Comfort-Aware Home Energy Management Under Market-Based Demand-Response

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***Abstract*—To regulate energy consumption and enable Demand-Response programs, effective demand-side management at home is key and an integral part of the future Smart Grid. In essence, the home energy management is a mix between discrete appliance scheduling problem with deadlines and continuous Heating, Ventilation and Cooling (HVC) device control problem. In this paper, we present near-optimal algorithm designs for energy management at home that is incentive-compatible with market-based Demand-Response programs under explicit user comfort constraints. Theoretical analysis aside, we also show the effectiveness of our algorithms through simulation studies based on real energy pricing and consumption data in South Korea.**

***Index Terms*—smart grid, demand-response, energy management**

I. Introduction

Around the globe today the increasing rate of energy demand far out-strides the growth of energy production capability. A consequence of this discrepancy is the noticeable rise in electricity price over the past decade and the increasingly frequent power curtailment and blackouts during peak demand. Effective demand-side energy management is an integral part of the Smart Grid, and home energy consumption accounts for more than one third of the total energy consumption in the US alone. At the edge of the Smart Grid infrastructure, a home energy management solution should address the dual issues of: effective energy management for the users and serving as the enabler of Demand-Response (DR) programs.

Demand-Response programs have been active in the US since 1999. It played a major role in mitigating electrical system emergencies in several regions in the US during summer 2001 [1]. A key objective of DR programs is to achieve **peak shaving**. Peak demand introduces unpredictability to energy management and is a major contributor of electric grid faults. Moreover, large number of backup power plants (nearly 20% of total plants [1]) have to be maintained to meet peak demand. DR programs belong to two general categories: peak capping and market-based pricing. In peak capping, each home is allocated an energy quota. In market-based pricing, the price of energy varies based on market supply-demand. Market-based DR program is advocated as the more economically efficient and long-term viable way of regulating energy consumptions. We work with day-ahead market pricing in this paper as it is the likely DR regime in the near future. Indeed studies [2][3] show that households adjust their demand in response to DR prices and this in turn curbs the long-term market-clearing prices to equilibrium level. However survey [4] conducted in the state of New York in the US told a rather startling story: 54% of the home owners reported that they were unable to curtail their electricity usage. Key detriments identified in the studies are:

- **Lack of smart planning**: the consumers do not have the necessary technical skills to monitor hourly electricity price and are not aware of the detailed breakdowns of their power consumption.
- **Concerns over discomfort**: the consumers are unwilling to change living habits.
- **Customers are risk-averse**: the customers do not trust DR programs in fear of unpredictable prices and unexpected power outages.

Therefore, we believe it is imperative for the demand-side management to address these user concerns. At POSTECH, we built a Smart Home (Figure 1) equipped with energy monitors and power line communication (PLC) based controllers. Energy scheduling and appliance control are automatically performed by the Grid Home Service (GHS) which used a discrete appliance scheduling algorithm for reducing aggregate energy cost [5]. In this paper, we take a more comprehensive treatment to the home energy management problem by formulating home energy management as a mixed problem between discrete appliance scheduling with deadlines and continuous Heating, Ventilation and Cooling (HVC) device control, under market-based DR programs. We establish a minmax over cost objective as an effective enabler of global energy peak shaving and is incentive-compatible with the households. We use electricity as the case study and the terms *electricity* and *energy* are used interchangeably. Our contributions are as follows: first, we establish our minmax over cost objective and show that it is incentive-compatible, peak shaving and risk reducing. Second, we present the $minMax$ algorithm as

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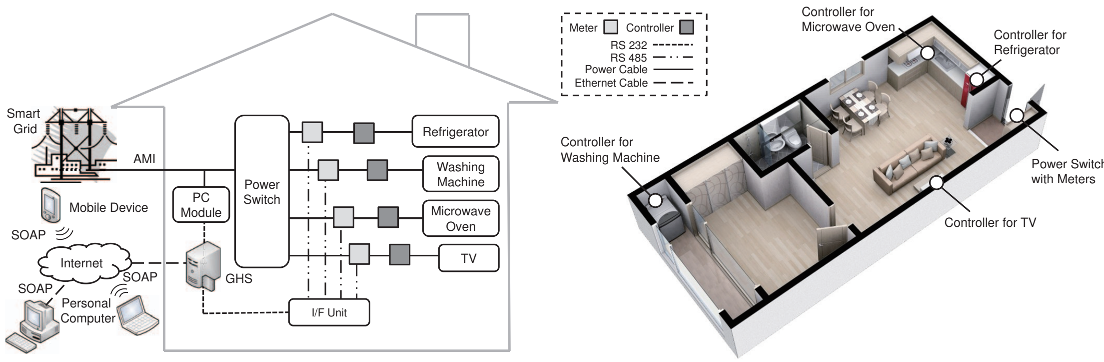

Fig. 1. Grid-Home Service Deployment and Test House at POSTECH

a near-optimal solution for the NP-hard appliance scheduling problem. Thirdly, we extend our algorithm to cover Heating, Ventilation, and Cooling (HVC) devices that are best modeled as continuous control setting problem. Using A/C as an example, we present the $minMax_{HVC}$ algorithm that combines both discrete appliance scheduling and continuous HVC control into an integrated near-optimal solution. We model user comforts explicitly by placing constraints on both the $minMax$ and the $minMax_{HVC}$ algorithms. Theoretical analysis aside, we also show the performance of our algorithms based on real energy pricing and consumption data in Korea. To the best of our knowledge, few work in literature examines the home energy management problem as a mixed problem between discrete appliance scheduling and continuous HVC control under explicit user comfort constraints; and we are one of the first to investigate the relation between user comfort and energy peak shaving.

