New ILP Formulations For Multicast Routing in Sparse-splitting Optical Networks

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***Abstract*—This paper investigates the route optimization of multicast session in a sparse-splitting optical network. The main objective is to minimize the total cost of wavelength channels utilized by the light-tree while satisfying required QoS parameters. In this paper, both the optical-layer constraints (e.g., optical signal power) and application-layer requirements (e.g., end-to-end delay and inter-destination delay variation) are considered as the QoS parameters. Integer Linear Programming (ILP) formulations to solve the optimal multicast routing problem with the given QoS parameters are presented.**

I. INTRODUCTION

Multicast technology is central to the efficient support of many point-to-multipoint applications such as high-definition television (HDTV), e-learning, on-line multi-party gaming, and so on. The main mechanism for optical multicasting capability is a light-tree. To establish a light-tree, the optical switches in the network must split the incoming light signal to form two (or more) outgoing light signals at the cost of reduced optical power per outgoing signal. Optical amplifiers may be used to offset the power loss. As it requires both optical power splitters and amplifiers, the light-splitting multicast capable switch (MC node) is usually more expensive to build compared to a multicast-incapable optical switch (MI node). To contain the whole network cost, a *sparse-splitting* approach may be adopted so as to reduce the number of (expensive) MC nodes in the network [1]. A small percentage of MC nodes mixed with MI nodes in the same network may suffice to support a number of multicast applications.

In this paper, a route optimization of multicast session with QoS requirements is investigated in a sparse-splitting optical network. The main objective is to minimize the total cost of utilized wavelength channels in the sparse-splitting optical network while satisfying required QoS parameters of the application. End-to-end delay and inter-destination delay variation are considered, as both factors are important application-layer requirements in interactive real-time applications such as remote operation, tele-surgery [2]. In addition to these requirements, optical-layer constraints such as optical signal power and wavelength continuity are also deterministic metrics for constructing a light-tree. Therefore, in this paper, both the application-layer requirements and optical-layer constraints are considered as the QoS parameters. To construct a light-tree with guaranteed QoS, Integer Linear Programming (ILP) formulations are presented. The ILP is a popular technique used to solve multicast routing optimization problems. To verify the ILP formulations, simulation results are presented in terms of the cost of utilized wavelength channels in NSFNET network topology (14 nodes and 21 links).

This paper is organized as follows. The related work of the problem of multicast routing in a sparse splitting optical network is described in Section II. The ILP formulations to solve this problem are presented in Section III and simulation results are presented in Section IV. Finally, Section V concludes the paper.

II. RELATED WORK

Since the optimal multicast routing problem in sparse-splitting optical networks is known to be NP-complete, the ILP is widely used to solve multicast routing optimization problems. In [3], the static multicast advance reservation (MCAR) problem is investigated for all-optical wavelength-routed optical networks. The authors proved the MCAR problem is NP-complete and formulated the problem mathematically as an integer linear program. In [4], multicast routing and wavelength assignment (MC-RWA) in optical networks with heterogeneous capabilities is addressed. Specifically, it investigated an extended MC-RWA with delay constraints that incorporates delay constraints in sparse-splitting optical network.

III. ILP FORMULATION FOR MULTICAST ROUTING

In this section, the ILP formulations for QoS-driven multicast routing are presented in a sparse-splitting optical network. The notations used in our ILP formulations are introduced in Table I.

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TABLE I
NOTATION FOR ILP FORMULATION.

$G(V, E, \Lambda, D)$	A directed graph (V, E) representing a mesh WDM network topology with available number of wavelengths (Λ) and delay (D) on each link.
$r(s, M, \Delta, \phi, \rho)$	A multicast session request with required QoS parameters.
θ	The maximum split-capacity of the MC node.
L_{dB}^{tap}	The power loss (dB) that indicates a fraction of the input signal tapped into the local station.
c_e^{λ}	The binary value indicating wavelength channel of link e in which wavelength λ is used for a multicast session.
$In(v)$	The set of input links of node v.
$Out(v)$	The set of output links of node v.
M_v	If node v is a destination node of a multicast session.
$P_e^{\lambda,m}$	The binary value indicating wavelength channel of link e in which wavelength λ is used for the light-path from source s to destination m.
F_v^{λ}	The fanout of node v in which wavelength λ is used for a multicast session.
$f_e^{\lambda,m}$	The integer value indicating the flow in link e used by wavelength λ to the destination m.
$r_v^{\lambda,m}$	The binary value indicating whether node v is used to tap the signal power for the light-path from source to destination m.

