object (irrespective of its size or length) or a flat subscription rate (irrespective of the amount of media downloaded). We herein chose a price per unit throughput for CP subscribers as that may more directly relate to the CP's own operational expenditures. The net utility of each CPi,

$$U_i = \sum_{k \in S_i} (\pi_i \theta_k - c_{p,i} p_k) \qquad (1)$$

is to be maximized subject to

$$\theta_k \le B \log_2(1 + \mathsf{SINR}_k) \qquad (2)$$

$$\sum_{k \in S} p_k \le p_{\max} \quad \text{and} \quad p_k \le p_{\max,k}, \qquad (3)$$

where

- θ_k is subscriber k's transmission rate (throughput) bounded by the Shannon channel capacity constraint (2), SINR_k is their signal-to-interference-plus-noise ratio, and B is the total channel bandwidth.
- p_k is subscriber k's transmission power which should satisfy the power constraints (3), $p_{\max}$ is the total power of the base station of the cell under consideration, $p_{\max,k}$

is maximum power for each user, and $c_{p,i}$ per-unit power cost paid by CPi.

- Since CP1 is the eyeball ISP's subsidiary, take the effective costs per-unit-power for CP2 as

$$c_{p,2} \ = \ c_{p,1} + \phi \ > \ c_{p,1}, \tag{4}$$

where $\phi > 0$ is effectively a side-payment from CP2 to the ISP.

We assume subscriber k's throughput demand response to price is linear,

$$\theta_k \ = \ (\theta_{\max,k} - \delta\pi_i)^+, \quad \text{when } k \in S_i, \tag{5}$$

where $\theta_{\max,k}$ is the subscriber k's maximum throughput demand, δ is their price sensitivity parameter, and $(\cdot)^+ \equiv \max(\cdot, 0)$.

To further simplify the model for tractibility, we replace the set of users S_i with a single equivalent user so that the net-utilities (1) can be written as

$$U_i = s_i \overline{u}_i \tag{6}$$

where $s_i = |S_i|$ is the number of CPi's users (in S_i). Assume the fractions of subscribers divided is inversely proportional to subscription price with competition parameter b (e.g., [8]),

$$s_i = \frac{\pi_{3-i}^b}{\pi_1^b + \pi_2^b} s \quad \text{and} \quad s = s_1 + s_2 = |S|, \tag{7}$$

So, all subscribers go to the lowest-priced provider as $b \to \infty$. Let $\overline{u}_i$ be the net-utility of CPi per subscriber,

$$\overline{u}_i = \pi_i \overline{\theta}_i - c_{p,i} \overline{p}_i \tag{8}$$

where $\overline{\theta}_i$ is the average throughput, and $\overline{p}_i$ is the average transmission power of a subscriber of CPi, so that the total power constraint of (3) becomes

$$\sum_{i \in \{1,2\}} s_i \overline{p}_i \leq p_{\max}. \tag{9}$$

Assume $\overline{\theta}_i = B \log_2(1 + \overline{\mathsf{SINR}}_i)$ in (2), where $\overline{\mathsf{SINR}}_i$ is a kind of "mean-field" approximation of signal-to-interference-plus-noise ratio (SINR), i.e., for a single downlink (e.g., streaming video) channel of a single cell under CDMA,

$$\overline{\mathsf{SINR}}_i = \frac{\overline{p}_i}{\varepsilon_i \overline{\left(\frac{N}{\alpha}\right)}_i + g(s_{3-i}\overline{p}_{3-i} + (s_i - 1)\overline{p}_i)} \tag{10}$$

where α is the attenuation from the transmitter to the receiver, N is the noise power (including shadowing effects), g is a constant orthogonality factor in CDMA, and ε_i models the delay jitter through the network such that

$$\varepsilon_i \begin{cases} = 1 & \text{CP1} \\ > 1 & \text{CP2} \end{cases} \tag{11}$$

That is, CP2's packets experience more delay jitter which may result in more underflow of the (client-side) playout buffer, and thus reduced throughput - we capture this effect by an equivalent increase in noise levels in the physical channel of CP's subscribers.

Remark: In summary, compared to CP1 which is vertically integrated with the cellular ISP, the disadvantages of CP2 are manifest in the side-payment $\phi > 0$ in (4) and service-quality degradation modeled using parameter $\varepsilon_2 > 1$. Such simple (parsimoniously parameterized) models of complex economic systems are important to government regulators attempting to fairly regulate them (e.g., set guidelines for direct or effective side-payment ϕ) while considering the interests of the disputing parties (ISP/CP1 v. CP2) and the welfare of the subscribers, *cf.*, Sections III-B and IV.