The remainder of the paper is organized as follows: Section II presents related works, and Section III formulates the home energy management problem and presents the $minMax$ algorithm. We incorporate HVC devices in Section IV and present the $minMax_{HVC}$ algorithms. Section V reports on our simulation studies. and we conclude in Section VI.

II. Related work

Optimization techniques have been applied to solve energy management problems. For instance, the work in [6] focuses on the supplier aspect of power generation, noting that the use of renewable energy from microgrids (e.g. local wind and solar generators) can reduce the reliance on wholesale power, but the power quality must be controlled. Works on optimizing domestic energy usage [7][8] focus on the optimal on/off switching of thermal appliances such that energy consumption is minimized while a consistent level of comfort is maintained. Others such as [9] propose energy cost optimization on the demand and supply market agent-based auctioning. They also study consumer-side energy optimization via time shifting but do not guarantee deadlines. The work on energy consumption scheduling [10] focuses on constructing an incentive-based schedule for neighborhood-wise energy usage derived from home-wise energy scheduling. Each user's schedule is assumed to be flexible without deadlines. In their later work [11], they modeled the home energy scheduling problem as a game where the consumption rate of each appliance is a range of real values and reasoned about the goodness of the global state. User comfort and task shifting based on strict deadlines are not modeled and the appropriate price settings to obtain steady state are also not determined. We explicitly address both of these issues in this paper.

We observe that consumer side task schedule is often accompanied by hard deadlines and requires task atomicity. Furthermore, once a task is committed it cannot be aborted in the middle. The particularities of these requirements favor the optimization of demand-side energy consumption at the home level as a scheduling problem. Few works focus on this aspect of task scheduling as defined by classical scheduling problems such as Longest Process Time (LPT) and Just-In Time (JIT) known to be NP-complete [12]. Work such as [13] attempts to provide an approximation algorithm for this problem, but the proposed solution does not account for variability in task lengths or the variability in electricity prices.

To date, some issues in home energy management have been studied. Hobby et. al. [14] investigated the electricity consumption patterns in residential area. They find that home energy consumption can be segregated into appliance and HVC/lighting consumptions. Pedrasa et. al. [15] modeled the home scheduling problem as a decision making process where utility maximization is the objective. A heuristic is developed based on particle swarm. Conejo et. al. [16] postulated the demand-side energy scheduling as a utility maximization problem under fixed daily energy limit. Linear programming algorithms are proposed to determine the best distribution. Barbato et. al. [17] proposed an optimization model for dealing with appliance scheduling in household with battery buffering. They follow a total cost reduction model. Since energy consumption is modeled as a generic splitable value, there is no concept of atomic tasks. Du and Lu [18] addressed the home water heater control as a two-step scheduling process. They considered user comfort explicitly. Our HVC model has a more general and aggregate form so as to fit into an integrated solution with our discrete appliance scheduling.

III. Algorithm for Home Energy Management

A. Problem formulation

We consider the scheduling horizon of a home (e.g., 1 day) to be divided into n time slots wherein each time slot can have different energy price (e.g., 1 hour). Household's appliance requirements can be represented tasks to be scheduled into these slots. Some of the tasks are persistent, as they consume electricity throughout the day (e.g., refrigerator), while others are flexible within a time interval (e.g., washer/dryer). Once a task has been started (e.g., washer or oven), it must be scheduled as a whole and carried out to completion uninterrupted. Thus it is natural to model them as discrete atomic tasks. The problem parameters are summarized in Table I. Given an appliance task i, we describe its demand attributes as $d_i = (s_i, f_i, r_i, l_i) \in D$, $1 \le i \le m$. s_i and f_i are the start and end time constraints of task i respectively, r_i is the expected energy consumption rate (e.g., hourly) of task i and l_i is the time length of task i. We assume r_i is fixed for non-HVC appliances. s_i and f_i are collectively referred to as the *deadline* in that task i must be scheduled within this interval. In this way, we introduce user comfort as explicit constraints in the problem formulation. $(f_i - s_i)/l_i \ge 1$ is the *task shift-ability* factor. Intuitively households with higher task shift-ability can better optimize their energy consumptions. The demand set d_i is either given explicitly by the household as input, or is extrapolated from historical appliance usage patterns. We follow the latter approach in our investigation. Our goal is to optimally assign $d_i \in D$ such that each d_i occupies consecutive time slots $t_j, t_{j+1}, ...t_{j+l_i}$ and the deadline is obeyed (i.e. $j \ge s_i$ and $j + l_i \le f_i$). This results in our discrete appliance task scheduling formulation.