A. Problem Description

A sparse-splitting optical network can be modeled as a directed graph $G(V, E, \Lambda, D)$. $V (= V_{MC} \bigcup V_{MI})$ denotes the set of all nodes, and E, the set of edges, corresponds to the set of communication links connecting the nodes. Suppose that a set $\Lambda = \{\lambda_1, \lambda_2, \cdots, \lambda_k\}$ of wavelengths is supported by every link in the network. Let $\Lambda(e) \in \Lambda$ be the set of available wavelengths on the link $e \in E$. We define a delay function $D : E \mapsto R^+$, which assigns a nonnegative weight to each link in the network. The value $D(e)$ associated with link e is the delay time of transmitting an optical signal to a destination node. In a multicast scenario, a source node $s \in V$ sends the packets to a set $M \subseteq V - s$ of destination nodes. $M = \{m_1, m_2, \cdots, m_n\}$ denotes the multicast group members. The multicast packets are routed from s to the members of M via the links of light-tree LT. $P_t(s, m)$ denotes the path from source s to destination $m \in M$ in LT. The total delay from s to m is then denoted as $d(P_t(s, m)) = \sum_{e \in P_t(s,m)} D(e)$. In addition, $SP(s, m)$ denotes the shortest path from source s to destination $m \in M$ using Dijkstra's algorithm.

Our objective is to minimize the total cost of utilized wavelength channels in a sparse-splitting optical network while satisfying required QoS parameters for establishing a multicast session.

Objective function:

$$Minimize \sum_{e \in E} \sum_{\lambda \in \Lambda} c_e^{\lambda}. \qquad (1)$$

The set of constraints is categorized into two types. First, multicast routing and wavelength continuity constraints are required to ensure that the constructed light-tree will not contradict the capability of nodes and flows. Second, QoS parameters must be satisfied to ensure that the constructed light-tree meets the application requirements. Each category is detailed in the following.

B. Multicast Routing and Wavelength Continuity Constraints

1) Node constraints:

$$\sum_{\lambda \in \Lambda} \sum_{e \in In(s)} P_e^{\lambda,m} = 0, \qquad \forall m \in M. \qquad (2)$$

$$\sum_{\lambda \in \Lambda} \sum_{e \in Out(s)} P_e^{\lambda,m} \geq 1, \qquad \forall m \in M. \qquad (3)$$

$$\sum_{e \in In(v)} c_e^{\lambda} \leq 1, \qquad \forall \lambda \in \Lambda, \forall v \in V. \qquad (4)$$

$$\sum_{e \in Out(v)} c_e^{\lambda} \leq \theta, \qquad \forall \lambda \in \Lambda, \forall v \in V_{MC}, v \neq s. \qquad (5)$$

Constraints (2)-(5) ensure that the light-tree will not contradict the capability of a node. Specifically, constraints (2) and (3) ensure that no wavelength enters into the source node and at least one wavelength branch should leave from the source node to the destination node. Constraint (4) guarantees that only one wavelength channel comes into the node in the light-tree. The capacity constraint (5) ensures that the maximum light splitting capacity of the MC node will be less than or equal to θ.

2) Flow constraints:

$$\sum_{e \in In(v)} P_e^{\lambda,m} - \sum_{e \in Out(v)} P_e^{\lambda,m} = 0,$$
$$\forall m \in M, \forall \lambda \in \Lambda, \forall v \in V, v \neq s, v \neq m. \qquad (6)$$

$$c_e^{\lambda} \geq P_e^{\lambda,m}, \qquad \forall m \in M, \forall \lambda \in \Lambda, \forall e \in E. \qquad (7)$$

$$\sum_{e \in In(v)} f_e^{\lambda,m} - \sum_{e \in Out(v)} f_e^{\lambda,m} \geq \sum_{e \in Out(v)} P_e^{\lambda,m},$$
$$P_e^{\lambda,m} \leq f_e^{\lambda,m} \leq P_e^{\lambda,m} \times |V|, \forall m \in M, \forall \lambda \in \Lambda, \forall v \in V, v \neq s. \qquad (8)$$

Constraint (6) is a wavelength continuity constraint, and guarantees that only one wavelength is used by one multicast tree, as there is no wavelength converter in the network. This constraint also ensures one light-path from the source node to the destination node will be established. Inequality (7) is a wavelength usage constraint; it counts the wavelengths used on each link. Constraint (8) is derived from the conservation of the flow property. The input flow should be larger than the summation of all the output flows. Generally if only the end-to-end delay is considered, constraints (2)-(6)

and the minimization of cost function could guarantee that one multicast tree will be created. Unfortunately, when inter-destination delay variation is a concern, an isolated wavelength circle may arise in the network to compensate the delay difference between different light-paths. Constraint (8) ensures that an isolated light-path circle will not occur in the network.