So, under linear demand-response assuming the Shannon capacity is met,

$$\overline{\theta}_i = \ B \log_2(1 + \overline{\mathsf{SINR}}_i) = (\theta_{\max,i} - \delta\pi_i)^+ \tag{12}$$

$$\Rightarrow \quad \overline{\mathsf{SINR}}_i = 2^{\frac{(\theta_{\max,i} - \delta\pi_i)^+}{B}} - 1 \ =: \Phi(\pi_i) \tag{13}$$

Also let $\mathcal{N}_i = \varepsilon_i \overline{\left(\frac{N}{\alpha}\right)}_i > 0$. Substituting (10) into (13) gives

$$\begin{cases} \overline{p}_1 = \dfrac{\Phi(\pi_1)\,(\mathcal{N}_1 + g s_2 \overline{p}_2)}{1 - g\Phi(\pi_1)(s_1 - 1)} \\ \overline{p}_2 = \dfrac{\Phi(\pi_2)\,(\mathcal{N}_2 + g s_1 \overline{p}_1)}{1 - g\Phi(\pi_2)(s_2 - 1)} \end{cases} \tag{14}$$

To further simplify our model toward a close-form Nash Equilibrium, assume $(\theta_{\max,i} - \delta\pi_i)/B$ in (13) is small and B large so that,

$$2^{\frac{(\theta_{\max,i} - \delta\pi_i)^+}{B}} - 1 \approx (\frac{(\theta_{\max,i} - \delta\pi_i)^+}{B}) \ln 2. \tag{15}$$

Also consider the channel in a congested state so that the interference power equals $p_{\max} - \overline{p}_i$ and that for all i, $\overline{p}_i \ll p_{\max}$, so that (10) becomes

$$\overline{\mathsf{SINR}}_i = \frac{\overline{p}_i}{\mathcal{N}_i + g p_{\max}}. \tag{16}$$

Thus (13) can be written as

$$\frac{\overline{p}_i}{\mathcal{N}_i + g p_{\max}} = (\frac{(\theta_{\max,i} - \delta\pi_i)^+}{B}) \ln 2 \tag{17}$$

and the average transmission power per subscriber of CPi is

$$\overline{p}_i(\pi_i) = \frac{\ln 2}{B} (\mathcal{N}_i + g p_{\max})(\theta_{\max,i} - \delta\pi_i)^+ \tag{18}$$

In the following, we take $b = 1$ in the subscriber competition model (7) (so that not all users go to the CP with the lowest subscription price π).

To simply the notation in the following, let

$$\mathcal{F}_i = c_{p,i} \frac{\ln 2}{B} (\mathcal{N}_i + g p_{\max}) \ > 0 \tag{19}$$

To summarize, by (6), (7), (12) and (18), the utility of CPi is approximated as

$$U_i = \frac{\pi_{3-i} s}{\pi_1 + \pi_2} (\theta_{\max} - \delta\pi_i)(\pi_i - \mathcal{F}_i) \tag{20}$$

where

$$\forall i, \ \mathcal{F}_i \ \leq \ \pi_i \ \leq \ \theta_{\max,i}/\delta. \tag{21}$$

III. Nash and Stackelberg Equilibria

A. Nash equilibrium

Proposition 1. *For a non-cooperative game among the CPs with net utilities (20), there exists a unique interior Nash equilibrium that is stable under best-response dynamics.*

Proof. One can show that, $\forall i,\ \frac{\partial^2 U_1}{\partial \pi_1^2}, \frac{\partial^2 U_2}{\partial \pi_2^2} < 0$, i.e., U_i is a strictly concave function of π_i. Let us denote the best response functions by π_1^* and π_2^* for CP1 and CP2 respectively. We find that

$$\pi_1^*(\pi_2) = -\pi_2 + \sqrt{(\pi_2 + \mathcal{F}_1)(\pi_2 + \frac{\theta_{\max,1}}{\delta})} \qquad (22)$$

$$\pi_2^*(\pi_1) = -\pi_1 + \sqrt{(\pi_1 + \mathcal{F}_2)(\pi_1 + \frac{\theta_{\max,2}}{\delta})} \qquad (23)$$

from the first order necessary condition

$$\frac{\partial U_1}{\partial \pi_1} = 0 = \frac{\partial U_2}{\partial \pi_2}. \qquad (24)$$