We associate a pricing function $Prc(t_j)$ that varies in time. In this paper, we deal with day-ahead pricing with hourly prices. It is then reasonable to minimize the total energy cost $\sum_{j \in n} Prc(t_j)c_j$ which is obtained if we greedily schedule each task i within deadline such that $\sum_{d_i \to t_k, k=j,...,j+l_i-1} Prc(t_k)r_i$ is minimal. However, two major issues arise: one, the objective of global **peak shaving** is not readily obtainable. This problem can be demonstrated with an example. Since a uniform pricing function does not provide the incentive for households to shift energy consumption patterns, we consider a non-uniform pricing function $Prc()$ computed based on market-clearing price of the demand-supply condition. Let time t_j be the time of peak energy pricing and time t'_j be some off-peak pricing. If we consider the simple case where there are no deadlines for tasks (i.e., tasks are perfectly shift-able), then a solution that minimizes the total energy cost will schedule all of the tasks from time t_j to time t'_j, resulting in a displacement of the energy peak rather than shaving it. We also observe this effect in Section V. Two, minimizing the total energy cost is not **risk averse**. We consider the energy price to be known for the day since we deal with day-ahead pricing. Accordingly we can define risk aversion as the minimization of uncertainty caused by two exterior factors: power outage and unexpected appliance usages. To the households, unexpected power outage results in loss of scheduled work. If we consider the probability of power outage to be perfectly random, then risk aversion can be defined as the minmax of energy scheduled across the time slots, which is energy peak shaving of a home. To the provider, unexpected appliance usages can be the result of household deviating from expected energy consumption behaviour or error in demand forecast. If we consider the probability of unexpected appliance usage to be perfectly uniform, risk aversion can be defined as the minmax of aggregate energy peak. Again, minimizing the total energy cost is not the most suitable optimization objective. Accordingly, we define the appliance scheduling problem as: given a set of task demands $d_i = (s_i, f_i, r_i, l_i) \in D$ where $1 \le i \le m$, find an assignment of each task to time slots: $d_i \to (t_j, t_{j+1}, ...t_{j+l_i-1})$ obeying deadlines $j \ge s_i$ and $j + l_i - 1 \le f_i$, such that,

$$\min_{\{d_i \in D \to t_{j=1..n}\}} \{ \max_{k=1..n} \{ c_k = \sum_{\forall d_i \to t_k} Prc(t_k) r_i \} \}$$

This minmax objective satisfies both the peak shaving and the risk aversion requirements. Since we optimize our objective in the cost domain, our approach is incentive-compatible with the household. The role of DR price setting is then to induce desired demand shaping in the energy domain. We investigate the role of price setting in Section V. If we do not care about the atomicity of a task and assume a uniform $Prc()$, then this problem can be reduced to assigning multiple computing tasks with deadlines to a set of identical processors, which is known to be NP-hard.

TABLE I
Problem Parameters

n	the number of time slots
D	the set of appliance task demands
m	the size of D
d_i	the demand tuple of task i
s_i	the start time constraint of task i
f_i	the end time constraint of task i
r_i	the energy consumption of task i
l_i	the time length of task i
t_j	a time slot with index j
$Prc()$	the price function
$Prc()_{min}$	the minimum price over all time slots
$Prc()_{max}$	the maximum price over all time slots
c_j	total energy consumption at t_j
$Eng()$	the energy consumption function for A/C
HVC_LIM	the operations limit of HVC device (max. ΔT)
$Dip()$	the heat dissipation function
ΔT	change in temperature
T_j	room temperature that start of time slot t_j
$T_{ext,j}$	average outdoor temperature of time slot t_j
T_{tgt}	the target thermostat setting
T_{max}	the household's desired temperature
T_{min}	the household's tolerable temperature

B. Home energy management for appliance tasks

Appliance task scheduling is realized through the $minMax$ algorithm (Algorithm 1) which greedily schedules household appliance tasks according to their deadlines. Initially, the tasks with fixed schedule are scheduled (Line 4). Then, the algorithm greedily schedules tasks in descending energy consumption r_i (major order) and in descending task length l_i

(minor order). For each task, it examines all of the possible time slot assignments for the task within the specified deadline and assigns the task such that the maximum among the cumulative energy cost $c_{1..n}$ is minimized.

Algorithm 1 $minMax$ Scheduling

Input:
$t_k \in T$ where $1 \leq k \leq n$
$d_i = (s_i, f_i, r_i, l_i) \in D$ where $1 \leq i \leq m$
$Prc()$
Output:
Scheduled start time of appliance tasks $sch_i = t_k$ where $1 \leq i \leq m$, $1 \leq k \leq n$
$c_{1..n}$

$sch_{1..n} = 0$
$c_{1..n} = 0$
$tmpC_{1..n} = 0$
For all d_i where $(f_i - s_i)/l_i = 1$: $sch_i = s_i$ and update $c_{1..n}$ accordingly
Sort remaining tasks D^- in descending order using r as the major key and l as the minor key
for each $d_i \in D^-$ **do**
  $minC_i \leftarrow \infty$
  for $j = s_i$ to f_i **do**
    $tmpC_j = c_j + Prc(t_j)r_i$
  end for
  for $k = s_i$ to $f_i - l_i + 1$ **do**
    if $minC_i > max\{tmpC_{k..k+l_i-1}\}$ **then**
      $sch_i = k$
      $minC_i = max\{tmpC_{k..k+l_i-1}\}$
    end if
  end for
  for $j = sch_i$ to $sch_i + l_i - 1$ **do**
    $c_j = c_j + Prc(t_j)r_i$
  end for
end for

The complexity of $minMax$ is $O(nm + mlogm)$ with efficient sorting of the tasks. In the remainder of this section, we show that the $minMax$ algorithm is an approximation algorithm for the appliance schedule problem we have defined in Section III-A. To do so, we first examine a simple version of our problem without deadlines.