C. Required QoS Parameters

In this paper, three QoS parameters are considered. These parameters that can be used to characterize the quality of the light-tree as required by the application.

1) End-to-end delay tolerance, Δ: This parameter represents an upper bound on the end-to-end delay. To provide distributed multimedia application, this parameter should be guaranteed.

$$\sum_{e \in E} (P_e^{\lambda,m} \times D(e)) \leq \Delta, \qquad \forall m \in M, \forall \lambda \in \Lambda. \tag{9}$$

2) Inter-destination delay variation tolerance, ϕ: This is the maximum difference between the end-to-end delays along the paths from the source node to any two destination nodes. Since this parameter provides synchronization among the various receivers, it should be guaranteed to support interactive real-time multicasting of media-oriented applications (e.g. on-line game and e-learning) [5].

$$-\phi \leq \sum_{e \in E} (P_e^{\lambda,m_i} \times D(e)) - \sum_{\ell \in E} (P_\ell^{\lambda,m_j} \times D(\ell)) \leq \phi, \qquad \forall m_i, m_j \in M, \forall \lambda \in \Lambda, i \neq j. \tag{10}$$

3) Source-destination loss tolerance, ρ: This parameter represents an upper bound on the acceptable end-to-end power loss among any path from the source to a destination node as described in [6]. $P_{in}(s,m)$ and $P_{out}(s,m)$ denote the power of the optical signal injected into the network by the source node s and received by the destination node $m \in M$, respectively. In (11), $L^{atten}(s,m)$, $L^{split}(s,m)$, and $L^{tap}(s,m)$ account for the power loss due to attenuation, splitting, and tapping, respectively.

$$P_{out}(s,m) = L^{atten}(s,m) \times L^{split}(s,m) \times L^{tap}(s,m) \times P_{in}(s,m). \tag{11}$$

In this paper, the losses due to the multiplexing and demultiplexing of signals as well as the propagation of light at the switching elements are ignored. In addition, $L^{split}(s,m)$ and $L^{tap}(s,m)$ can be denoted as $\prod_{v \in P_t(s,m)} \frac{1}{F_\lambda(v)}$ and $\prod_{v \in P_t(s,m)} L^{tap}(v)$, respectively, where $F_\lambda(v)$ is the fanout of node v, in which wavelength λ is used for the light-tree and $L^{tap}(v)$ is defined as

$$L^{tap}(v) = \begin{cases} L^{tap}, & \text{if the input sigal is tapped at node } v \\ 1, & \text{otherwise.} \end{cases}$$

To recover the information carried by the optical signal without error, the residual power of the signal received at the destination node must be larger than ρ. Therefore, (11) has to be converted into (12) .

$$\frac{P_{out}(s,m)}{P_{in}(s,m)} = \prod_{v \in P_t(s,m)} \left(\frac{1}{F_v^\lambda} \times L^{tap}(v) \right) \geq \rho, \qquad \forall \lambda \in \Lambda, \forall m \in M. \tag{12}$$

Since it cannot be directly used as an ILP formulation due to the product term, (12) has to be converted into dB scale as follows:

$$\sum_{v \in P_t(s,m)} \{10log_{10} F_v^\lambda + (r_v^\lambda \times L_{dB}^{tap})\} \leq 10log_{10}\frac{1}{\rho}, \qquad \forall m \in M, \forall \lambda \in \Lambda. \tag{13}$$

However, since F_v^λ is a variable, incorporating this term in the formulation makes it non-linear as well. Here, a way to approximate (13) is introduced. Since a logarithm function is a strictly increasing concave function in (0, + ∞), it can be seen that the convex envelope of $10log_{10}F_v^\lambda$ over [1, θ] is a line running through two points (1, 0), (θ,$10log_{10}\theta$), as shown in (14).

$$10log_{10}F_v^\lambda \approx \frac{10log_{10}\theta}{\theta - 1} \times (F_v^\lambda - 1) \tag{14}$$

Therefore, the signal quality constraint (12) can be approximated to (15), which is a linear function.

$$\sum_{v \in P_t(s,m)} \{(\frac{10log_{10}\theta}{\theta - 1} \times (F_v^\lambda - 1)) + (r_v^\lambda \times L^{tap})\} \leq 10log_{10}\frac{1}{\rho}, \qquad \forall m \in M, \forall \lambda \in \Lambda. \tag{15}$$

IV. EXPERIMENTS

A. Simulation Environment

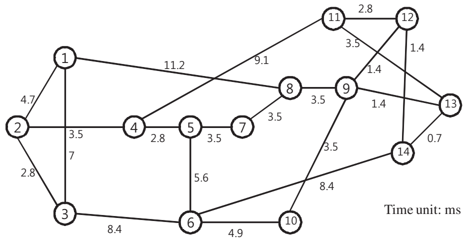

Fig. 1. A 14-node and 21-link NSFNet network.