By direct calculation, one can show that these best responses are strictly concave. Thus,

$$(\pi_i^*)^{-1}(\pi_i) = \frac{\pi_i^2 - \mathcal{F}_i\theta_{\max,i}/\delta}{\mathcal{F}_i + \theta_{\max,i}/\delta - 2\pi_i} \qquad (25)$$

is convex, and

$$f_2(\pi_2) := \pi_1^*(\pi_2) - (\pi_2^*)^{-1}(\pi_2) \qquad (26)$$

is concave. The latter is subject to

$$\sqrt{\mathcal{F}_i\theta_{\max,i}/\delta} \le \pi_i \le \tfrac{1}{2}(\mathcal{F}_i + \theta_{\max,i}/\delta) \qquad (27)$$

i.e., between the (tighter) geometric and arithmetic means of the limits of (21). Additionally, $f_2(\sqrt{\mathcal{F}_2\theta_{\max,2}/\delta}) > 0$, $f_2'(\sqrt{\mathcal{F}_2\theta_{\max,2}/\delta}) > 0$, and $\lim_{\pi_2 \uparrow (\mathcal{F}_2+\theta_{\max,2}/\delta)/2} f_2(\pi_2) = -\infty$. Thus, f_2 has a unique root. Let us denote the root by $\hat{\pi}_2$. Then

$$(\hat{\pi}_1, \hat{\pi}_2) = (\pi_1^*(\hat{\pi}_2), \hat{\pi}_2)$$

is the unique Nash equilibrium of our two player game (i.e., where the two best-response curves meet). Concavity also gives stability of the equilibrium under best-response dynamics: $(\pi_1, \pi_2) \to (\pi_1^*(\pi_2), \pi_2^*(\pi_1))$, i.e., convergence to the Nash equilibrium $(\hat{\pi}_1, \hat{\pi}_2)$ beginning from any feasible initial play-action (π_1, π_2).

□

Remarks regarding the proof: U_1 and U_2 are concave in π_1 and π_2 respectively, and so a Nash equilibrium exists [19]. Also, the unconstrained best-responses $\pi_1^*(\pi_2)$ and $\pi_2^*(\pi_1)$ are "standard" functions that satisfy positivity ($\pi_i^*(\pi_{3-i}) > 0$), monotonicity ($\pi_{3-i} \ge \pi'_{3-i} \Rightarrow \pi_i^*(\pi_{3-i}) > \pi_i^*(\pi'_{3-i})$), and scalability ($\forall \mu > 1$, $\mu\pi_i^*(\pi_{3-i}) > \pi_i^*(\mu\pi_{3-i})$). Thus, the Nash equilibrium is unique for this model (as [21]).

B. Stackelberg game

Consider a 2-step Stackelberg game with the regulator (the leader) and two CPs (followers), determining own strategy sequentially.

In the first step, when $c_{p,1}$ is fixed, the regulator might determine the per-unit transmission power side-payment ϕ from CP2 to ISP/CP1 to optimize the social welfare objective function Ω. After observing ϕ chosen by the regulator, the CP followers decide their responses to subsequently maximize their net utilities. We have proved there is a unique Nash equilibrium $(\hat{\pi}_1, \hat{\pi}_2)$ which depends in particular on lumped parameter $\mathcal{F}_2(\phi)$. According to (4) and (19), when we set ϕ as a variable and all other parameters are given, each ϕ can map to a unique Nash equilibrium, that is $(\hat{\pi}_1(\phi), \hat{\pi}_2(\phi))$.

For this leader-followers game, firstly we solve the second step that followers determine their strategies given what the leader has chosen in the first step, thereafter the first step with the supposition that the leader determine the side-payment with the knowledge that followers will react to it according to their best response functions in the second step (i.e., followers' responses and their utilities could be regarded as functions in terms of ϕ for the regulator).

Considering both customers and companies, the socioeconomic cost function can be modeled as

$$\begin{aligned}\Omega(\phi) \ &= \frac{s_1}{s_1+s_2}\hat{\pi}_1(\phi) + \frac{s_2}{s_1+s_2}\hat{\pi}_2(\phi) - \xi(\beta\hat{U}_1(\hat{\pi}_1(\phi), \hat{\pi}_2(\phi)) \\ &\quad + (1-\beta)\hat{U}_2(\hat{\pi}_1(\phi), \hat{\pi}_2(\phi), \phi)),\end{aligned} \qquad (28)$$

where $\hat{U}_1(\hat{\pi}_1(\phi), \hat{\pi}_2(\phi))$ and $\hat{U}_2(\hat{\pi}_1(\phi), \hat{\pi}_2(\phi), \phi)$ are net utilities under the Nash equilibrium for CP1 and CP2 respectively. $\frac{s_1}{s_1+s_2}\hat{\pi}_1(\phi) + \frac{s_2}{s_1+s_2}\hat{\pi}_2(\phi)$ is the average price paid by all subscribers. ξ weights the providers' net revenues against the subscribers' costs, and β indicates a preference between CP1 and CP2.