THEOREM 1: *Given a task set D without deadlines (i.e., $s_i = 1$ and $f_i = n$), the $minMax$ algorithm produces a solution at most $Prc()_{max}(r_{max})$ more than the optimal solution.*

Considering an optimal schedule of the problem without deadlines, let $\Phi^* = max\{c_{1..n}\}$ denote the highest cumulative cost among the time slots of the optimal schedule. In the simple case where the optimal schedule does not contain more than 1 task per time slot (i.e., no overlapping), it follows that the $minMax$ algorithm can produce the optimal solution. If overlap exists, then we have the following condition:

$$\Phi^* \geq 2Prc()_{min}(r_q), \textit{where } r_q \textit{ is the last task scheduled}$$

We can consider the perfect cost average where the costs are evenly spread across the time slots regardless of their energy consumptions (i.e., perfect peak shaving), thus producing an absolute (and generous) lower bound. The optimal schedule is then lower bounded by the distribution below:

$$\Phi^* \geq \frac{Prc()_{min}}{\sum_{j=1}^{n} \frac{Prc()_{min}}{Prc(t_j)}} \sum_{i=1}^{n} r_i l_i \tag{1}$$

Now, consider the schedule produced by our $minMax$ algorithm, let Φ denote the highest cumulative cost among the time slots. The last task d_k scheduled into the timeslot of Φ must be done when Φ has the least cumulative cost due to the greedy property. Furthermore, we know that the tasks are scheduled in descending order of consumption and therefore the average distribution strictly increases with each additional scheduled task. We arrive at the following inequality:

$$\Phi - Prc(t_\Phi)r_k < \frac{Prc()_{min}}{\sum_{j=1}^{n} \frac{Prc()_{min}}{Prc(t_j)}} \sum_{i=1}^{n} r_i l_i \tag{2}$$

Combining Equations 1 and 2 yields:

$$\Phi < \Phi^* + Prc()_{max}(r_{max})$$

$Prc()_{max}$ is the highest hourly price and r_{max} is the largest energy consuming appliance. Therefore, the $minMax$ algorithm produces a solution at most $Prc()_{max}(r_{max})$ more than the optimal solution.

□

The above bound is rather generous as in practice, the time slot of Φ generally has low hourly price ($\sim Prc()_{min}$). Our simulation study (Section V) shows that the performance of $minMax$ in practice is much better than this upper bound. Next, we analyze our problem with deadline constraints. The proof builds on the results of Theorem 1.

THEOREM 2: *Given an appliance task scheduling problem with deadlines, the $minMax$ algorithm produces a solution that is upper-bounded by $\gamma(\Phi^* + Prc()_{max}r_{max})$ where $\gamma = \frac{max\{Overlap_{1..n}\}}{min_{i=1..m}\{\lfloor (f_i - s_i)/l_i \rfloor\}}$.*

For a time slot t_j, we denote the number of tasks that can be scheduled to this time slot as $Overlap_j$. It is sufficient to examine the behavior of $Overlap_\Phi$ where Φ is the time slot containing the maximum cumulative cost produced by the $minMax$ algorithm.

As each task $d_i \in Overlap_\Phi$ is scheduled from the largest energy consuming task to the least. it is always scheduled to the least cumulative cost time slot constrained by the specific deadline time frame. This produces a constrained sub-problem spanning from time slot s_i to f_i, under which we can bound the marginal loss of optimality by applying Theorem 1. Therefore, each time Φ is raised, a marginal loss occurs on the sub-problem, resulting in the total optimality loss of $\gamma(\Phi^* + Prc()_{max}r_{max})$.

γ is upper bounded by $max\{Overlap_{1..n}\}$, the maximum number of tasks that can be scheduled into a single time slot. However not every task that can, will be scheduled into it. It follows from the greedy property that once a task is scheduled into a time slot, it is guaranteed that the subsequent tasks will not be scheduled into the same time slot until each of the other non-overlapping time slots have been assigned at least one task. The minimum number of non-overlapping time slots is dependent on the task shift-ability factor $\frac{f_i - s_i}{l_i}$. Therefore, γ is upper bounded by:

$$\gamma = \frac{max\{Overlap_{1..n}\}}{min_{i=1..m}\{\lfloor \frac{f_i - s_i}{l_i} \rfloor\}}$$

□

We observe that the optimality of $minMax$ is strongly dependent on both how "crowded" tasks are scheduled around a time slot and on how shift-able are the tasks. Generally higher number of tasks overlapping in the same time slot result in higher optimality loss. Similarly, the more shift-able tasks are, the more optimal the $minMax$ solution will be.