In this section, to verify the presented ILP formulations, the NSFNet with 14 nodes and 21 links is used as shown in Fig. 1. The placement of MC nodes that support light splitting capability is very important to reduce the utilized wavelength channel for constructing light-tree. To determine the MC nodes, three heuristic algorithms are used.

1) Maximum path count first (MPCF) [7]: The MPCF uses the maximum count of each node for all the source-destination node pairs in the network that the signal passes through.

2) Minimum path delay first (MPDF): The MPDF heuristic algorithm determines the candidate MC nodes based on the delay. First, compute the priority of each node using $Pri_v = \sum_{u \in V, u \neq v} d(SP(v,u))$. The nodes are then sorted in ascending order of Pri_v.

3) Minimum path hop-count first (MPHF): The MPHF heuristic algorithm determines the candidate MC nodes based on the hop count. To this end, compute the hop count $C_{hop}(SP_{v,u})$ between all pairs. It then compute the priority of each node using $Pri_v = \sum_{u \in V, u \neq v} C_{hop}(SP(v,u))$. Finally, the nodes are sorted in ascending order of Pri_v.

In this simulation, the placement of MC nodes are determined based on different three strategies and fifty sets of destination nodes are selected randomly. The QoS parameters are preset as $\rho = 0.125$, $\Delta = 30ms$, and $\phi = 10ms$.

B. Performance Evaluation in NSFNet

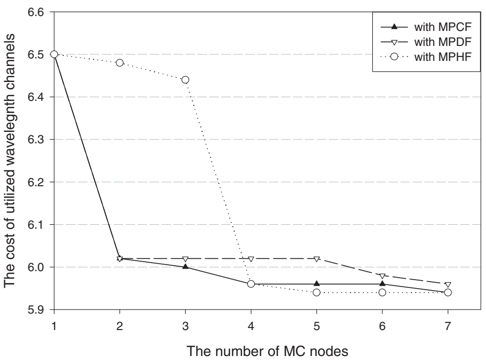

Fig. 2. The average number of utilized wavelength channels with three different MC node placement strategies ($|M|$ = 4).

Fig. 2 shows the average number of utilized wavelength channels for constructing light-trees by using the ILP formulations when $|M| = 4$. As can be seen, the number of utilized wavelength channels is decreased as the number of MC nodes increases for all strategies. Specifically, the cost of ILP with MPHF is larger than or equal to that of ILPs with MPDF and MPHF, respectively, when the number of MC nodes is less than 4. But, ILP with MPHF shows the better performance compared with the others when the number of MC nodes is 5 and 6. From this figure, it can be inferred that MPCF shows the reasonable performance compared with the others regardless of the number of MC nodes.

Fig. 3 shows the average number of utilized wavelength channels for constructing light-trees by using ILP solution under MPCF strategy. As can be seen, the number of utilized wavelength channels is increased as the number of multicast group members increases.

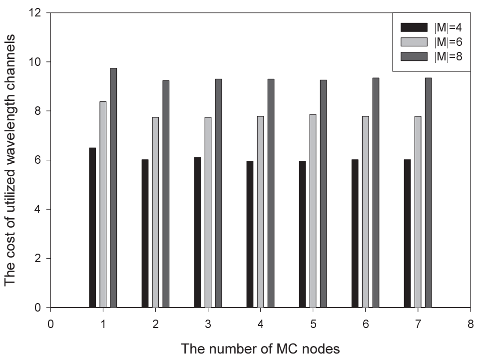

Fig. 3. The average number of utilized wavelength channels with MPCF strategy

V. CONCLUSION

In this paper, the new ILP formulations were presented to optimize a route of multicast session in the sparse splitting optical network with two sets of QoS constraints (e.g., optical layer and application layer constraints). The ILP solution are compared under different MC node placement strategies: MPCF, MPDF, and MPHF in simulation results. From these results, it can be verified that ILP with MPCF shows the reasonable performance compared with the others regardless of the number of MC nodes.

The computation complexity of ILP grows exponentially with the network size. Therefore, an efficient heuristic algorithm should be explored in future research to obtain solution in a large-scale network.

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