These parameters may be set by a regulator/leader in crafting their objective Ω to determine the side payment $\phi > 0$ from CP2 to ISP/CP1. The Stackelberg equilibrium is $(\hat{\pi}_1(\hat{\phi}), \hat{\pi}_2(\hat{\phi}), \hat{\phi})$ where $\hat{\phi}$ minimizes Ω subject to (9) and (27) .

IV. Numerical results

In the following section, we present a typical set of numerical results for our simplified model; specifically, for specified $\phi > 0$ and $\varepsilon_2 > 1$ determined via the $\mathcal{N}$ lumped parameters. We consider cases where the constraints allow for a unique interior Nash equilibrium or where there are plural Nash equilibria on the constraint boundaries. Finally, we give the Stackelberg equilibria for these cases.

A. Nash equilibrium

Consider the example with $\mathcal{F}_1 = 2$, $\mathcal{F}_2 = 5$, $c_{p,1} = 1$, $\phi = c_{p,2} - c_{p,1} = 0.5$, $p_{\max} = 9000$, $\theta_{\max} = 5$, and $\delta = 0.05$. From (19), we have $\mathcal{N}_2 = \frac{5}{3}\mathcal{N}_1 + 6000g$ which satisfies $\mathcal{N}_2 > \mathcal{N}_1$ when $\varepsilon_2 > 1 \equiv \varepsilon_1$ as (11). For example, if $\mathcal{N}_1 = 90$, $\mathcal{N}_2 = 270$ and $g = 0.02$, then $B = 135 \ln 2$ so that assumption (15) holds (RHS ≤ 0.05).

Fig. 3 shows constrained best-response curves bounded by asymptotes (27). We numerically verify the existence and uniqueness of Nash equilibrium that two best-response curves have only one intersection point which here is in the feasible area.

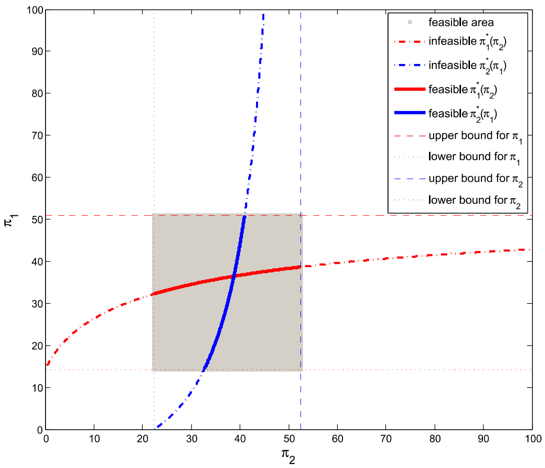

Fig. 3. Constrained best-response curves for $\varepsilon_2 > 1 \equiv \varepsilon_1$, $\phi = 0.5$, and otherwise the same parameters for the two players CP1/ISP and CP2, under constraints (27) but not under power constraint (9)

Furthermore, taking the power constraint (9) into account when $p_{\max} = 9000$, Fig. 4 shows how part of the constrained best-response curves in Fig. 3 are out of the contracted feasible area which is now bounded by the asymptotes (27) and the power constraint boundary (9). Since U_i is strictly concave in π_i, the power constraint boundary is the suboptimal response curve for the infeasible part. The figure shows there exits a unique feasible Nash equilibrium where the best-response curves meet.

Reducing the value of $p_{\max} = 7500$ in (9), congestion ensues at Nash equilibrium. The intersection point of unconstrained best-response curves is now outside of the feasible area in Fig. 5. In this case, the two constrained best-response curves overlap (bold solid green line in the figure), with all overlapping points being (boundary) Nash equilibria. This is a Nash Bargaining scenario where the regulator/leader may use a socioeconomic objective to decide on one Nash equilibrium.

B. Minimal socioeconomic cost without constraints

For the Stackelberg game, we considered the side-payments ϕ in the range from 0 to 5 (i.e., $c_{p,2} \in [1, 6]$ when $c_{p,1} = 1$), and all other parameters are the same as the previous section.