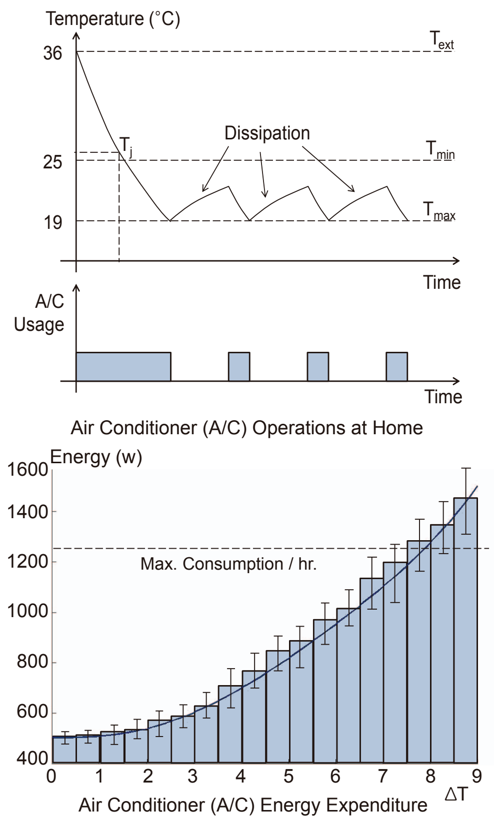

Fig. 2. Air Conditioner Operations and Energy

IV. Home Energy Management with HVC

We now extend the $minmax$ scheduling algorithm to consider Heating, Ventilation and Cooling (HVC) devices. Compared with the discrete appliance scheduling problem, HVC devices such as air conditioner operate with varied energy consumption patterns best modeled as a continuous function. Figure 2 depicts the general operation cycles of an A/C appliance at home. The top figure shows how A/C is turned on to bring the room temperature to a target temperature setting and then shuts off. It also periodically turns on to offset dissipation. Accordingly, there are two components to model: the cooling function $Eng()$ and the dissipation function $Dip()$. We first discuss the cooling function $Eng()$. As our algorithm operates in discrete time slots (e.g., 1 hour), we only need to account for the aggregate energy consumption at each time interval. The bottom of Figure 2 shows the aggregate consumption for cooling the household by a set number of Celsius degrees. This graph is collected based on the air conditioner operation report in [19] and normalized by the Korean air conditioner energy consumption data [20] for mansions (Korean apartment-sized condos). The solid line shows a polynomial interpolation function based on operations measurement of our test home, which is a one-bedroom apartment located at ground floor corner unit. The maximum consumption line shows the change in room temperature if the A/C is to operate non-stop for 1 hour. This is denoted as the A/C's *operations threshold*, HVC_LIM. The dissipation function $Dip()$ can be modeled based on Newtonian law of heat exchange as follows:

$$Dip(T_j) = T_j + (T_{ext,j} - T_j)e^{-\frac{60}{\tau}}$$

□

Here T_j denotes the room temperature at start of time slot t_j, $T_{ext,j}$ denotes the average outdoor temperature of time slot t_j, and τ is the heat dissipation rate. A lower τ value is achievable by having better home insulation solutions. $Dip()$ is an exponential function depending on the temperature difference. Based on measurements in the test home, we find $\tau \cong 48.5$ at per minute rate. This is approximately 27% loss per hour, as Korean apartments generally have low insulation. By incorporating $Dip()$, we can find the temperature change ΔT required to maintain a household at target thermostat setting T_{tgt} in a time slot t_j as:

$$\Delta T = T_j - T_{tgt} + (T_{ext,j} - T_{tgt})e^{-\frac{60}{\tau}}$$

And the amount of energy consumed by A/C in the time interval $\{t_j, t_{j+1}\}$ is then $Eng(\Delta T)$. User comfort is represented by the thermostat setting T_{tgt}. Following the classic want-need psychological definition, we consider two T_{tgt} settings: T_{max} is the desirable room temperature the household would like to maintain, and T_{min} is the threshold temperature the household can tolerate without excess discomfort. In the A/C cooling case, $T_{max} \leq T_{min}$, and Figure 2 shows an example of these two settings. Work such as [18] also use similar user comfort constraints. The objective of our $minMax_{HVC}$ algorithm is to find an optimal operations schedule for the HVC devices such that energy expenditure is minimal with respect to the following properties: $\forall t_j, T_{min} \geq T_j \geq T_{max}$ and $T_j \cong T_{max}$. The $minMax_{HVC}$ is shown in Algorithm 2 for the cooling case in summer days. It can be generalized to handle heating case in winter days with minor modifications. The complexity of $minMax_{HVC}$ remains at $O(nm + mlogm)$ because the HVC scheduling with $O(n)$ time is not the dominating complexity term.

Algorithm 2 $minMax_{HVC}$ Scheduling for Cooling

Input:
$t_k \in T$ where $1 \leq k \leq n$
$Eng(), Prc(), HVC\_LIM$
$T_1, T_{ext,1..n}, T_{min}, T_{max}, \tau$
Output:
T_{n+1}
$c_{1..n}$
The HVC energy consumption schedule $schHVC_{1..n}$

1: $c_{1..n} = 0$ //Replace init. in $minMax$
2: $schHVC_{1..n} = 0$
3: **for** $j = 1$ to n **do**
4:   **if** $Dip(T_j) \leq T_{min}$ **then**
5:     $T_{j+1} = Dip(T_j)$
6:     $schMINC_j = 0$
7:   **else**
8:     $\Delta T = max\{HVC\_LIM, T_j - T_{min} + (T_{ext,j} - T_{min})e^{-\frac{60}{\tau}}\}$
9:     $T_{j+1} = max\{HVC\_LIM, T_{min}\}$
10:     $c_j = Prc(t_j)Eng(\Delta T)$
11:     $schMINC_j = c_j$
12:   **end if**
13: **end for**
14: $c_{1..n} = minMax()$
15: $Ref = max\{c_{1..n}\}$
16: $c_{1..n} = c_{1..n} - schMINC_{1..n}$
17: **for** $j = 1$ to n **do**
18:   **if** $Dip(T_j) > T_{max}$ **then**
19:     $\Delta T = max\{HVC\_LIM, T_j - T_{max} + (T_{ext,j} - T_{max})e^{-\frac{60}{\tau}}\}$
20:     $UsedC = Prc(t_j)Eng(\Delta T)$
21:     **if** $UsedC + c_j > Ref$ **then**
22:       $UsedC = Ref - c_j$
23:       $\Delta T = Eng^{-1}(\frac{UsedC}{Prc(t_j)})$
24:       $T_{j+1} = \frac{T_j - \Delta T + T_{ext,j}(e^{-\frac{60}{\tau}})}{1 + e^{-\frac{60}{\tau}}}$
25:       $c_j = Ref$
26:     **else**
27:       $T_{j+1} = max\{HVC\_LIM, T_{max}\}$
28:       $c_j = c_j + UsedC$
29:     **end if**
30:     $schHVC_j = Eng^{-1}(\frac{UsedC}{Prc(t_j)})$
31:   **else**
32:     $T_{j+1} = Dip(T_j)$
33:     $schHVC_j = 0$
34:   **end if**
35: **end for**

In the $minMax_{HVC}$ algorithm, we first schedule the HVC device at tolerable threshold T_{min} (Line 3-13). The rationale is two folds: one, T_{min} must be maintained throughout the day, it is the minimal energy consumption even at peak hours; two, since HVC scheduling at T_{min} is not flexible, it is equivalent to the fixed appliance scheduling case. We then run $minMax$ algorithm for appliance scheduling and obtain a reference peak point Ref (Line 14-15) considering that T_{min} is maintained at peak hours. We then erase our minimal HVC consumption (Line 16) and the HVC devices are rescheduled at desirable threshold T_{max} with respect to the energy consumption ceiling Ref (Line 17-35). Therefore, $minMax_{HVC}$ ensures that at least T_{min} is maintained, while the room temperature is brought up to as close to T_{max} as possible without compromising the minmax objective. We now show that $minMax_{HVC}$ is an approximation algorithm for the combined energy scheduling problem (i.e., appliance and HVC) defined in Section III-A.

THEOREM 3: *$minMax_{HVC}$ is at most $Prc()_{max}Eng(T_1 - T_{min}) + \gamma(\Phi^* + Prc()_{max}r_{max})$.*

Let Φ^* be the peak aggregate cost of the optimal solution. The HVC scheduling strictly increases the energy consumption at this time slot by the amount that is required to maintain the room temperature at T_{min}. Let ΔT^* denote the optimal amount of temperature change required. in $minMax_{HVC}$, after the scheduling at T_{min} operations and the $minMax$ operations, we arrive at a Ref point, with ΔT being the amount of temperature change required during peak Φ. It must be that $\Delta T \geq \Delta T^*$. In the worst case, $\Delta T^* = 0$. This may happen if the HVC devices were operating at near T_{max} before the time slot of Φ^*, and therefore do not need to be turned on at all during peak time of Φ^*. It follows then we can bound the loss of optimality due to HVC scheduling by at most $Prc()_{max}Eng(T_1 - T_{min})$ which is the upper bound on the energy cost required to bring the household from initial temperature T_1 to tolerable temperature T_{min}. The scheduling at T_{max} operations of $minMax_{HVC}$ operates with the Ref limitation and therefore do not change this upper bound. The overall optimality of $minMax_{HVC}$ is then at most $Prc()_{max}Eng(T_1 - T_{min}) + \gamma(\Phi^* + Prc()_{max}r_{max})$ worse than the optimal.

V. SIMULATION STUDY

In this section, we present the performance of $minMax$ and $minMax_{HVC}$ algorithms through simulations. In order to obtain realistic projections, we have constructed a home task generator which generates the appliance task demands based on the probability of appliance usage by time-of-day (see Figure 5). In our study, a household demand is approximately 60 tasks with shift-ability varying from 0 to 4 hours. We have also constructed a simple market pricing function based on the energy consumption patterns in Korea: by using the Korean home energy price as the mean, we set energy price in peak hours to be exponentially expensive, while energy price in early hours of a day are significantly discounted. For the HVC scheduling, we utilize the air conditioning statistics obtained from our test home as described in Section IV and the reported Korean summer temperatures [21].