Initially, we found both components of an instance of the objective Ω given by (28), the mean subscriber costs $\frac{s_1\hat{\pi}_1 + s_2\hat{\pi}_2}{s_1+s_2}$ and the total net losses of the CPs $-(\hat{U}_1 + \hat{U}_2)$, are monotonically increasing with $\phi \in [0, 5]$. Figures 6 and 7 illustrate how $\beta\hat{U}_1 + (1-\beta)\hat{U}_2$ is

- unimodal for $\beta \in (0.66, 0.71)$, so that optimal $\hat{\phi}$ is such that $0 < \hat{\phi} < 5$, and
- monotonic otherwise, so that $\hat{\phi} = 0$ or $\hat{\phi} = 5$.

For $\beta = 0.69$, Fig. 8 shows the Stackelberg equilibrium $\hat{\phi}$ which minimizes the cost-objective Ω in (28) for different weight parameters $\xi \in [0, 1]$. A side-payment $\phi > 0$ corresponds to $\xi > 0.01$. When $\xi > 0.57$, Ω prefers the two CPs' revenues over achieving a balance between subscribers' costs and the CPs' revenues. So an "effective" range for ξ is $(0.01, 0.57)$ for our example. If $\xi = 0.1$, then (28) becomes

$$\Omega(\phi) = \frac{s_1\hat{\pi}_1(\phi) + s_2\hat{\pi}_2(\phi)}{s_1 + s_2} - 0.1(0.69\hat{U}_1(\phi) + 0.31\hat{U}_2(\phi)) \tag{29}$$

which can be used to select a unique Nash equilibrium under the Nash Bargaining scenario for the regulator/leader.

C. Stackelberg equilibrium

For objective (29) and associated parameters, we numerically verified that each $\phi \in [0, 5]$ corresponds to a unique Nash equilibrium, see Fig. 9. Furthermore, under the power constraint (9) with $p_{\max} = 9000$ and asymptotes (27), projections of $(\hat{\pi}_1(\phi), \hat{\pi}_2(\phi), \phi)$ on the π_1-π_2 plane (the latter being the Nash equilibria) remain strictly within the feasibility area, see the corresponding Fig. 10. In this case, the unique, interior Stackelberg equilibrium is

$$(\hat{\pi}_1(\hat{\phi}), \hat{\pi}_2(\hat{\phi}), \hat{\phi}) = (37.27, 43.28, 2).$$

Moreover, at Stackelberg equilibrium,

- the average subscriber prices (the first term of (29) is 40.05,
- $(\hat{U}_1(\hat{\pi}_1(\hat{\phi}), \hat{\pi}_2(\hat{\phi})), \quad \hat{U}_2(\hat{\pi}_1(\hat{\phi}), \hat{\pi}_2(\hat{\phi}), \hat{\phi})) = (59441, 43668)$, and
- among 1000 subscribers, 537 users subscribe to CP1 and the other 463 subscribe to CP2.

Note that at Stackelberg equilibrium, CP2 must charge more than CP1 (to accommodate its side payment ϕ), but nevertheless retains subscribers according to the competition model (7) which does not assume all end-users subscribers to the cheapest CP (e.g., owing to subscriber inertia). CP2 may initially (i.e., as a market entrant) engage in "noneconomic" pricing to try to acquire subscribers before eventually increasing its prices to Nash equilibrium to become profitable, e.g., [8].

Considering a reduced $p_{\max} = 7500$, the projections of unconstrained $(\hat{\pi}_1(\phi), \hat{\pi}_2(\phi), \phi)$ on π_1-π_2 plane are given in Fig. 11 and Fig. 12, where some of them are infeasible. The feasible, boundary $(\hat{\pi}_1(\phi), \hat{\pi}_2(\phi), \phi)$ corresponding to a Nash bargaining scenario where a particular Nash equilibrium among plural can be determined by minimizing an objective Ω as in (29), otherwise there is a unique interior Nash equilibrium corresponding to each $\phi > 0$.

More specifically, using Ω (29) for this case, we get the unique Stackelberg equilibrium is at side-payment $\hat{\phi} = 0$:

$$(\hat{\pi}_1(\hat{\phi}), \hat{\pi}_2(\hat{\phi}), \hat{\phi}) = (40.29, 44.80, 0),$$