Figure 3 illustrates the effects of our $minMax$ and $minMax_{HVC}$ algorithms as applied to an example Korean household on a hot summer day. This household operates a mixture of appliance tasks with varied shift-ability ranging from 0 to 3 hours and operates A/C for cooling. The dark bars illustrate the results of scheduling appliance tasks. The first two graphs show the effect on cost, we see that we can achieve significant cost reduction and cost peak shaving by applying the $minMax$ algorithm. The third and forth graphs show the same result in terms of energy consumption. It is interesting to see that we have the peak shaving effect is not nearly as pronounced and rather we have a peak shifting effect. To understand this behaviour, we plot the DR pricing

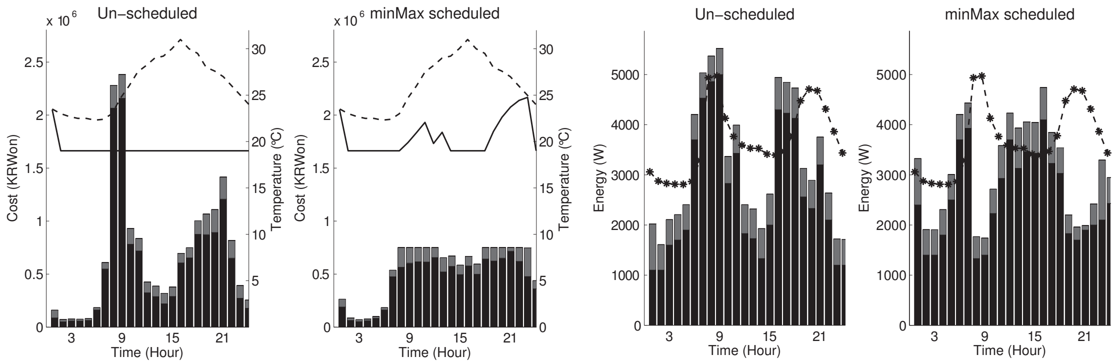

Fig. 3. Example scheduling with $minMax_{hvc}$

function (dotted line with '*'). We see that the DR price function is constructed to dissuade peak time energy usage by applying a high peak energy price (i.e., morning peak and night peak in the unscheduled graph). In response, to reduce energy cost the $minMax$ schedule shifts energy consumptions to off peak times and therefore produces the peak shifting effect. Therefore, we observe that setting the right DR price is of paramount importance in controlling/smoothing the energy demand of the households, as to bring about convergence between cost peak shaving (what the households care about) and energy peak shaving (what the operators want).

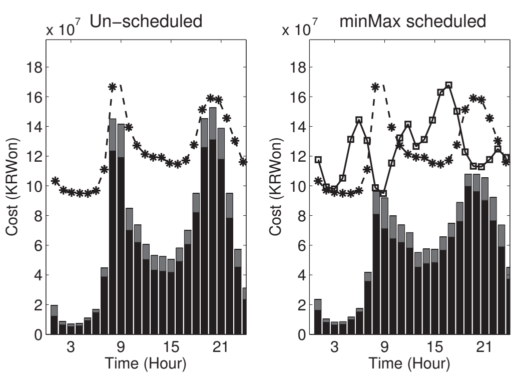

Fig. 4. Energy Consumption of Scheduling

Now we look at the effect of $minMax_{HVC}$ scheduling. The grey bars of the graphs show the consumption due to A/C. For clarity, we focus on the first two graphs of Figure 3. The dotted line shows the external temperature as recorded on June 30th, 2004 in Gwangju city, South Korea [21]. We set the desirable room temperature T_{max} to be 19 Celsius and the tolerable room temperature T_{min} to be 25 Celsius. Furthermore, we assume the house has occupant throughout the day. The solid line shows the room temperature $T_{1..n}$ at the start of each hour. For the unscheduled case, the thermostat is set to 19 and we see that room temperature is maintained around 19 throughout the day. This adds significant cost to the household energy bill especially during peak pricing period when energy is expensive, and during early afternoon when outside temperature is high. In comparison, with the $minMax_{HVC}$ generated A/C schedule, we observe that the desirable room temperature (i.e., 19 Celsius) is still achievable during the off-peak hours of the day. During the peak hours when energy is expensive, the A/C operations are cut back which causes the room temperature to rise, but still below the tolerable threshold T_{min}. For the few hours during prime night price peak (especially 9pm), the A/C is barely turned on at all. The resulting cost reduction is quite significant, while still making the household comfortable for the occupants. The curvature we see in the evening time is the combined effects of non-linear heat dissipation and the rapid increase in energy price in those hours.

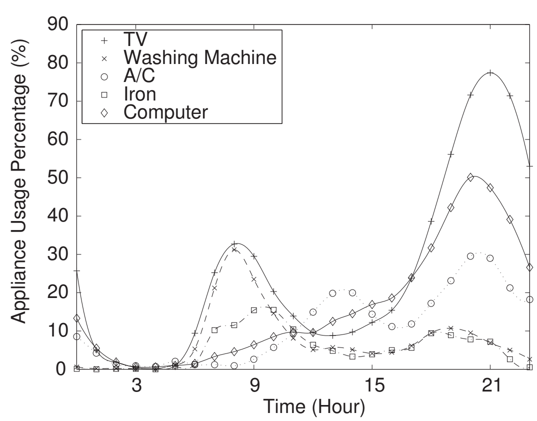

Fig. 5. Energy consumption patterns of example appliances in Korea [20]

We also examine the effectiveness of $minMax$ and $minMax_{HVC}$ in the aggregate across many households. We simulated 100 households where each home ran the $minMax_{HVC}$ algorithm. To investigate the optimality of the $minMax$ and $minMax_{HVC}$ algorithms, 10 of the 100 homes in this experiment are randomly selected and their optimal schedules are computed using exponential branch-and-bound. We found that the peak cost of the schedule produced by the $minMax_{HVC}$ is at most 6% more than that of the optimal, much better than our worst-case theoretical

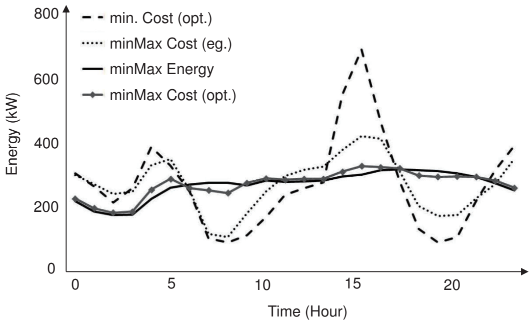

Fig. 6. Comparison of Different Optimization Objectives

bound. Figure 4 illustrates the aggregate effect of adopting our algorithms in 100 households. The dotted line with '*' shows the DR price function. We see that the peak time price corresponds to time of the day where energy demand is high. The effect of applying $minMax_{HVC}$ is apparent at the aggregate level, as we see significant energy shaving in the peak hours. Since tasks have varied shift-ability, we could not achieve perfect minmax smoothing. The grey bars on the graphs depicts consumption due to HVC. When examining the percentage total cost reduction, we find that household can save on average 29.26% of the total energy cost by utilizing our $minMax_{HVC}$ under the example market-based pricing.

We also want to study the effect of applying different optimization objectives on peak shaving, and the importance of price setting. Figure 6 shows a comparison of running $minMax$ under different optimization objectives and with different price settings. Two price settings are used: our example market-based pricing function (minMax Cost (eg.)), and we used a near-optimal price setting (minMax Cost (opt.)) that we have computed based on gradient descent over the aggregate energy demand. Due to space limitation, we could not elaborate on the technique. The aggregate energy expenditure of 100 households are represented on the graph. We exclude HVC in this case because the energy expenditure of HVC under $minMax_{HVC}$ varies significantly depending on the price setting and introduces unwanted noise for our comparison. From the figure, we observe that if we minimize total cost, we cannot obtain peak shaving, and this line is practically the same under both example price function and the near-optimal price setting. We also show the result when minimizing the maximum/peak energy (minMax Energy). The minmax energy achieves the best possible peak shaving, but is not incentive-compatible with household users. Its plot cannot be perfectly smooth due to the user comfort constraints. The goal of a good price setting is therefore to converge the effect of minmax over cost (user incentive-compatible) towards minmax over energy (operator goal but not user incentive-compatible). In examining the plot of minmax over cost objective under two pricing regimes (eg. and opt.), we can observe: 1) even under non-optimal price setting, minmax over cost objective achieves much better peak shaving than minimizing total cost; 2) with good price setting, we can indeed converge the minmax over cost objective close to the minmax over energy objective.

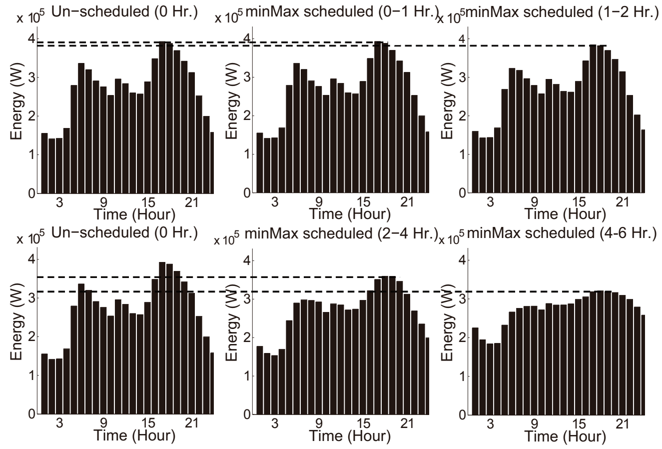

Fig. 7. $minMax$ under varying task flexibility

Since the degree of peak shaving is strongly related to task-shiftability, we study their relationship. A task set for 100 homes is generated as the control input, and the near-optimal price function is computed. We performed multiple runs of $minMax$ on the control input each with varied task shift-ability. HVC is again excluded as to reduce noise. Figure 7 illustrates our findings. As we are interested in the effect of peak shaving, aggregate energy consumptions are shown. We can observe two clear trends: 1) the degree of peak shaving is highly dependent on task-shiftability and it is non-linear; 2) If the tasks are only shiftable under 2 hours, we do not obtain significant peak shaving. Since shifting daily appliance tasks by 2 hours would require significant alteration to households' life style, we postulate that it is difficult to achieve good peak shaving with market-price based incentive mechanism alone. It may be necessary to combine market-based DR pricing with mandatory energy consumption capping.

VI. Conclusion

In this paper, we have investigated the problem of effective home energy management under market-based demand-response programs. We have established minmax over cost as a household user incentive-compatible optimization objective. We discussed the design of $minMax$ and $minMax_{HVC}$ algorithms, and showed that they are near-optimal. In modeling user comfort explicitly as constraints, we found that it is possible to achieve significant energy saving while still maintaining comfortable living. Through simulation studies, we have demonstrated the effectiveness of the algorithms and found that: 1) significant cost saving can be obtained using our algorithms; 2) global energy peak shaving is sensitive to DR price setting; and 3) task shift-ability is a major determining factor of peak energy shaving. Currently we have adopted the algorithms in our GHS server, in the future, we will investigate efficient computation techniques for DR price setting and design user-friendly interfaces to enable efficient energy management at home.

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