and

- the average subscriber prices (the first term of (29) is 42.42,

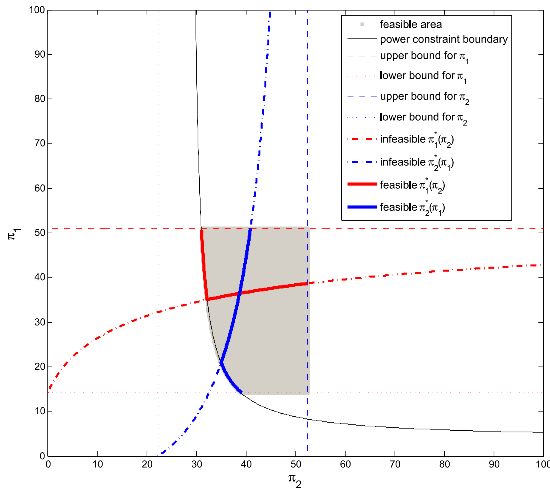

Fig. 4. Constrained best-response curves for $\varepsilon_2 > 1 \equiv \varepsilon_1$, $\phi = 0.5$, and otherwise the same parameters for the two players CP1/ISP and CP2, under constraints (27) and under power constraint (9) with $p_{\max} = 9000$ - note unique interior Nash equilibria

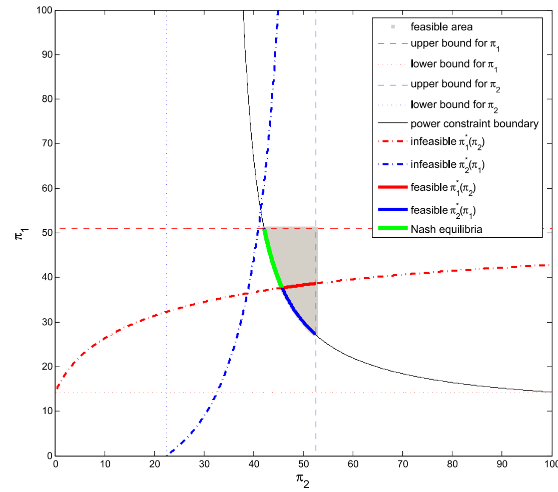

Fig. 5. Constrained best-response curves for $\varepsilon_2 > 1 \equiv \varepsilon_1$, $\phi = 0.5$, and otherwise the same parameters for the two players CP1/ISP and CP2, under constraints (27) and under power constraint (9) with $p_{\max} = 7500$ - note non-unique boundary Nash equilibria

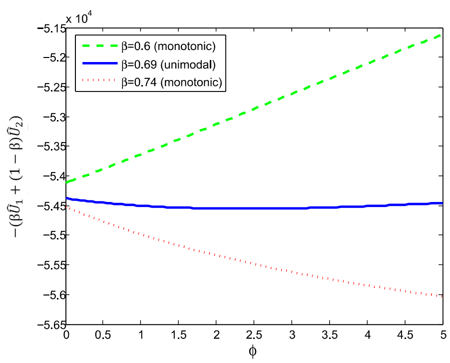

Fig. 6. $-(\beta\hat{U}_1 + (1-\beta)\hat{U}_2)$ versus ϕ for different weights β illustrating monotonic and unimodal cases, for $\varepsilon_2 > 1 \equiv \varepsilon_1$, and otherwise the same parameters for the two players CP1/ISP and CP2

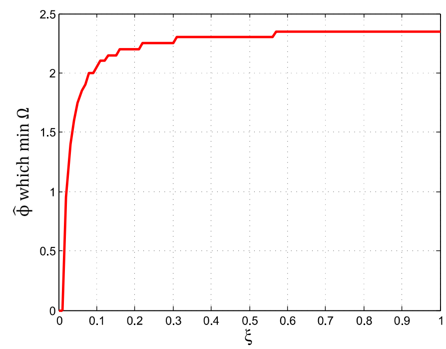

Fig. 8. Optimal side-payment $\hat{\phi}$ as a function of Ω-objective parameter ξ weighting CP revenues averaged using $\beta = 0.69$ - found by direct grid search where the ripple in the curve is due to the coarseness of the grid

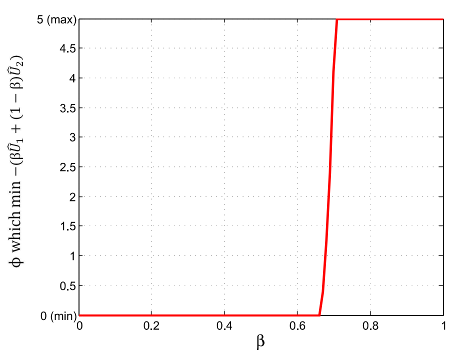

Fig. 7. Optimal side-payment $\hat{\phi}$ maximizing weighted-averaged CP revenue $\beta\hat{U}_1 + (1-\beta)\hat{U}_2$ versus β, for $\varepsilon_2 > 1 \equiv \varepsilon_1$, and otherwise the same parameters for the two players CP1/ISP and CP2

- $(\hat{U}_1(\hat{\pi}_1(\hat{\phi}), \hat{\pi}_2(\hat{\phi})), \quad \hat{U}_2(\hat{\pi}_1(\hat{\phi}), \hat{\pi}_2(\hat{\phi}), \hat{\phi})) = (60187, 54189)$, and
- among 1000 subscribers, 527 users subscribe to CP1, and the other 473 subscribe to CP2.

Interestingly here, the Stackelberg equilibrium side-payment $\hat{\phi} = 0$ - note that a component of the cost function Ω $(-U_2)$ increases with ϕ. Also note that the average subscriber price is higher owing to the constrained power resource. Finally, note that the CP2's price $\hat{\pi}_2$ remains higher than that of CP1 (even though the side-payment is zero) because CP2 still needs relatively more power to overcome its relatively higher noise factor $\varepsilon_2 > 1 \equiv \varepsilon_1$.

V. Discussion: Queueing Based Models and Future Work

In this section, we describe queueing based models of CP utility that are relevant to 4G LTE. Data-traffic multiplexing into a single channel is suggested in, e.g., [1] for machine-to-machine communication, i.e., a plurality of very short, low-volume sessions that individually may not warrant an entire channel. Consider two CPs sharing a transmission queue to access a physical wireless channel, where CP2's traffic suffers additional delay jitter as it traverses the Internet. CPi's throughput is θ_i and the service rate of the access queue (corresponding to the access bandwidth of the physical channel) is μ_a. If the total throughput is $\lambda = \theta_1 + \theta_2$,

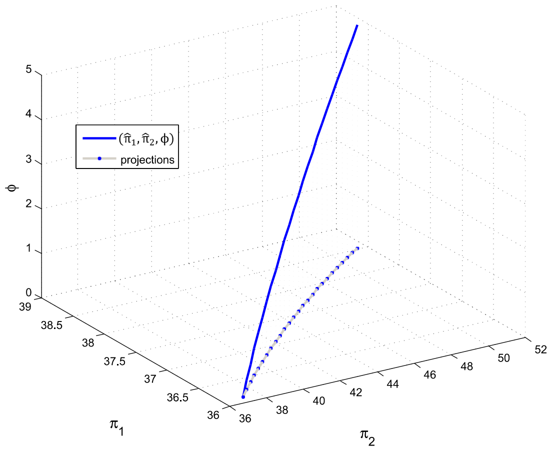

Fig. 9. $(\hat{\pi}_1(\phi), \hat{\pi}_2(\phi), \phi)$ projections on π_1-π_2 plane with maximum power $p_{\max} = 9000$ - unique interior Nash equilibrium for each ϕ

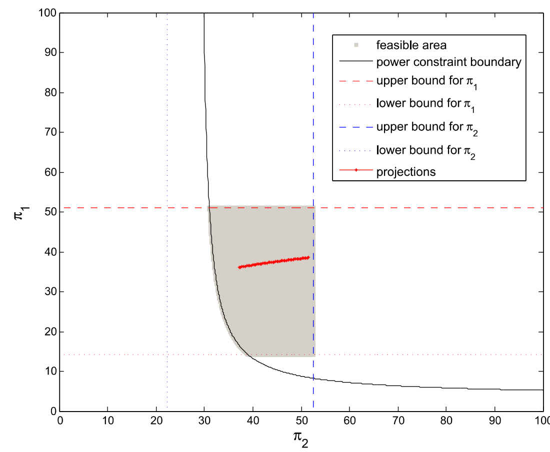

Fig. 10. $(\hat{\pi}_1(\phi), \hat{\pi}_2(\phi), \phi)$ projection onto the π_1-π_2 plane with maximum power $p_{\max} = 9000$ - unique interior Nash equilibrium for each ϕ

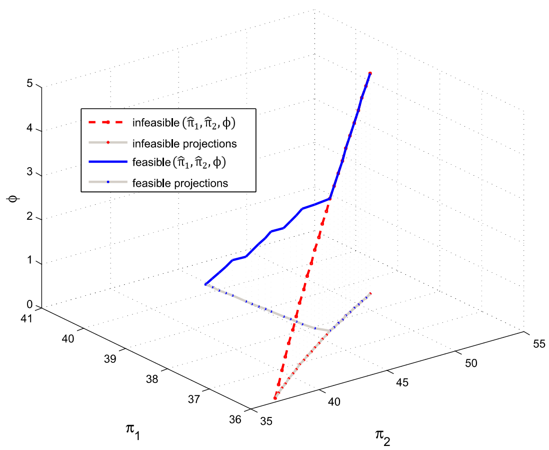

Fig. 11. $(\hat{\pi}_1(\phi), \hat{\pi}_2(\phi), \phi)$ projections on the π_1-π_2 plane with reduced maximum power $p_{\max} = 7500$ - plural boundary Nash equilibria for low ϕ when power bound is tight (congested regime)

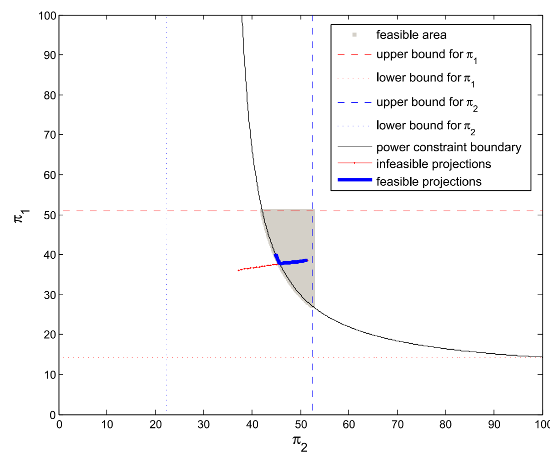

Fig. 12. $(\hat{\pi}_1(\phi), \hat{\pi}_2(\phi), \phi)$ projections on the π_1-π_2 plane with reduced maximum power $p_{\max} = 7500$ - plural boundary Nash equilibria for low ϕ when power-bound is tight(congested regime)

the PDF of the sojourn time through a M/M/1 queue is exponential, $f(t) = (\mu_a - \lambda)\mathrm{e}^{t(\mu_a - \lambda)}u(t)$ where u is the unit step function. So, the variance of delay through the M/M/1 queue is $(\mu_a - \lambda)^{-2}$. By PASTA, both CPs experience the same delay variance. We can simply model the utility of CPi as the throughput penalized by the standard-deviation of delay jitter,

$$U_i = \pi_i\theta_i - \gamma_i\sqrt{\frac{1}{(\mu_a - \theta_1 + \theta_2)^2} + \frac{\varepsilon_i}{(\mu_n - \theta_2)^2}} - c_i\theta_i$$

where: the net side-payment $\phi = c_2 - c_1 > 0$ is such that $c_1 < \min\{\pi_2 - \phi, \pi_1\}$ as above but here in terms of dollars per unit aggregate throughput, $\gamma_i > 0$ is a jitter penalty parameter, μ_n corresponds to the bottleneck or effective *network* bandwidth experienced by CP2, and $\varepsilon_1 = 0$ but $\varepsilon_2 = 1$, the latter to account for additional delay jitter of CP2's traffic acquired by transmission across the Internet. For stability of the queues, we require

$$\theta_1 + \theta_2 < \mu_a \quad \text{and} \quad \theta_2 < \mu_n \tag{30}$$

The Stackelberg equilibrium of this constrained system can be analyzed as above for CDMA using a linear demand-response model (5).

An even simpler model is formulated when one considers CPs that do not share channels but each employ multiple channels simultaneously. That is, the utilities $U_1 = (\pi_1 - c_1)\theta_1$ and $U_2 = (\pi_2 - c_2)\theta_2 - \gamma_2/(\mu_n - \theta_2)$ are maximized under (30) where here μ_a is the total cellular wireless capacity (or number of channels).

In the future, we will explore other simple model variations where, e.g., there is more than one (different) CP that is not vertically integrated with the ISP, or there are QoS tiers, e.g., by a priority or processor-sharing access queue. We will also explore multi-period CP competition models, e.g., as [8], particularly for CPs that are entrants to the marketplace.

VI. Summary

We modeled a non-cooperative game between affiliated (vertically integrated with ISP) and unaffiliated (remote) CPs with a cellular wireless ISP using CDMA, considering increased network delay jitter and effects of reduced responsiveness (owing to increased RTT) modeled as amplified noise, and considering an additional side-payment for the unaffiliated CP. For our simplified model without power constraints, we proved the existence and uniqueness of a Nash equilibrium. With different power constraints, numerical results show the unique Nash equilibrium or a Nash Bargaining scenario with plural boundary equilibria. Finally, a fair side-payment from unaffiliated CP to ISP is set by the regulator via a Stackelberg game to minimize a socioeconomic cost function, again considering different power constraints.

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